Transient Stability Analysis of Multiple Converter Based Microgrid

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Abstract

The analysis of transient stability of conventional power systems is well established, but for inverter based microgrids there is a need to establish how circuit and control features gave rise to particular oscillatory modes and which of these have poor damping. This paper develops the modeling and stability analysis of autonomous operation of inverter based microgrids. Each sub-module is modeled in state-space form and all are combined together on a common reference frame. The model captures the detail of the control loops of the inverter but not the switching action. Some inverter modes are found at relatively high frequency and so a full dynamic model of the network (rather than an algebraic impedance model) is used. High gain angle droop control ensures proper load sharing, especially under weak system conditions, but has a negative impact on the overall stability. It has been shown that real power modes get affected with the real power droop coefficients, while the reactive power modes are sensitive to reactive power droop coefficients. Transient stability results have been obtained from a microgrid of three inverters. The abstract should summarize the content of the paper. Try to keep the abstract below 150 words. Do not have references or displayed equations in the abstract. It is imperative that the margins and style described below be adhered to carefully. This will enable us to maintain uniformity in the final printed copies of the Journal. Papers not made according these guidelines will not be published although its content has been accepted for publication. Paper form is a necessary condition for its publication, as well as its content.

Keywords: Inverter, inverter model, microgrid, power control, Transient Stability.

1. Introduction

Recent innovations in small-scale distributed power generation systems combined with technological advancements in power electronic systems led to concepts of future network technologies such as microgrids. These small autonomous regions of power systems can offer increased reliability and efficiency and can help integrate renewable energy and other forms of distributed generation (DG) [1]. Many forms of distributed generation such as fuel-cells, photo-voltaic and micro-turbines are interfaced to the network through power electronic converters [2–5]. These interface devices make the sources more flexible in their operation and control compared to the conventional electrical machines. However, due to their negligible physical inertia they also make the system potentially susceptible to oscillations resulting from network disturbances.

A microgrid can be operated either in grid connected mode or in stand-alone mode. In grid connected mode, most of the system-level dynamics are dictated by the main grid due to the relatively small size of micro sources. In stand-alone mode, the system dynamics are dictated by micro sources themselves, their power regulation control and, to an unusual degree, by the network itself.

One of the important concerns in the reliable operation of a microgrid is transient stability. In conventional power systems, stability analysis is well established and for the different frequency ranges (or time horizons) of possible concern there are models which include the appropriate features. The features have been established on the basis of decades of experience so that there are standard models of synchronous machines, governors and excitation systems of varying orders that are known to capture the important modes for particular classes of problem. This does not yet exist for microgrids and may be difficult to achieve because of the range of power technologies that might be deployed. However, we can begin by developing full-order models of inverters and the inverter equivalents of governors and exciters. Examination of these models applied to various systems will develop that body of
experience that allows reduced order models to be selected for some problems.

In this paper, a systematic approach to modeling an inverter-based microgrid is presented [6-18]. Each DG inverter will have an outer power loop based on droop control to share the fundamental real and reactive powers with other DGs. Inverter internal controls will include voltage and current controllers which are designed to reject high frequency disturbances and damp the output LC filter to avoid any resonance with the external network. The small-signal state-space model of an individual inverter is constructed by including the controllers, output filter and coupling inductor on a synchronous reference frame whose rotation frequency is set by the power controller of that inverter. An arbitrary choice is made to select one inverter frame as the common reference frame and all other inverters are translated to this common reference frame using the simple transformation techniques familiar in synchronous machine systems. It is considered that state-less impedance models of the network are inadequate for use with full-order inverter models which include high frequency modes. Instead a dynamic (state-space) model of the network is formed on the common reference frame.

Once the small-signal model has been formed, eigenvalues (or modes) are identified that indicate the frequency and damping of the oscillatory terms of the system transient response. The analytical nature of this examination then allows further investigation so that the relation between system stability and system parameters, such as the gains of controllers is established. This represents a systematic approach to finding appropriate models and avoids the danger of neglecting a system feature that later turns out to be important.

2. CONVERTER STRUCTURE AND CONTROL

All the DGs are assumed to be an ideal dc voltage source supplying a voltage of $V_{dc}$ to the VSC. The structure of the VSC is as shown in fig.1. In this, $u \cdot V_{dc}$ represents the converter output voltage, where $u = \pm 1$. The main aim of the converter control is to generate $u$.

The following state vector is chosen

$$ x^T = [i_2 \ i_{cf} \ v_{cf}] $$

where converter output voltage is the same as the voltage across the filter capacitor $V_{cf}$. The control action results in perfect tracking when the error is within limit. The switching function $u$ is then generated as

- If $uc > h$ then $u = +1$
- elseif $uc < -h$ then $u = -1$

where $h$ is a small number.

Each inverter is modeled on its individual reference frame whose rotation frequency is set by its local power sharing controller. The inverter model includes the power sharing control dynamics, output filter dynamics, coupling inductor dynamics and voltage and current controller dynamics. These last two elements introduce high frequency dynamics which are apparent at peak and light load conditions and during large changes in load.

3. DROOP CONTROL AND DG REFERENCE GENERATION

The same control strategy is applied to all the DGs. It is assumed total power demand in the microgrid can be supplied by the DGs and no load shedding is required. The output voltages of the converter are controlled to share this load proportional to the rating of the DGs. As an output inductance is connected to each of the VSCs the real and reactive power injection from the DG source to the microgrid can be controlled by changing voltage magnitude and its angle. It is evident that the reference for all the elements of the states, given in (1), is required for state feedback. Since $V_{d}$ and $V_{q}$ are obtained from the droop equation the reference for the capacitor voltage and current are given by

![Fig1: Single-phase equivalent circuit of VSC (LCL filter).](image)
\[ v_{c\text{ref}} = V \cos(\omega t + \delta) \]
\[ i_{c\text{ref}} = V \omega C_f \sin(\omega t + \delta) \]
\[ (3) \]

The reference for the current \( i_2 \) can be calculated as

\[ i_{2\text{ref}} = \frac{v_c - v_t}{j \omega f} \]
\[ (4) \]

The above calculation will need a phase shifter for the instantaneous current reference. This may not be desirable. Hence the measured values of the average real and reactive power output of the VSC can be used to find magnitude and phase angle of the reference rms current.

\[ |I_{2\text{ref}}| = \frac{\sqrt{P^2 + Q^2}}{v_c} \quad \text{and} \quad \angle I_{2\text{ref}} = \delta - \tan^{-1} \left( \frac{Q}{P} \right) \]
\[ (5) \]

Hence the current reference can be given as

\[ i_{2\text{ref}} = |I_{2\text{ref}}| \cos(\omega t + \angle I_{2\text{ref}}) \]
\[ (6) \]

4. STATE SPACE MODEL OF AUTONOMOUS MICROGRID

The stability of a microgrid needs to be studied through the analysis of state-space models, and so suitable models of converters are needed to complement the well-established models of rotating machines. As machine models include features such as automatic voltage regulators and wash-out functions, the converter model should also include control loops. In an autonomous microgrid that contains converter-based DGs only, the fast switching action can influence the network dynamics. Hence the network is modeled by differential equations rather than algebraic equations for stability investigation. So far we have presented the single-phase control of the converter. However, for the analysis of the total microgrid system, a common reference frame is chosen and the system voltages and currents are converted in a DQ reference frame.

Fig. 2 shows the block diagram of the complete microgrid system containing \( Z \) number of DGs. It is assumed that the model of each of DGs is same. This includes the VSC with its state feedback controller, droop controller and the interface block that connects the converter to the network. The system equations are nonlinear and thus they are linearized to perform eigenvalue analysis. The linear quantities are denoted by the prefix \( \Delta \). The measured real and reactive power output (\( \Delta P, \Delta Q \)) of converter is fed to the droop controller, while voltage reference (\( \Delta v_{c\text{ref}}, \Delta \delta_{\text{ref}} \)), set by droop controller, is fed back to the converter. The DGs are connected to the network through the interface block which converts the input/output signal from DG reference frame to the common reference frame and vice versa. Each DG block has, current output to the network, which is converter output current (\( \Delta i_{2D}, \Delta i_{2q} \)) and network voltage as input (\( \Delta v_{TD}, \Delta v_{TQ} \)). Similarly, the input to the load model is the network voltage at the connected nodes (\( \Delta v_{TD}, \Delta v_{TQ} \)) and its output is the load current (\( \Delta i_{\text{LoadDQ}} \)). The state-space equations of the DG-VSC, load and network are derived separately in a modular fashion. These are then combined together depending on the network structure to get the overall microgrid system model.

5. CONVERTER MODEL

From equivalent circuit shown in Fig. 1 the following equations are obtained for each of the phases of the three-phase system
Equations (7-9) are translated into a d-q reference frame of converter output voltages, rotating at system frequency $\omega$, where a-b-c to d-q transformation matrix $P$ is given by:

$$
P = \frac{2}{3} \begin{bmatrix}
\cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\
\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right)
\end{bmatrix}
$$

Defining a state vector as $x = [i_{d1} \ i_{d2} \ i_{q1} \ v_{cdfd} \ v_{cfrd}]^T$,

the state equation in the d-q frame is given by

$$
\dot{x} = A_1 x + B_1 u_{cdq} + B_2 v_{tdq}
$$

In (10), the matrices are

$$
A_1 = \begin{bmatrix}
-R_f & \omega & 0 & 0 & -\frac{1}{L_f} & 0 \\
\frac{R_f}{L_f} & 0 & 0 & 0 & 0 & 0 \\
-\omega & \frac{R_f}{L_f} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega & 0 \\
0 & 1 & -\frac{1}{C_f} & 0 & 0 & \omega \\
0 & 0 & -\frac{1}{C_f} & 0 & 0 & \omega
\end{bmatrix}
$$

and

$$
B_1 = \begin{bmatrix}
\frac{1}{L_f} \\
0 \\
\frac{1}{L_f} \\
0 \\
0 \\
0
\end{bmatrix}
$$

$$
B_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

It is assumed here that the tracking is perfect and hence, in the limit, $u$ can be represented by $u_c \cdot u_{cdq}$ can be expressed as

$$
u_{cdq} = -k_1(i_{d2q} - i_{2refd}) - k_2(i_{d1q} - i_{2dq} - i_{cfrd}) - k_3(v_{cdfd} - v_{cfrd})$$

$$
= -k_2i_{d1q} + (k_2 - k_3)i_{2dq} - k_2v_{cdfd} - k_3v_{cfrd}
$$

(11)

The above equation can be written in matrix form as

$$
[u_d^T \ u_q^T] = G_i x_i + H_i y_{refdq}
$$

(12)

Where,

$$
G_i = \begin{bmatrix}
-k_2 & 0 & (k_2 - k_3) & 0 & 0 & -k_3 \\
0 & -k_2 & 0 & (k_2 - k_1) & 0 & 0 \\
0 & 0 & -k_2 & 0 & 0 & -k_2
\end{bmatrix}
$$

$$
H_i = \begin{bmatrix}
-k_1 & 0 & -k_2 & 0 & -k_3 & 0 \\
0 & -k_1 & 0 & -k_2 & 0 & -k_2
\end{bmatrix}
$$

$$
y_{refdq} = [i_{2refd} \ i_{2refq} \ i_{cfrd} \ i_{cfrq} \ v_{cdfd} \ v_{cfrd}]^T
$$

Substituting (11) into (12) we get
Since $V_w$ is assumed to be constant, the linearization (13) will not alter $B_1$. This linearization results in
\[
\Delta x_i = A_{\text{CONV}} \Delta x_i + B_{\text{CONV}} \Delta y_{\text{ref}d} + B_{\text{2}} \Delta v_{tdq}
\]
(14)

Where,
\[A_{\text{CONV}} = A_1 + B_1 G_i\]
and
\[B_{\text{CONV}} = B_1 H_i\]

The current references can be expressed in terms of the voltage reference as
\[
\begin{bmatrix}
\Delta i_{\text{ref}f} \\
\Delta i_{\text{ref}q}
\end{bmatrix} =
\begin{bmatrix}
0 & -\omega_f \\
\omega_f & 0
\end{bmatrix}
\begin{bmatrix}
\Delta v_{\text{ref}f} \\
\Delta v_{\text{ref}q}
\end{bmatrix}
\]
(15)

\[
\begin{bmatrix}
\Delta i_{\text{ref}f} \\
\Delta i_{\text{ref}q}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
-\frac{1}{\omega_f} & 1
\end{bmatrix}
\begin{bmatrix}
\Delta v_{\text{ref}f} \\
\Delta v_{\text{ref}q}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta v_{\text{ref}d} \\
\Delta v_{\text{ref}q}
\end{bmatrix}
\]
(16)

Combining (15) and (16), the reference vector is given as
\[
y_{\text{ref}d} = M_1 \Delta v_{\text{ref}f} + M_2 \Delta v_{\text{ref}d}
\]
(17)

Where
\[
M_1 =
\begin{bmatrix}
0 & -\frac{1}{\omega_f} \\
\frac{1}{\omega_f} & 0 \\
\omega_f & 0 \\
0 & 1
\end{bmatrix}
\quad M_2 =
\begin{bmatrix}
0 & -\frac{1}{\omega_f} \\
\frac{1}{\omega_f} & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

Combining (14) and (17) we get the converter model as
\[
\dot{x}_i = A_{\text{CONV}} \dot{x}_i + B_2 \Delta v_{\text{ref}f} + B_{\text{bus}} \Delta v_{\text{ref}d}
\]
(18)

$B_2 = B_{\text{CONV}} M_2$ and $B_{\text{bus}} = B_{\text{CONV}} (M_2 + B_2)$

6. DROOP CONTROLLER

Droop controller sets the references for converter output voltage magnitude and its angle. The output voltage of the converter is equal to voltage across the capacitor $C_f$. The measured instantaneous real and reactive power are passed through two low-pass filters to obtain the average values of $P$ and $Q$ respectively. The basic idea behind the droop control is to mimic the governor of a synchronous generator. In a conventional power system, synchronous generators will share any increase in the load by decreasing the frequency according to their governor droop characteristic. This principle is implemented in inverters by decreasing the reference frequency when there is an increase in the load. Similarly, reactive power is shared by introducing a droop characteristic in the voltage magnitude. These can be expressed as
\[
P = \frac{\omega_c}{s+\omega_c} \hat{P} = \frac{\omega_c}{s+\omega_c} (v_{cf}\dot{i}_{2d} + v_{cfq}\dot{i}_{2q})
\]
(19)
\[ Q = \frac{\omega_e}{\omega_{e0}} \dot{Q} = \frac{\omega_e}{\omega_{e0}} (v_{cfd} i_{2q} - v_{cfq} i_{2d}) \]

(20)

The real power sharing between inverters is obtained by introducing an artificial droop in the inverter frequency as in (19),(20).

### TABLE-1: NOMINAL SYSTEM PARAMETERS

#### 7. COMPLETE MODEL OF AN INDIVIDUAL INVERTER

To connect an inverter to the whole system the output variables need to be converted to the common reference frame. The obtained overall equations for individual inverter

\[ \frac{di_{d}}{dt} = \frac{-v_{cfd}}{L_{f}} i_{d} + \omega_{e} i_{q} - \frac{1}{L_{f}} v_{cfd} i_{d} + \frac{v_{dc}}{L_{f}} u_{d} \]

(21)

\[ \frac{di_{q}}{dt} = -\omega * i_{sd} + \frac{-v_{cfd}}{L_{f}} i_{q} - \frac{1}{L_{f}} v_{cfd} + \frac{v_{dc}}{L_{f}} u_{q} \]

(22)

\[ \frac{di_{2q}}{dt} = \omega i_{2q} + \frac{1}{L_{f}} v_{cfd} - \frac{1}{L_{f}} v_{td} \]

(23)

\[ \frac{di_{2d}}{dt} = -\omega i_{2d} + \frac{1}{L_{f}} v_{cfd} - \frac{1}{L_{f}} v_{td} \]

(24)

\[ \frac{dv_{cfd}}{dt} = \frac{1}{c_{f}} i_{ld} - \frac{1}{c_{f}} i_{td} + \omega * v_{cfd} \]

(25)

\[ \frac{dv_{cfq}}{dt} = \frac{1}{c_{f}} i_{lq} - \frac{1}{c_{f}} i_{2q} - \omega * v_{cfd} \]

(26)

\[ p = v_{cfd} * i_{2d} + v_{cfq} * i_{2q} \]

(27)

\[ q = v_{cfd} * i_{2q} - v_{cfq} * i_{2d} \]

(28)

#### 8. TRANSIENT STABILITY RESULTS

![Fig 4. Output Currents of three inverters](image)

Fig 4. Output Currents of three inverters

Figures 4 shows output current response of three inverters. Figures 5 shows output voltages of three inverters. Figure 6 shows the real power load sharing among the inverter and DGs fundamental output power response. Figure 7 shows reactive power load sharing. Figs. 4–7 show the response of state variables and of all the three inverters.

![Fig 5. Output Voltages of three inverters](image)

Fig 5. Output Voltages of three inverters
9. Conclusions

In this paper, transient stability analysis of a microgrid presented. The converter models illustrated in this paper allow microgrids to be designed to achieve the stability margin required of reliable power systems. This paper clearly shows the real power sharing and reactive power sharing between the inverters. The results obtained gives clear transient stability analysis.

References