Star Attached Divisor Cordial Graphs

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Abstract

A divisor cordial labeling of graph G with vertex set V is bijection from V to {1,2,……..V(G)} such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

In this paper, it is proved that $C_3 \ominus K_{1,n}$, $C_3 * S_n$, $< K_{1,n}:n >$, $K_{1,3}*K_{1,n}$, $D_2(S_n)$, $K_{1,1,n}$ are divisor cordial graphs.

Keywords: Divisor cordial labeling, Divisor cordial graph

2010 Mathematics subject classification Number: 05C78

1.Introduction:

A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each e = {uv} of vertices in E is called an edge or a line of G.

2.Preliminaries:

Definition:2.1

Let G be a graph and we define the concept of divisor cordial labeling as follows:

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,…..V(G)} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise , then

the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition:2.2

$\mathbf{C_3 \ominus K_{1,n}}$ is a graph obtained by joining each vertex of a star having n edges, to one of the vertex of a cycle of length 3.

Definition:2.3

$\mathbf{C_3 * S_n}$ is a graph obtained by attaching central vertex of a star $\mathbf{S_n}$ at one of the vertex of $\mathbf{C_3}$.

Definition:2.4

$< \mathbf{K}_{1,n} : \mathbf{n} >$ is a graph obtained as one point union of n paths of path length 2.

$< \mathbf{K}_{1,n} : \mathbf{n} >$ is also called subdivided star graph.

Definition:2.5

$\mathbf{K_{1,3} * K_{1,n}}$ is a graph obtained from a star $\mathbf{K_{1,3}}$ by attaching root of a star $\mathbf{K_{1,n}}$ at each pendentvertex of$\mathbf{K_{1,3}}$.

Definition:2.6

Let G be connected graph. A graph, constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex v in G_2 , that is for every vertex u in G_1 there exists v in G_2 such that N(u)= N(v). The resulting graph is known as **Shadow Graph** and it is denoted by $\mathbf{D_2(G)}$.

Definition:2.7

$\mathbf{K_{1,1,n}}$ is a graph obtained by attaching root of a star $\mathbf{K_{1,n}}$ at one end of P_2 is joined with each pendent vertex of $\mathbf{K_{1,n}}$.

3.Main Results.

THEOREM:3.1

$C_3 \ominus K_{1,n}$is a divisor cordial graph.

Proof:

Let $V(C_3 \ominus K_{1,n}) = [u,v,w,\{u_i: 1 \le i \le n\},\{v_i:1 \le i \le n\},\{w_i:1 \le i \le n\}]$

Let $E(C_3 \ominus K_{1,n}) = [(uv)U(vw)U(wu)U\{(uu_i):1 \le i \le n\}U\{(vv_i):1 \le i \le n\}U\{(ww_i):1 \le i \le n\}]$

Define f: $V(C_3 \ominus K_{1,n}) \rightarrow \{1,2,3…………3n+3\}$

The vertex labeling are

f(u) = 1

f(v) = 2

f(w) = 3

$f(u_i) = 3i+3$, $1 \le i \le n$

$f(v_i) = 3i+2, \quad 1 \le i \le n$

$f(w_i) = 3i+1, \quad 1 \le i \le n$

The induced edge labeling are

$f^*(uv) = 1$

$f^*(vw) = 0$

$f^*(wu) = 1$

$f^*(uu_i) = 1, \quad 1 \le i \le n$

$$f^*(vv_i) = \begin{cases} 1 & \text{if } i \equiv 0 \bmod 2 \\ 0 & \text{otherwise} \end{cases}, \quad 1 \le i \le n.$$

$f^*(ww_i)=0, \quad 1 \le i \le n$

Here, $e_f(1)=e_f(0)$ if 'n' is odd.

$e_f(1)=e_f(0)+1$ if 'n' is even.

Clearly, it satisfies the conditions $|e_f(1)-e_f(0)| \le 1$.

Hence, the induced edge labeling shows that $C_3 \ominus K_{1,n}$ is a divisor cordial graph.

For Example, $C_3 \ominus K_{1,5}$ is a divisor cordial graph as shown in the figure3.2

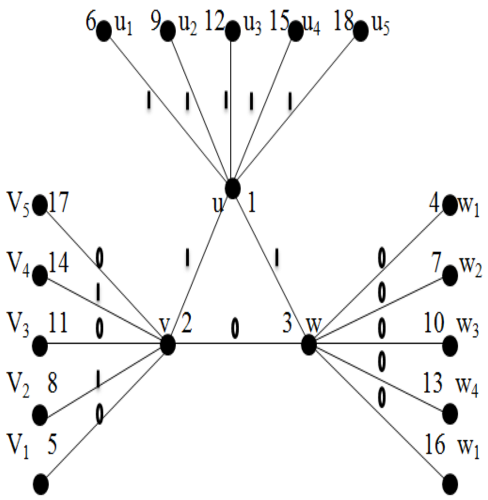

Figure 3.2

$C_3 \ominus K_{1,4}$ is a divisor cordial graph as shown in the figure3.3

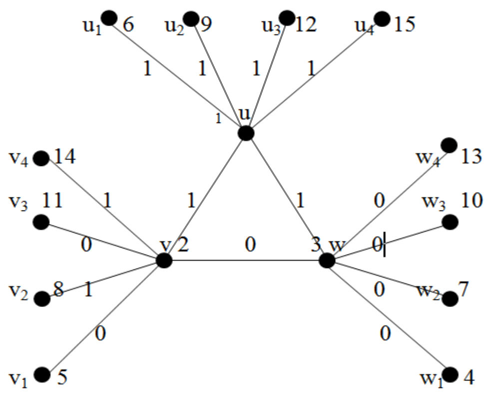

Figure 3.3

THEOREM:3.4

$C_3 * S_n$ is a divisor cordial graph.

Proof:

Let $V(C_3 * S_n) = [u,v,w,u_i: 1 \le i \le n]$

Let $E(C_3 * S_n) = [\{uv\} \cup \{vw\} \cup \{wu\} \cup \{(uu_i): 1 \le i \le n\}]$

Define $f: V(C_3 * S_n) \rightarrow \{1,2,3 \ldots\ldots\ldots n+3\}$

The vertex labeling are

$f(u) = 2$

$f(v) = 4$

$f(w) = 3$

$f(u_1) = 1$

$f(u_i) = 2i+3, \quad 1 < i \le n$

The induced edge labeling are

$$f^*(uu_i) = \begin{cases} 1 & \text{if } i \equiv 0 \bmod 2 \\ 0 & \text{otherwise} \end{cases}, \quad 1 \le i \le n$$

$f^*(uv) = 1$

$f^*(vw) = 0$

$f^*(wu) = 0$

Here, $e_f(1)=e_f(0)$ if 'n' is odd.

$e_f(1)=e_f(0)-1$ if 'n' is even.

Clearly, it satisfies the conditions $|e_f(1)-e_f(0)| \le 1$.

Hence, the induced edge labeling shows that $C_3 * S_n$ is a divisor cordial graph.

For Example, $C_3 * S_5$ is a divisor cordial graph as shown in the figure 3.5.

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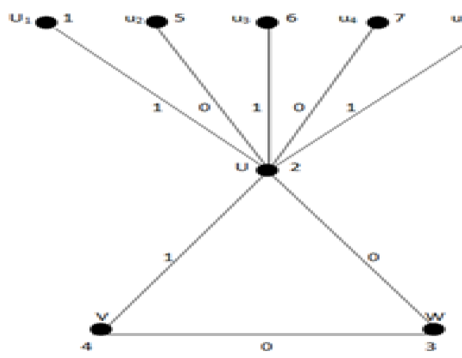

Figure 3.5.

$C_3 * S_4$ is a divisor cordial graph as shown in the figure 3.6.

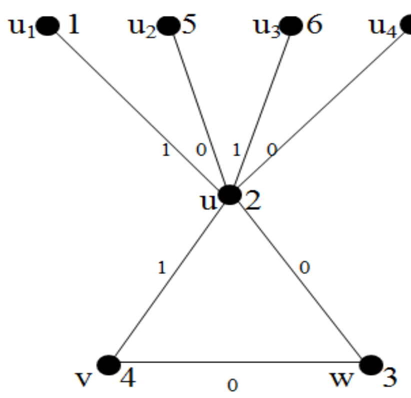

Figure 3.6

THEOREM: 3.7

$< K_{1,n}:n >$ is a divisor cordial graph.

Proof:

Let V($< K_{1,n}:n >$) = [$u, u_i: 1 \leq i \leq n, v_i: 1 \leq i \leq n$]

Let E($< K_{1,n}:n >$) = [{$(uu_i): 1 \leq i \leq n$}U{$(u_iv_i): 1 \leq i \leq n$}]

Define f: V($< K_{1,n}:n >$)→{1,2,3………2n+1}

The vertex labeling are

f(u) = 1

$f(u_i)$ = 2i , $1 \leq i \leq n$

$f(v_i)$ = 2i+1 , $1 \leq i \leq n$

The induced edge labeling are

$f^*(uu_i)$ = 1 , $1 \leq i \leq n$

$f^*(vv_i)$ = 0 , $1 \leq i \leq n$

Here, $e_f(1) = e_f(0)$

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.

Hence, the induced edge labeling shows that $< K_{1,n}: n >$ is a divisor cordial graph.

For Example, $< K_{1,5}: 5 >$ is a divisor cordial graph as shown in the figure 3.8

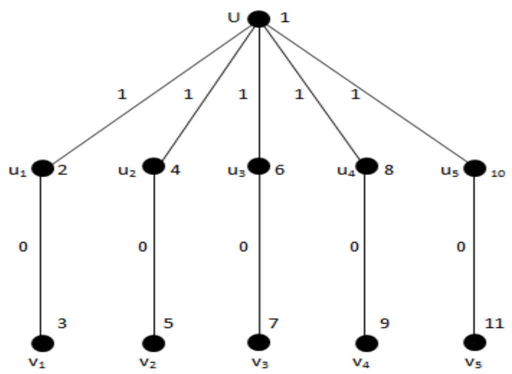

Figure 3.8

THEOREM: 3.9.

$K_{1,3} * K_{1,n}$ is a divisor cordial graph.

Proof:

Let V($K_{1,3} * K_{1,n}$) = [$u, u_i: 1 \leq i \leq n$, $u_{ij}: 1 \leq i \leq 3$, $1 \leq j \leq n$]

Let E($K_{1,3} * K_{1,n}$) = [{(uu_i): $1 \leq i \leq n$}U{(u_iu_{ij}): $1 \leq i \leq 3, 1 \leq j \leq n$}]

Define f:V($K_{1,3} * K_{1,n}$)→{1,2,3………3n+4}

The vertex labeling are

f(u) = 4

$f(u_i)$ = i , $1 \leq i \leq 3$

$f(u_{1i})$ = 3i+3 , $1 \leq i \leq n$

$f(u_{2i})$ = 3i+2 , $1 \leq i \leq n$

$f(u_{3i})$ = 3i+4 , $1 \leq i \leq n$

The induced edge labeling are

$f^*(uu_1)$ = 1

$f^*(uu_2)$ = 1

$f^*(uu_3)$ = 0

$f^*(u_1u_{1i})$ = 1 , $1 \leq i \leq n$

$f^*(u_2u_{2i}) = \begin{cases} 1 & \text{if } i \equiv mod\ 2 \\ 0 & \text{otherwise} \end{cases}, \quad 1 \le i \le n$

$f^*(u_3u_{3i})=0$, $1 \le i \le n$

Here ,$e_f(1)=e_f(0)$ if 'n' is odd.

$e_f(1)=e_f(0)+1$ if 'n' is even.

Clearly, it satisfies the conditions

$|e_f(1) – e_f(0)| \le 1$.

Hence, the induced edge labeling shows that

$K_{1,3}$* $K_{1,n}$ is a divisor cordial graph.

For Example, $K_{1,3}$* $K_{1,5}$ is a divisor cordial graph as shown in the figure 3.10

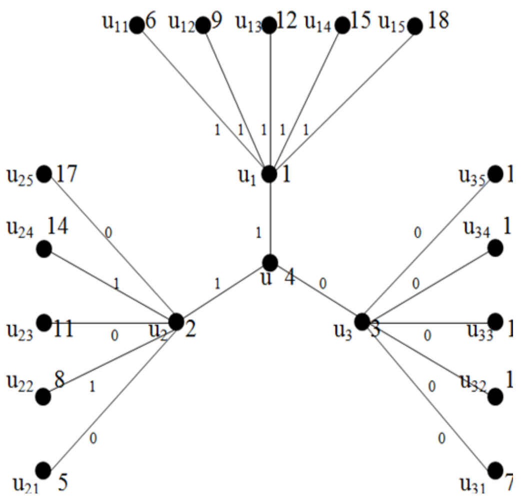

Figure 3.10

$K_{1,3}$* $K_{1,4}$is a divisor cordial graph as shown in the figure 3.11

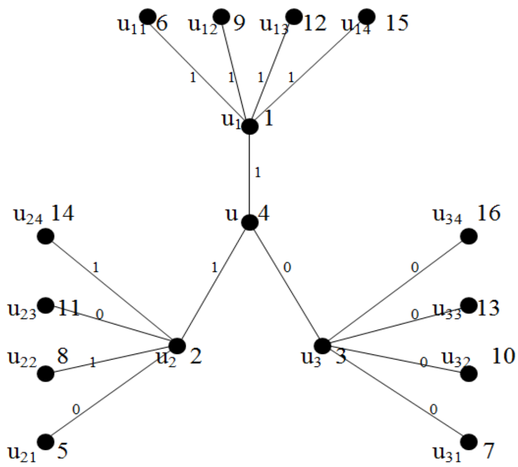

Figure 3.11

THEOREM: 3.12.

$D_2(S_n)$ is a divisor cordial graph.

Proof:

Let $V(D_2(Sn)) = [u,v, w_i:1\le i \le 2n]$

Let $E(D_2(S_n)) = [\{(uw_i):1\le i \le 2n\}U\{(vw_i):1\le i \le 2n\}]$

Define f: $V(D_2(S_n))\rightarrow\{1,2,3……….2n+2\}$

The vertex labeling are

<u>Case 1:</u>$n \equiv 1\ mod\ 2$

<u>Subcase 1a:</u>If $n \ne 6k+1$, $k \in N$

f(u) = 1

f(v) = n+2

$f(w_i)= \begin{cases} i+1 , & 1\le i \le n \\ i+2 , & n+1 \le i \le 2n \end{cases}$

<u>Subcase 1b:</u> If $n = 6k+1$, $k \in N$

f(u) = 1

f(v) = n+4

$f(w_i)= \begin{cases} i+1 , & 1 \le i \le n+2 \\ i+2 , & n+3 \le i \le 2n \end{cases}$

<u>Case 2:</u>$n \equiv 0\ mod\ 2$

<u>Subcase 2a:</u> If $n \ne 6k$, $k \in N$

f(u) = 1

f(v) = n+3

$f(w_i) = \begin{cases} i+1 , & 1\le i \le n+1 \\ i+2 , & n+2 \le i \le 2n \end{cases}$

<u>Subcase 2b:</u> If $n = 6k$, $k \in N$

f(u) = 1

f(v) = n+5

$f(w_i) = \begin{cases} i+1 , & 1\le i \le n+3 \\ i+2 , & n+4 \le i \le 2n \end{cases}$

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The induced edge labeling in both cases are

$f^*(uw_i) = 1$, $1 \leq i \leq 2n$

$f^*(vw_i) = 0$, $1 \leq i \leq 2n$

Here , $e_f(1)=e_f(0)$

Clearly, it satisfies the conditions

$|e_f(1) - e_f(0)| \leq 1$.

Hence, the induced edge labeling shows that $D_2(S_n)$ is a divisor cordial graph.

For Example, $D_2(S_5)$ is a divisor cordial graph as shown in the figure 3.13

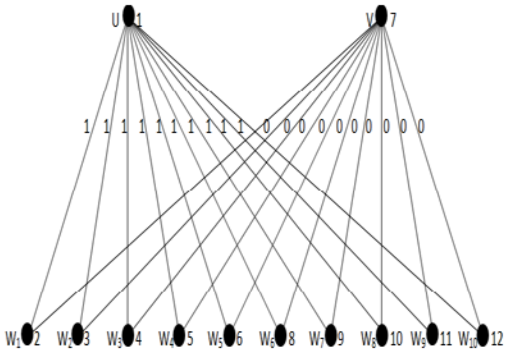

Figure 3.13

$D_2(S_7)$ is a divisor cordial graph as shown in the figure 3.14

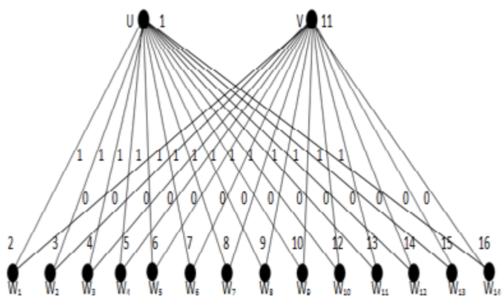

Figure 3.14

$D_2(S_4)$ is a divisor cordial graph as shown in the figure 3.15

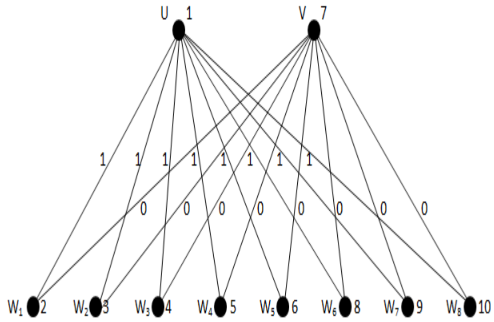

Figure 3.15

$D_2(S_6)$ is a divisor cordial graph as shown in the figure 3.16

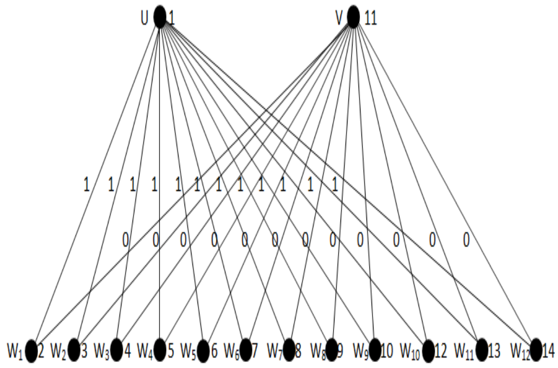

Figure 3.16

THEOREM: 3.17.

$K_{1,n,n}$ is a divisor cordial graph.

Proof:

Let $V(K_{1,n,n}) = [u, v, w_i : 1 \leq i \leq n]$

Let $E(K_{1,n,n}) = [(uv) \cup \{(uw_i) : 1 \leq i \leq n\} \cup \{(vw_i) : 1 \leq i \leq n\}]$

Define $f : V(K_{1,n,n}) \rightarrow \{1,2,3 \ldots\ldots\ldots n+2\}$

The vertex labeling are

Case 1: $n \equiv 1 \bmod 2$

Subcase 1a: If $n \neq 6k+1$, $k \in N$

$f(u) = 1$

$f(v) = n+2$

$f(w_i) = i+1$, $1 \leq i \leq n$

Subcase 1b: If $n = 6k+1$, $k \in N$

$f(u) = 1$

$f(v) = n$

$$f(w_i) = \begin{cases} i+1, & 1 \le i \le n-2 \\ i+2, & n-1 \le i \le n \end{cases}$$

Case 2: $n \equiv 0 \bmod 2$

Subcase 2a: If $n \neq 6k+2$, $k \in N$

$f(u) = 1$

$f(v) = n+1$

$$f(w_i) = \begin{cases} i+1, & 1 \le i < n \\ i+2, & i = n \end{cases}$$

Subcase 2b: If $n = 6k+2$, $k \in N$

$f(u) = 1$

$f(v) = n-1$

$$f(w_i) = \begin{cases} i+1, & 1 \le i \le n-3 \\ i+2, & n-2 \le i \le n \end{cases}$$

The induced edge labeling in both cases are

$f^*(uv) = 1$

$f^*(uw_i) = 1$, $1 \le i \le n$

$f^*(vw_i) = 0$, $1 \le i \le n$

Here, $e_f(1) = e_f(0)+1$

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \le 1$.

Hence, the induced edge labeling shows that $K_{1,n,n}$ is a divisor cordial graph.

For Example, $K_{1,1,5}$ is a divisor cordial graph as shown in the figure 3.18

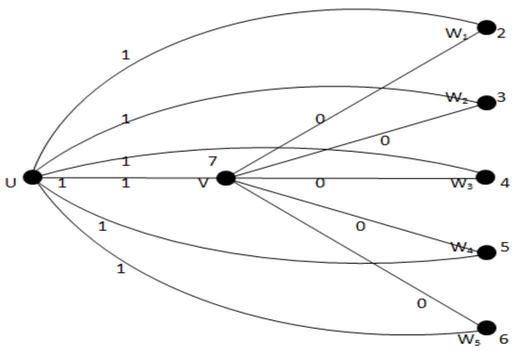

Figure 3.18

$K_{1,1,7}$ is a divisor cordial graph as shown in the figure 3.19

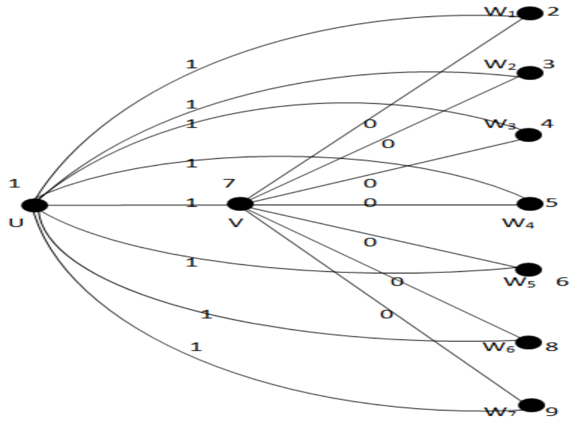

Figure 3.19

$K_{1,1,4}$ is a divisor cordial graph as shown in the figure 3.20

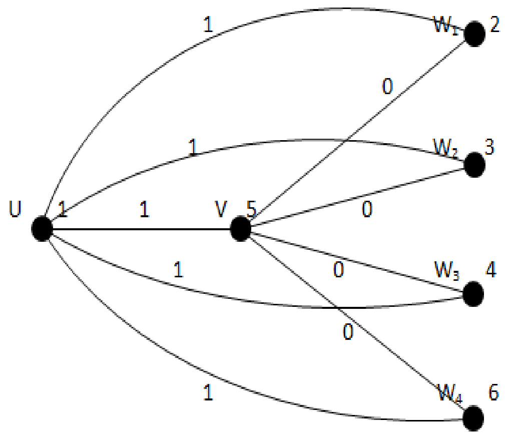

Figure 3.20

$K_{1,1,8}$ is a divisor cordial graph as shown in the figure 3.21

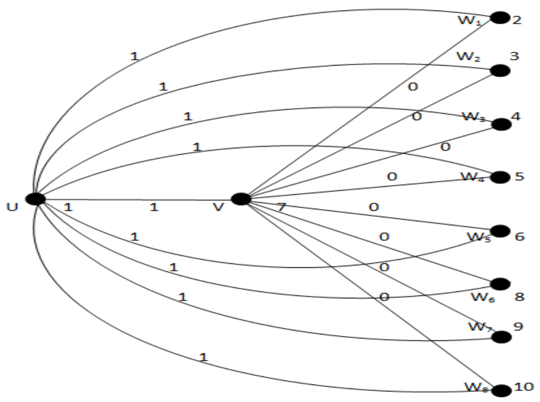

Figure 3.21

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