Star Attached Divisor Cordial Graphs

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Abstract
A divisor cordial labeling of a graph G with vertex set V is a bijection from V to \{1,2,\ldots,V(G)\} such that if each edge uv is assigned the label 1 if \( f(u)/f(v) \) or \( f(v)/f(u) \) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

In this paper, it is proved that \( C_3 \Theta K_{1,n}, C_3 \ast S_n, <K_{1,a}:n>, K_{1,1,n}, D_2(S_n) \), \( K_{1,1,n} \) are divisor cordial graphs.

Keywords: Divisor cordial labeling, Divisor cordial graph

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1. Introduction:
A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each \( e = \{uv\} \) of vertices in E is called an edge or a line of G.

2. Preliminaries:

Definition 2.1
Let \( G \) be a graph and we define the concept of divisor cordial labeling as follows:

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to \{1,2,\ldots,V(G)\} such that if each edge uv is assigned the label 1 if \( f(u)/f(v) \) or \( f(v)/f(u) \) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition 2.2
\( C_3 \Theta K_{1,a} \) is a graph obtained by joining each vertex of a star having \( n \) edges, to one of the vertex of a cycle of length 3.

Definition 2.3
\( C_3 \ast S_n \) is a graph obtained by attaching central vertex of a star \( S_n \) at one of the vertex of \( C_3 \).

Definition 2.4
\( < K_{1,a}:n > \) is a graph obtained as one point union of \( n \) paths of path length 2.

Definition 2.5
\( K_{1,3} \ast K_{1,a} \) is a graph obtained from a star \( K_{1,3} \) by attaching root of a star \( K_{1,a} \) at each pendent vertex of \( K_{1,3} \).

Definition 2.6
Let G be connected graph. A graph, constructed by taking two copies of G say \( G_1 \) and \( G_2 \) and joining each vertex \( u \) in \( G_1 \) to the neighbours of the corresponding vertex \( v \) in \( G_2 \), that is for every vertex \( u \) in \( G_1 \) there exists \( v \) in \( G_2 \) such that \( N(u) = N(v) \). The resulting graph is known as Shadow Graph and it is denoted by \( D_2(G) \).

Definition 2.7
\( K_{1,1,a} \) is a graph obtained by attaching root of a star \( K_{1,a} \) at one end of \( P_2 \) is joined with each pendent vertex of \( K_{1,a} \).

3. Main Results.

THEOREM 3.1
\( C_3 \Theta K_{1,a} \) is a divisor cordial graph.

Proof:
Let \( V(C_3 \Theta K_{1,a}) = \{u,v,w,\{u_i: 1 \leq i \leq n\}, \{v_i: 1 \leq i \leq n\}, \{w_i: 1 \leq i \leq n\}\} \)

Let \( E(C_3 \Theta K_{1,a}) = \{(uv)U(vw)U(wu)U\{uu_i: 1 \leq i \leq n\}U\{vv_i: 1 \leq i \leq n\}U\{ww_i: 1 \leq i \leq n\}\} \)

Define \( f: V(C_3 \Theta K_{1,a}) \rightarrow \{1,2,3,\ldots,3n+3\} \)

The vertex labeling are
\[
\begin{align*}
f(u) & = 1 \\
f(v) & = 2 \\
f(w) & = 3 \\
f(u_i) & = 3i+3, \quad 1 \leq i \leq n
\end{align*}
\]
The induced edge labeling are

\[ f^*(uv) = 1 \]
\[ f^*(vw) = 0 \]
\[ f^*(wu) = 1 \]
\[ f^*(u_iu_j) = 1, \quad 1 \leq i \leq n \]
\[ f^*(ww_i) = 0, \quad 1 \leq i \leq n \]

Here, \( e_f(1) = e_f(0) \) if ‘n’ is odd.
\[ e_f(1) = e_f(0) + 1 \] if ‘n’ is even.

Clearly, it satisfies the conditions \( |e_f(1) - e_f(0)| \leq 1 \).

Hence, the induced edge labeling shows that \( C^\ast_{3\theta K_{1,n}} \) is a divisor cordial graph.

For Example, \( C^\ast_{3\theta K_{1,5}} \) is a divisor cordial graph as shown in the figure 3.2

\[ f(v_i) = 3i+2, \quad 1 \leq i \leq n \]
\[ f(w_i) = 3i+1, \quad 1 \leq i \leq n \]

\[ f^*(vv_i) = \begin{cases} 1 & \text{if } i = 0 \mod 2 \\ 0 & \text{otherwise} \end{cases}, \quad 1 \leq i \leq n. \]

\[ f^*(ww_i) = 0, \quad 1 \leq i \leq n \]

THEOREM: 3.4

\( C^\ast_{3\bigcirc S_n} \) is a divisor cordial graph.

**Proof:**

Let \( V(C^\ast_{3\bigcirc S_n}) = \{u,v,w,u_i: 1 \leq i \leq n\} \)
Let \( E(C^\ast_{3\bigcirc S_n}) = \{uv\} \cup \{vw\} \cup \{wu\} \cup \{u_iu_j: 1 \leq i \leq n\} \}

Define \( f: V(C^\ast_{3\bigcirc S_n}) \rightarrow \{1, 2, 3, \ldots, n+3\} \)

The vertex labeling are

\[ f(u) = 2 \]
\[ f(v) = 4 \]
\[ f(w) = 3 \]
\[ f(u_i) = 1 \]
\[ f(u_j) = 2i+3, \quad 1 < i \leq n \]

The induced edge labeling are

\[ f^*(uv) = 1 \]
\[ f^*(vw) = 0 \]
\[ f^*(wu) = 0 \]

Here, \( e_f(1) = e_f(0) \) if ‘n’ is odd.
\( e_f(1) = e_f(0) - 1 \) if ‘n’ is even.

Clearly, it satisfies the conditions \( |e_f(1) - e_f(0)| \leq 1 \).

Hence, the induced edge labeling shows that \( C^\ast_{3\bigcirc S_n} \) is a divisor cordial graph.

For Example, \( C^\ast_{3\bigcirc S_4} \) is a divisor cordial graph as shown in the figure 3.5.
Theorem 3.7

\(< K_{1,n:n} >\) is a divisor cordial graph.

**Proof:**

Let \(V(< K_{1,n:n} >) = [u, u_i:1 \leq i \leq n, v, v_i:1 \leq i \leq n]\)

Let \(E(< K_{1,n:n} >) = \{(uu_i):1 \leq i \leq n\} \cup \{(u_iu_i):1 \leq i \leq n\}\)

Define \(f: V(< K_{1,n:n} >) \rightarrow \{1,2,3,\ldots,2n+1\}\)

The vertex labeling are

\[f(u) = 4\]
\[f(u_i) = i, \quad 1 \leq i \leq 3\]
\[f(u_{1i}) = 3i+3, \quad 1 \leq i \leq n\]
\[f(u_{2i}) = 3i+2, \quad 1 \leq i \leq n\]
\[f(u_{3i}) = 3i+4, \quad 1 \leq i \leq n\]

The induced edge labeling are

\[f^*(uu_i) = 1, \quad 1 \leq i \leq n\]
\[f^*(uv_i) = 0, \quad 1 \leq i \leq n\]

Here, \(c_0(1)=c_0(0)\)

Clearly, it satisfies the conditions \(|e_d(1) - e_d(0)| \leq 1\).

Hence, the induced edge labeling shows that \(< K_{1,n:n} >\) is a divisor cordial graph.

For Example, \(< K_{1,5:5} >\) is a divisor cordial graph as shown in the figure 3.8.

Theorem 3.9

\(K_{1,3} \ast K_{1,n}\) is a divisor cordial graph.

**Proof:**

Let \(V(K_{1,3} \ast K_{1,n}) = [u, u_i: 1 \leq i \leq n, u_{ij}: 1 \leq i \leq 3, 1 \leq j \leq n]\)

Let \(E(K_{1,3} \ast K_{1,n}) = \{(uu_i): 1 \leq i \leq n\} \cup \{(u_iu_j): 1 \leq i \leq 3, 1 \leq j \leq n\}\)

Define \(f: V(K_{1,3} \ast K_{1,n}) \rightarrow \{1,2,3,\ldots,3n+4\}\)

The vertex labeling are

\[f(u) = 4\]
\[f(u_i) = i, \quad 1 \leq i \leq 3\]
\[f(u_{1i}) = 3i+3, \quad 1 \leq i \leq n\]
\[f(u_{2i}) = 3i+2, \quad 1 \leq i \leq n\]
\[f(u_{3i}) = 3i+4, \quad 1 \leq i \leq n\]

The induced edge labeling are

\[f^*(uu_i) = 1\]
\[f^*(uu_{ij}) = 1\]
\[f^*(uu_{ij}) = 0\]
\[f^*(u_iu_{ij}) = 1, \quad 1 \leq i \leq n\]
\[ f^*(u_i, u_3) = \begin{cases} 1 & \text{if } i \equiv \text{mod } 2 \\ 0 & \text{otherwise} \end{cases} , 1 \leq i \leq n \]

Here, \( e_f(1) = e_f(0) \) if ‘n’ is odd.

\( e_f(1) = e_f(0) + 1 \) if ‘n’ is even.

Clearly, it satisfies the conditions

\[ |e_f(1) - e_f(0)| \leq 1. \]

Hence, the induced edge labeling shows that

\( K_{1, 3} \leq K_{1, n} \) is a divisor cordial graph.

For Example, \( K_{1, 3} \leq K_{1, 5} \) is a divisor cordial graph as shown in the figure 3.10

\[ f(u) = 1 \\ f(v) = n+2 \]

\[ f(w_i) = \begin{cases} i+1 & , 1 \leq i \leq n \\ i+2 & , n+1 \leq i \leq 2n \end{cases} \]

Subcase 1a: If \( n \neq 6k+1 , k \in N \)

\( f(u) = 1 \\ f(v) = n+4 \)

\[ f(w_i) = \begin{cases} i+1 & , 1 \leq i \leq n+2 \\ i+2 & , n+3 \leq i \leq 2n \end{cases} \]

Subcase 1b: If \( n = 6k+1 , k \in N \)

\( f(u) = 1 \\ f(v) = n+5 \)

\[ f(w_i) = \begin{cases} i+1 & , 1 \leq i \leq n+1 \\ i+2 & , n+2 \leq i \leq 2n \end{cases} \]

Case 2: \( n \equiv 0 \mod 2 \)

Subcase 2a: If \( n \neq 6k , k \in N \)

\( f(u) = 1 \\ f(v) = n+3 \)

\[ f(w_i) = \begin{cases} i+1 & , 1 \leq i \leq n+1 \\ i+2 & , n+2 \leq i \leq 2n \end{cases} \]

Subcase 2b: If \( n = 6k , k \in N \)

\( f(u) = 1 \\ f(v) = n+5 \)

\[ f(w_i) = \begin{cases} i+1 & , 1 \leq i \leq n+3 \\ i+2 & , n+4 \leq i \leq 2n \end{cases} \]

THEOREM: 3.12.

\( D_2(S_n) \) is a divisor cordial graph.

Proof:

Let \( V(D_2(S_n)) = [u, v, w_i:1 \leq i \leq 2n] \)

Let \( E(D_2(S_n)) = [(u, w_i:1 \leq i \leq 2n) \cup (v, w_i:1 \leq i \leq 2n)] \)

Define \( f: V(D_2(S_n)) \rightarrow \{1, 2, 3, \ldots, 2n+2\} \)

The vertex labeling are

\[ f^*(u_1, u_3) = \begin{cases} 1 & \text{if } i \equiv \text{mod } 2 \\ 0 & \text{otherwise} \end{cases} , 1 \leq i \leq n \]

\[ f^*(u_i, u_3) = 0 , 1 \leq i \leq n \]

\[ f^*(u_i, u_3) = 0 , 1 \leq i \leq n \]
The induced edge labeling in both cases are
\[ f^*(uw_i) = 1, \quad 1 \leq i \leq 2n \]
\[ f^*(vw_i) = 0, \quad 1 \leq i \leq 2n \]

Here, \( e_f(1) = e_f(0) \) clearly satisfies the conditions
\[ |e_f(1) - e_f(0)| \leq 1. \]

Hence, the induced edge labeling shows that \( D_2(S_n) \) is a divisor cordial graph.

For Example, \( D_2(S_5) \) is a divisor cordial graph as shown in the figure 3.13.

\[ D_2(S_7) \] is a divisor cordial graph as shown in the figure 3.14.

\[ D_2(S_4) \] is a divisor cordial graph as shown in the figure 3.15.

\[ D_2(S_6) \] is a divisor cordial graph as shown in the figure 3.16.

**THEOREM:** 3.17.
\( K_{1,n,n} \) is a divisor cordial graph.

**Proof:**
Let \( V(K_{1,n,n}) = \{u, v, w_i : 1 \leq i \leq n\} \)

Let \( E(K_{1,n,n}) = \{(uv) \cup \{(uw_i) : 1 \leq i \leq n\} \cup \{(vw_i) : 1 \leq i \leq n\}\} \)

Define \( f: V(K_{1,n,n}) \rightarrow \{1, 2, 3, \ldots, n+2\} \)

The vertex labeling are

\[ f(u) = 1 \]
\[ f(v) = n+2 \]
\[ f(w_i) = i+1, \quad 1 \leq i \leq n \]

**Subcase 1a:** If \( n \neq 6k+1 \), \( k \in N \)
\[ f(u) = 1 \]
\[ f(v) = n+2 \]
\[ f(w_i) = i+1, \quad 1 \leq i \leq n \]

**Subcase 1b:** If \( n = 6k+1 \), \( k \in N \)
\[ f(u) = 1 \]
\[ f(v) = n \]
\[ f(w_i) = \begin{cases} 
  i+1, & 1 \leq i \leq n-2 \\
  i+2, & n-1 \leq i \leq n 
\end{cases} \]

Case 2: \( n \equiv 0 \mod 2 \)

Subcase 2a: If \( n \neq 6k+2, k \in \mathbb{N} \)

\[
\begin{align*}
  f(u) &= 1 \\
  f(v) &= n+1 \\
  f(w_i) &= \begin{cases} 
    i+1, & 1 \leq i < n \\
    i+2, & i = n 
  \end{cases}
\end{align*}
\]

Subcase 2b: If \( n = 6k+2, k \in \mathbb{N} \)

\[
\begin{align*}
  f(u) &= 1 \\
  f(v) &= n-1 \\
  f(w_i) &= \begin{cases} 
    i+1, & 1 \leq i \leq n-3 \\
    i+2, & n-2 \leq i \leq n 
  \end{cases}
\end{align*}
\]

The induced edge labeling in both cases are

\[
\begin{align*}
  f^*(uv) &= 1 \\
  f^*(uw_i) &= 1, \quad 1 \leq i \leq n \\
  f^*(vw_i) &= 0, \quad 1 \leq i \leq n 
\end{align*}
\]

Here, \( e_f(1) = e_f(0) + 1 \)

Clearly, it satisfies the conditions \( |e_f(1) - e_f(0)| \leq 1 \).

Hence, the induced edge labeling shows that \( K_{1,n,n} \) is a divisor cordial graph.

For example, \( K_{1,1,5} \) is a divisor cordial graph as shown in the figure 3.18

\[ K_{1,1,7} \text{ is a divisor cordial graph as shown in the figure 3.19} \]

\[ K_{1,1,4} \text{ is a divisor cordial graph as shown in the figure 3.20} \]

\[ K_{1,1,8} \text{ is a divisor cordial graph as shown in the figure 3.21} \]
4. References


