The Golden Code in Asynchronous Distributed Networks using Maximum-mean strategy for the Relay Selection

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Abstract

A 2x2 linear dispersion algebraic space-time code called the Golden Code, have been introduced which acts an effective scheme to achieve asynchronous distributed network. Hence we are making use of relays in MIMO cooperative transmission network. In this paper, we realised multi-relay selection with maximum-mean strategy in order to improve the bit error performance. For decoding the Golden Code we adopt sphere detection approach within the asynchronous relay network. Simulation results based on end-to-end bit error rate confirm the efficacy of the approach.

Keywords: Golden codes, Sphere decoding, maximum-mean strategy

1. Introduction

Recently, a new 2x2 linear dispersion algebraic space-time code called the Golden Code, first proposed in [1], have been attracted by many researchers. For the improvement of the bit error performance and bit rates, this code which has the property of full-rate and full diversity along with full-rank, cubic shaping, non-vanishing minimum determinant and optimal diversity multiplexing gain tradeoff [1] and [2]. Therefore to get an improved link performance these properties should be exploited in cooperative transmission.

Previously research used to deal with synchronous transmission. Synchronization of transmission is a critical issue in exploiting cooperative diversity in wireless ad hoc, networks so that the signals arrive effectively at the destination simultaneously. The nodes are geographically dispersed in a wireless network, and hence the inter-node channels will have varying strengths and delays [3]. In the presence of such time delays, a simple Alamouti space-time transmission scheme for asynchronous cooperative systems is proposed in [4] based on an orthogonal frequency-division multiplexing (OFDM) scheme. In [5] and [6], a new 2 x 2 delay tolerant code based on the Golden Code and a bounded delay-tolerant STBCs for asynchronous cooperative networks have been proposed. However, in this paper, no change needed in the original code for transmission over a two-hop MIMO relay channel with asynchronous over OFDM for asynchronous cooperative networks.

Using all the relays may not obtain the optimal end-to-end performance of a relay network; moreover, using all the relays may create practical problems such as synchronization between the relays. Improved performance can be potentially achieved by selecting the cooperating relays to employ. In particular, selection can aim to find the best relay for solving the problem of multiple relay transmission by requesting only a single relay forwards the information from the source [7]. Such relay selection must be repeated as the channel conditions can change. Hence, we proposed a new relay selection called maximum-mean strategy.

In this paper, we build on and address asynchronous cooperative transmission with the Golden Code over frequency flat, quasi-static, Rayleigh fading channels. In Section II, we first develop the system model and analyze the basic fixed relay case for asynchronous cooperative systems, and then describe the sphere detection. We also consider maximum-mean strategy for relay selection in the asynchronous cooperative transmission. Section III includes the simulation results, and conclusions are drawn in Section IV.

II. Asynchronous distributed transmission based on the Golden code

A. Asynchronous distributed transmission with a fixed relay scheme

Due to the basic transmission environment of the Golden Code, we adopt a 2 x 2 MIMO channel scheme. Our cooperative scheme in general contains two phases of
transmission, a broadcast phase and a cooperation phase as shown in Fig. 1.

![Relay selection with the Golden Code in an asynchronous wireless network](image)

It is composed of one source, j-relays and one destination. There are respectively two antennas at the source and the destination, but only one antenna at each relay. Each node is half-duplex.

The information symbols are coded with the Golden Code in the source rather than in the relays. As mentioned, we assume these j-relay nodes are geographically dispersed, and a random time delay T in the cooperation phase. For instance, there is a T time difference between the signals from the RI relay and the signals from R2 to the destination node. Moreover, the length of the cyclic prefix is larger than or equal to T and the subcarriers are orthogonal to each other.

The other assumption is that the channels are quasi-static Rayleigh flat fading with coefficients hj and gj, which are independent identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and unit-variance, subscript j denotes the j-th relay.

The Golden Code is a full rate and full diversity 2 x 2 linear dispersion algebraic space-time code for two transmit antennas and two or more receive antennas MIMO system [1]. Based on the golden number ω=\(1 + \sqrt{5}\) can be written in block form,

\[
G = \frac{1}{\sqrt{2}} \begin{bmatrix}
\alpha(s_1 + s_2 \theta) & \alpha(s_3 + s_4 \theta)
\end{bmatrix} \begin{bmatrix}
\alpha(s_1 + s_2 \overline{\theta}) & \alpha(s_3 + s_4 \overline{\theta})
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

Where \(s_1, s_2, s_3, s_4\) and \(s_2, s_3, s_4\) are the vectors of the transmitted information symbol constellations. \(\alpha = 1 + i(1 - \theta)\) and \(\overline{\theta} = 1 + i(1 - \overline{\theta})\) where \(|\gamma|\) is set as unity for satisfying non-vanishing determinants and ensures the same average power is transmitted [1].

**Implementation at the source:**

At the source, the information bits are modulated as QPSK and restored as vectors S1, S2, S3 and S4, and then encoded with the Golden Code matrix. The encoded symbol vectors are X1, X2, X3 and X4. Each block of N modulated symbols is fed to an OFDM modulator of N subcarrier. We get four OFDM blocks are modulated by an N-Point IDFT.

\[
\mathbf{tx}_v = [tx_{0,v}, tx_{1,v}, tx_{2,v}, \ldots, tx_{N-1,v}]^T, \ v = 1, 2, \ldots, 4
\]

Then each block is preceded by a cyclic prefix (CP) with the length lcp. We set the CP length lcp to be not less than or equal to the time delay T, and lcp \(\leq (L - 1) + (L - 1) + T\max\) where L is the number of multipath. The first two vectors are broadcasted respectively by the source to the destination through two relay nodes and then the source broadcasts the other two vectors in the second symbol period. In the fixed relay case, we just use two relays R1 and R2. The signal received by the j-th relay can be expressed in the time domain

\[
\mathbf{tr}_j^k = \sqrt{P_r} (\mathbf{tr}_1^k \otimes \mathbf{g}_{2j-1} + (\mathbf{tr}_2^k \otimes \mathbf{g}_{2j})_r + \mathbf{tn}_{2j}^k), \quad (2)
\]

**Implementation at the destination:**

Due to the timing errors, there is a T sample delay between the signals from the R1 relay and the signals from R2 to the destination node. The signal received by the i-th antenna of the destination, during the relaying phase in the time domain.

\[
\mathbf{ty}_i^k = \sqrt{P_r} (\mathbf{tr}_1^k \otimes \mathbf{g}_{2i-1} + (\mathbf{tr}_2^k \otimes \mathbf{g}_{2i})_r + \mathbf{tn}_{2i}^k), \quad (3)
\]

The power of every transmitting antenna in the source and relays is given by

\[
P_r = \frac{P}{2A}, \quad (4)
\]

Where P the total is transmit power of the system and A denotes the number of relay used.

After the removal of the cyclic prefix and the DFT operation at the receiver, the time domain operation in can be equivalently written in the frequency domain.

\[
\mathbf{y}_i^k = \sqrt{P_r} (\mathbf{r}_1^k \odot \mathbf{g}_{2i-1} + \mathbf{r}_2^k \odot \mathbf{g}_{2i} \odot \mathbf{f}) + \mathbf{n}_i^k, \quad (5)
\]

Where \(\odot\) is the Hadamard product, and \(\mathbf{g}_{2i}^{\otimes-1}\) and \(\mathbf{g}_{2i}\) represent the DFTs of \(\mathbf{g}_{2i-1}\) and \(\mathbf{g}_{2i}\) and the time delay in the time domain corresponds to a phase change in the frequency domain \([4]\) which in vector form becomes

\[
\mathbf{f} = [1, e^{-j2\pi T/N}, \ldots, e^{-j2\pi (N-1)/N}]^T. \quad (6)
\]

Thus, the total received signal vector in the frequency domain for the asynchronous network \(\mathbf{y}_\alpha\) can be written as

\[
\mathbf{y}_\alpha = \mathbf{H}_\alpha \mathbf{x}_\alpha + \mathbf{n}_\alpha, \quad (7)
\]

Where \(\mathbf{y}_\alpha\) is the vector \([y_1^T, y_2^T, y_3^T, y_4^T]^T\) and \(\mathbf{x}_\alpha\) is the source vector \([s_1^T, s_2^T, s_3^T, s_4^T]^T\). \(\mathbf{H}_\alpha\) is a 4 x 4 block diagonal matrix of effective transmission channels which is given by
Where, 
\[ M = \begin{bmatrix}
    \text{diag}(h_j \circ g_j) + h_j \circ g_j' & \text{diag}(h_j \circ g_j) + h_j \circ g_j' \\
    \text{diag}(h_j \circ g_j) + h_j \circ g_j' & \text{diag}(h_j \circ g_j) + h_j \circ g_j'
\end{bmatrix}, \] 
\[ \Gamma \] represents the DFT of \( h_j \) and \( g_j' \). The total noise \( \mathbf{n}_a \) is the vector \([\mathbf{n}_1^T, \mathbf{n}_2^T, \mathbf{n}_3^T, \mathbf{n}_4^T]^T\). The elements of \( \mathbf{n}_a \) can be written as 
\[ \mathbf{n}_a^k = \sqrt{T_k} (a_{k-1} + i b_{k-1} + a_k + i b_k). \] 

Next, we consider methods to decode \( \mathbf{y}_a \) in (7).

3) Sphere Decoding: The sphere decoding algorithm is easily used for decoding the MIMO system. When received signals can be represented in a lattice structure, SD detection will search the closest lattice points to the received signal within a radius \( c \). Selection of \( c \) corresponds crucially to the speed of this algorithm. Once the radius \( c \) is decreased, the number of signal points which are searched can be reduced in order to increase the calculation speed, although the performance of the decoding will generally be decreased.

a) Initial Radius: The sphere detection of the Golden Code searches over only a radius \( c \) centered around the received signal vector based on the full maximum-minimum decoding, i.e., \( \| \mathbf{y} - \mathbf{Hs} \|^2 \leq c^2 \) [10]. From (1), we can get the matrix \( A \) [11]
\[ A = \frac{1}{\sqrt{5}} \begin{bmatrix}
    \alpha & \alpha \theta \\
    \alpha \overline{\theta} & \alpha \overline{\theta}
\end{bmatrix} \] (11)
Hence, the initial radius can be defined as [9]
\[ c^2 = \frac{1}{\beta} \| \mathbf{y} - \mathbf{A}_sf \|^2. \] (12)
\( S zf \) denotes the initial estimate signals, which comes from the zero-forcing (ZF) detection solution
\[ \mathbf{A}^+ = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \] (14)

b) Derivation: The channel is assumed to be perfectly known at the receiver. And then taking the QR factorization of the matrix \( A \) (11)
\[ A = Q \begin{bmatrix}
    \mathbf{R} & \mathbf{0} \\
    \mathbf{0} & \mathbf{0}
\end{bmatrix}, \] (15)
Where \( \mathbf{R} \) is an upper triangular matrix with positive diagonal elements, \( \mathbf{0} \) is a null matrix, and \( \mathbf{Q} \) is an unitary matrix and equals to \([\mathbf{Q1Q2}] \). Thus, we can obtain
\[ c^2 \geq \| \mathbf{y} - \mathbf{Q}_1 \mathbf{Q}_2 \|_2^2 = \| \mathbf{Q}_1 \mathbf{y} - \mathbf{R} \|_2^2 + \| \mathbf{Q}_2 \mathbf{y} \|_2^2, \] (16)
Where \( (\cdot)^* \) denotes Hermitian matrix transposition. Thereby, getting
\[ c^2 \geq \| \mathbf{Q}_1 \mathbf{y} - \mathbf{R} \|_2^2. \] (16)
Setting \( y' = \mathbf{Q}_1 \mathbf{y} \) and \( r^2 = \| \mathbf{Q}_2 \mathbf{y} \|_2^2 \), we can rewrite (16) as
\[ c^2 \geq \sum_{m=1}^{m} (y_a - \sum_{b=1}^{m} r_{a,b} s_b)^2, \] (17)
Where \( r_{a,b} \) denotes the \( (a, b) \) index of \( \mathbf{R} \). Expanding (17) it generates
\[ c^2 \geq \| \mathbf{y} - \mathbf{r} \|_2^2, \] (18)
From (18), the first term depends on \( \mathbf{y} \), the second term depends on \( \{s_m, s_{m-1}\} \) and so on
\[ \left[ \frac{r' + y_m}{r_{m,m}} \right] \leq s_m \leq \left[ \frac{r' + y_m}{r_{m,m}} \right], \] (19)
Where \( [\cdot] \) and \( \lfloor \cdot \rfloor \) denote respectively rounded up and rounded down.
Thereby, a necessary condition is \( r'^2 \geq \| \mathbf{y} - \mathbf{R} \|_2^2 \). In order to satisfy every \( s_m \) (19), we can have \( s_{m-1} \) in (18). According to the iteration, we can also easily get \( s_{m-1} \).

We look at the entire collection \( \{s_m, s_{m-1}, \ldots, s_2, s_1\} \), to compare the distance \( C_1 \) between point \( S_1 \) and the sphere initial radius \( c \). If \( C_1 < C \), the initial radius changes to \( C_1 \), and vice versa. Therefore, the processing must return to the last step to continue to calculate until the entire set \( \{s_m, s_{m-1}, \ldots, s_2, s_1\} \) is found. When the entire collection cannot be found, the initial radius needs to be increased.
We next consider relay selection.

B. Asynchronous distributed transmission with a relay selection scheme
Our aim is to select the best two relays from \( j \) relays for asynchronous Golden Code transmission in a cooperative network as in Fig. 1.
Then, the selection scheme which finds the mean of the maximum values among the channels is considered.

Maximum-mean selection: In this the selection approach is based on calculating the mean of the strengths of the channels connected to each relay. The mean of these four channels for the \( j \)-th relay is obtained as
\[ L_j^{mean} = \left( \frac{1}{4} \right)^{\frac{1}{2}} \sqrt{\sum_{k=1}^{4} |h_{2k-1}|^2 + |h_{2k}|^2 + |g_k|^2 + |h_{2k}|^2}, \] (21)
This mean value is then calculated for all relays and stored in \( L_{max} \). In contrast to maximum-minimum selection, this method can best balance the levels among the channels, while the maximum-minimum selection just depends on the minimum value of one channel.
III. Simulation results

This section shows the simulated performances of the relay selection with distributed transmission with sphere detection based on the Golden Code in asynchronous wireless networks. The performance is shown by bit error rate (BER) using quadrature phase-shift keying (QPSK) symbols. The total transmission power of the system is fixed as P. The transmitted symbol block size is 64. The assumptions are that the channel is quasi-static flat fading and perfectly known at the receiver. The speed of calculation of sphere decoding is faster. In simulation on a dual-core PC under Windows 7 and MATLAB 7.1, would take 3 hours with sphere decoding.

Fig. 2 shows the simulation of the BER performance of uncoded and coded maximum-mean relay selections. We set the constraint length equal to 3 at the coder and the code generator for the convolutional code is represented as [7 1] in octal. From Fig.4, we can observe that there is 2dB superiority generally based on selecting two best.

IV Conclusions

We realized distributed transmission using the Golden Code in asynchronous wireless relay networks and proposed a simple approach to overcome asynchronism within a two hop MIMO relay channel. As in synchronous wireless relay networks, the maximum of the channel parameter means selection was shown to still achieve the best performance for the relay selection through bit error rate simulations with computationally efficient sphere decoding. The improvement is because this approach performs an overall channel strength tradeoff at every relay node to select the best two relays. Further analysis of this scheme and extension to frequency selective channels is the subject of future work.

Acknowledgement

The author would like to thank the anonymous reviewers for their comments that helped to improve the presentation of this paper.

References