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Robust Fractional Order PID Control of a DC Motor with Parameter Uncertainty Structure

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Abstract

This paper presents a robust DC motor speed controller based on the concept of fractional-order PID (FOPID or $PI^{\lambda}D^{\mu}$) controllers. The method for tuning the $PI^{\lambda}D^{\mu}$ controller uses the Bode envelopes of the control systems with parametric uncertainty. The uncertainty caused by the parameter changes of DC motor are formulated as five different design specifications, which are used in the objective function in the design. Robust $PI^{\lambda}D^{\mu}$ controller is designed from the solution of fulfill five different design specifications. Simulation results are given to illustrate the effectiveness of this method.

Keywords: *dc motor, fractional order PID, robust control.*

1. Introduction

In the past decades, modern control theories have made great advances. Control techniques including optimal control, H_{∞}/H_2 control, fuzzy control, neural network control, predictive control, and so on, have been developed significantly. Nevertheless, the proportional-integral-derivative (PID) control technique has still been widely utilized in many industrial applications such as process control, motor drives, flight control, etc. Nowadays, more than 90% control loops in industry are PID control. This is mainly due to the fact that PID controller possesses robust performance to meet the global change of industry process, simple structure to be easily understood by engineers, and easiness to design and implement.

Recently, there are increasing interests to enhance the performance of PID controller by using the concept of fractional calculus, where the orders of derivatives and integrals are non-integer.

Fractional calculus is non-local, which makes it able to emphasize mathematically the long-memory. Fractional order PID controller proposed by Podlubny (1999) is a generalization of the PID controller using fractional calculus. A $PI^{\lambda}D^{\mu}$ controller is characterizing by five parameters, i.e., the proportional gain, the integrating gain, the derivative gain, the integrating order and the derivative order. Over the last years, $PI^{\lambda}D^{\mu}$ controllers find many applications in irrigation canal control (Domingues, Valerio, & da Costa, 2009), temperature tracking (Ahn, Bhambhani, & Chen, 2009), motion control of DC motor (Xue, Zhao, & Chen, 2006), boost converter control (Tehrani et al., 2010), hypersonic flight vehicle control (Changmao, Naiming, & Zhiguo, 2010), servo press control system (Fan, Sun, & Zhang, 2007). The above research results show that $PI^{\lambda}D^{\mu}$ controller has better performance and robustness than conventional PID controller. Though it is so, the parameter tuning of $PI^{\lambda}D^{\mu}$ controller is an important and critical issue. Compare to conventional PID controller, $PI^{\lambda}D^{\mu}$ controller has two extra parameters. On one hand, it enables people have more degrees of freedom to design $PI^{\lambda}D^{\mu}$ controller, and, on the other hand, it means that it is more complex in the synthesis of $PI^{\lambda}D^{\mu}$ controller. In the literature, many approaches have been documented to design $PI^{\lambda}D^{\mu}$ controller. These approaches can be classified into two classes: analytic methods and heuristic methods. In the analytic context, the parameters of $PI^{\lambda}D^{\mu}$ controller are tuned by minimizing a nonlinear objective function depending on the specifications imposed by the designers. Bouafoura and Braiek (2010) proposed a new analytic method to design $PI^{\lambda}D^{\mu}$ controller by expanding the control loop signal and reference model input and output over a piecewise orthogonal functions. In such a manner, the fractional differential calculus is replaced by the generalized operational matrices of differentiation related to these bases, and thus, the controller tuning is elaborated simply with a matrix manipulator manner. However, the tuning of orders of integrator and derivation are not considered in Bouafoura and Braiek (2010). In Padula and Visioli (2011), a set of tuning rules were devised based on a first order plus time model of the process by minimizing the integrated absolute error with a constraint on the maximum sensitivity. Ziegler–Nichols like tuning rules for $PI^{\lambda}D^{\mu}$ controller were given in Valéio and da Costa (2006). Evolutionary algorithms including genetic

algorithm (GA), particle swarm optimization (PSO), electromagnetism - like algorithm (EM), differential evolution are also used to design $PI^{\lambda}D^{\mu}$ controller. Genetic algorithms were adopted by Cao and Meng and Xue (2009) to design $PI^{\lambda}D^{\mu}$ controller by recasting the problem to an optimization problem. In Zamani, Karimi-Ghartemani, Sadati, and Parniani (2009), particle swarm optimization was used to design $PI^{\lambda}D^{\mu}$ controller for an Automatic Voltage Regulator (AVR) system via minimizing an objective function consisting of overshoot, rising time, settling time, steady-state error, the integral of absolute error (IAE), integral of squared input, gain margin and phase margin. An improved electromagnetism-like algorithm with genetic algorithm (IEMGA) technique was proposed by Lee and Chang (2010) for $PI^{\lambda}D^{\mu}$ controller design through minimizing the integrated-square-error (ISE). Biswas, Das, Abraham, and Dasgupta (2009) proposed to design $PI^{\lambda}D^{\mu}$ based on the root locus method using improved differential evolution.
In Cervera, Banos, Monje, and Vinagre (2006), tuning of $PI^{\lambda}D^{\mu}$ controller is recasted as a quantitative feedback theory (QFT) loop shaping problem, where the optimization objective is the high frequency gain of the nominal loop subjected to the restrictions given by the specifications.
Monje et al. (2004) proposed to tuning parameters of $PI^{\lambda}D^{\mu}$ controller by taking into account five_conditions about phase and gain margins specifications and constrains over the sensitivity functions. As far as the heuristic methods, rule-based methods and evolutionary algorithm based methods were explored by several authors. In C.Yeroglu, N. Tan (2011) , the Bode envelopes of the first-order and first-order plus dead time (FOPDT) systems with the parametric uncertainty structure are successfully combined with five design criteria, which Monje – Vinagre (2004) et al have used in their papers , to obtain robust $PI^{\lambda}D^{\mu}$ controller.
In this paper, we pay our attention to design a $PI^{\lambda}D^{\mu}$ controller for a DC motor system with uncertain parameter using C. Yeroglu, N. Tan (2011) method.
The rest of the paper is organized as follows. In Section 2 mathematical model of DC motor as a controlled object is briefly reviewed. Section 3 presents the basic of $PI^{\lambda}D^{\mu}$ controller. In Section 4 robust $PI^{\lambda}D^{\mu}$ controller is introduced. Design of robust $PI^{\lambda}D^{\mu}$ controller for dc motor systems with parameter uncertainty structure is given in Section 5. Results obtained from simulation and observations are described in Section 6. Finally, conclusion remarks are given in Section 7.

2. Model of DC Motor

We will consider the general model of the DC motor (DCM) which is depicted in Fig. 1. The applied voltage V_a controls the angular velocity $\omega(t)$. The relations for the armature controlled DC motor are shown schematically in Fig. 2.Transfer function (with $T_d(s) = 0$) has the form:

$$G_{DCM}(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s\left[(Ls+R)(Js+K_f)+K_bK_m\right]} \quad (1)$$

However, for many DCM the time constant of the armature is negligible and therefore we can simplify model (1). A simplified continuous mathematical model has the following form:

$$G_{DCM}(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s\left[R(Js+K_f)+K_bK_m\right]} = \frac{\left[K_m/R K_f + K_bK_m\right]}{s(\tau s+1)} = \frac{K_{DCM}}{s(\tau s+1)} \quad (2)$$

Where the time constant $\tau = RJ/(RK_f + K_bK_m)$ and $K_{DCM} = K_m/(R K_f + K_bK_m)$. It is of interest to note that $K_m = K_b$.

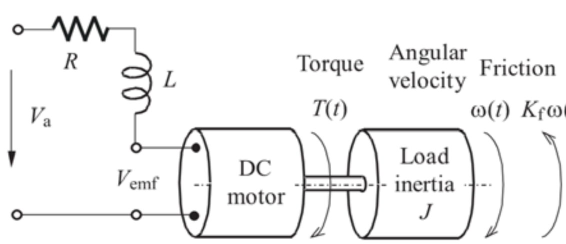

Fig. 1 General model of a DC motor.

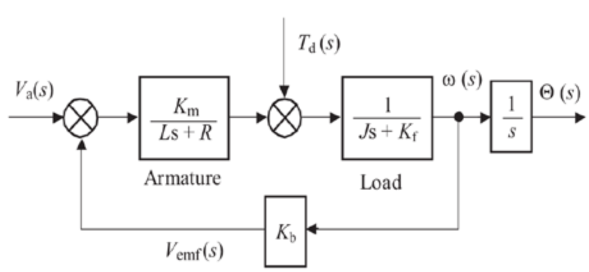

Fig. 2 Mathematical model of a DC motor.

For our mini DC motor the physical constants are:

R = 6 Ω*,*
$K_m = K_b = 0.1$,
$K_f = 0.2$ *N-m s,*
J = 0 .0 1 kgm2/ s2.
For these motor constants the transfer function (2) of the DC motor has the form:

$$G_{DCM}(s) = \frac{0.08}{s(0.05s+1)} \tag{3}$$

3. Fractional -order PID ($PI^{\lambda}D^{\mu}$) controller

The standard PID controller is defined in series (interacting) form as

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{4}$$

where K_p is the proportional gain, K_i is the integral time constant, K_d is the derivative time constant. The $PI^{\lambda}D^{\mu}$ controller is defined as a generalisation of the standard PID controller (4), namely,

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{5}$$

Where λ and μ are the non-integer orders of the integral and derivative terms respectively. It can be easily noted that, by selecting $\lambda = \mu = 1$, a standard PID controller is obtained. Indeed, in order to implement the fractional-order controller, the Oustaloup continuous integer-order approximation (Oustaloup et al. 1996) has been employed. It consists in using the following approximation based on a recursive distribution of zeros and poles:

$$s^{\upsilon} \cong k \prod_{n=1}^{N} \frac{1+\frac{s}{\omega_{z,n}}}{1+\frac{s}{\omega_{p,n}}}, \quad \upsilon > 0 \tag{6}$$

which is valid in a frequency range $[\omega_l, \omega_h]$ and where the gain k is adjusted so that the right side of (6) has unity gain at the gain crossover frequency of s^{υ}. In this paper the value $N = 2$ has been chosen, while $\omega_l = 0.001$ and $\omega_h = 1000$.

4. Design of robust $PI^{\lambda}D^{\mu}$ controller

4.1 Monje – Vinagre et al. method

Monje – Vinagre et al. (2004) proposed a $PI^{\lambda}D^{\mu}$ controller tuning method for the first-order and FOPDT systems. In their paper Bode envelopes of the first-order and FOPDT systems are combined with five design criteria to obtain the $PI^{\lambda}D^{\mu}$ controller. This five design criteria, such as magnitude at gain crossover frequency, phase margin, robustness to plant uncertain ties, high frequency noise attenuation and sensitivity functions are given as follows.

4.1.1. Phase margin (φ_m) and gain crossover frequency (ω_{cg}) specifications:

Gain and phase margins have always served as important measures of robustness. It is known that the phase margin is related to the damping of the system and therefore can also serve as a performance measure (see Franklin, Powell, & Naeini, 1986). The equations that define the phase margin and the gain crossover frequency are:

$$\left|C(j\omega_{cg})G(j\omega_{cg})\right|_{dB} = 0dB \tag{7}$$

$$\mathrm{Arg}(C(j\omega_{cg})G(j\omega_{cg})) = -\pi + \phi_{pm} \tag{8}$$

4.1.2 Robustness to variations in the gain of the plant:

The next constraint can be considered in this case (see Chen & Moore, 2005):
This condition forces the phase of the open-loop system

$$\left(\frac{d\left(\arg(C(j\omega)G(j\omega))\right)}{d\omega}\right)_{\omega=\omega_{cg}} = 0 \tag{9}$$

This condition forces the phase of the open-loop system to be flat at ω_{cg} and hence to be almost constant within an interval around ω_{cg}. It means that the system is more robust to gain changes and the overshoot of the response is almost constant within a gain range (iso-damping property of the time response). It must be remarked that the interval of gains for which the system is robust is not fixed with this condition. That is, the user cannot force the system to be robust for a particular gain range. This range depends on the frequency range around $\omega \geq \omega_t$ for which the phase of the open-loop system keeps flat. This frequency range will be longer or shorter, depending on the resulting controller and the plant.

4.1.3 High frequency noise rejection:

A constraint on the complementary sensitivity function *T* can be established:

$$\left| \mathrm{T}(\mathrm{j}\omega) = \frac{C(\mathrm{j}\omega)G(\mathrm{j}\omega)}{1+C(\mathrm{j}\omega)G(\mathrm{j}\omega)} \right|_{dB} \leq A dB \qquad (10)$$

Whit *A* the desired noise attenuation for frequencies $\omega \geq \omega_t$ rad/s.

4.1.4 To ensure a good output disturbance rejection:

A constraint on the sensitivity function *S* can be defined:

$$\left| \mathrm{S}(\mathrm{j}\omega) = \frac{1}{1+C(\mathrm{j}\omega)G(\mathrm{j}\omega)} \right|_{dB} \leq B dB \qquad (11)$$

With *B* the desired value of the sensitivity function for frequencies $\omega \leq \omega_s$ rad/s (desired frequency range).

For fractional order controllers such as a PI^{λ} or a PD^{μ}, three design specifications could be met (one for each parameter). Therefore, for the general case of a $PI^{\lambda}D^{\mu}$ controller the design problem is based on solving the system of five nonlinear equations (given by the corresponding design specifications) and five unknown parameters $k_p, k_i, k_d, \lambda, \mu$.

4.2 C. Yeroglu, N. Tan robust $PI^{\lambda}D^{\mu}$ controller tuning algorithm

Owing to the tuning method, which C. Yeroglu, N. Tan (2011) have used in their paper, we preferred to use 'C. Yeroglu, N. Tan method' as a title of this section. A robust controller is less sensitive to the parameter changes of the controlled system. The uncertainty can be caused by non-precise identification. The fractional order controllers are less sensitive to changes of controlled system parameters. C. Yeroglu and N.Tan (2011) proposed a robust $PI^{\lambda}D^{\mu}$ controller tuning algorithm for first-order and FOPDT system with parametric uncertainty structure. The FOPDT system with parametric uncertainty can be represented as

$$G(s) = \frac{[\underline{k}, \overline{k}]}{[\underline{\tau}, \overline{\tau}]s+1} e^{-[\underline{L}, \overline{L}]s} \qquad (12)$$

Where $\underline{k}, \underline{\tau}$ and $\underline{L}$ are lower limits, $\overline{k}, \overline{\tau}$ and $\overline{L}$ are upper limits of the parameters, respectively.

Minimum and maximum plots of the gain are shown by $G_{R1}(s), G_{R2}(s)$ and minimum and maximum plots of phase are shown by $G_{R3}(s), G_{R4}(s)$ respectively.

$$G_{R1}(s) = \frac{\underline{k}}{\overline{\tau}s+1} e^{-\overline{L}s}, \quad G_{R2}(s) = \frac{\overline{k}}{\underline{\tau}s+1} e^{-\overline{L}s} \qquad (13)$$

$$G_{R3}(s) = \frac{\overline{k}}{\underline{\tau}s+1} e^{-\underline{L}s}, \quad G_{R4}(s) = \frac{\underline{k}}{\overline{\tau}s+1} e^{-\overline{L}s} \qquad (14)$$

In order to design robust $PI^{\lambda}D^{\mu}$ controller, Equations (7), (8) , (9) , (10) , (11) should be satisfied with the transfer functions $G_{R2}(s)$, $G_{R4}(s)$, $G_{R4}(s)$, $G_{R1}(s)$, $G_{R2}(s)$ respectively, namely ω_{cg} must be taken at the point 'a ' in Fig. 3 and the point 'b ' in Fig. 3 should be the minimum phase margin of the system.

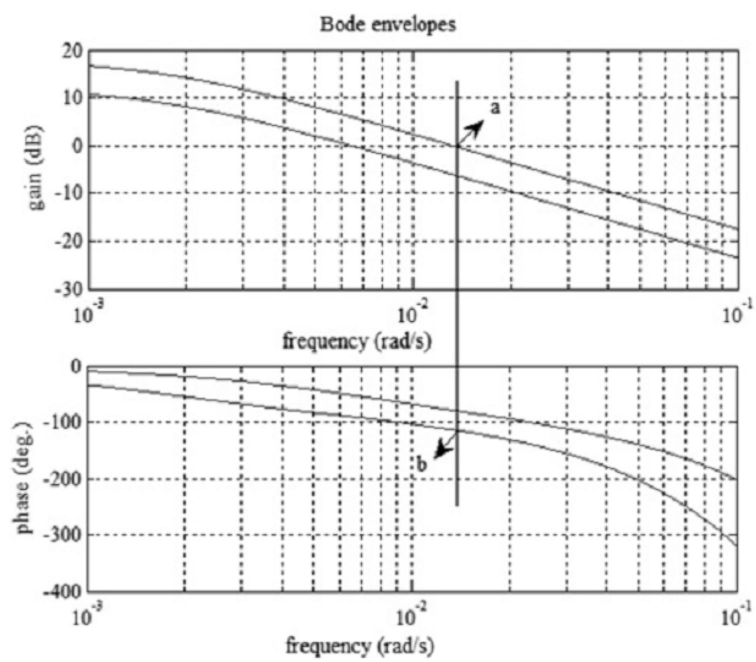

Fig. 3 Bode envelopes of a FOPDT plant

We leave the details of this subsection due to space limitation. For more details please refer to C.Yeroglu, N. Tan (2011).

5. Design of robust $PI^{\lambda}D^{\mu}$ controller for DC motor with uncertain parameter

This section presents a tuning method for robust $PI^{\lambda}D^{\mu}$ controller for dc motor systems with parameter uncertainty structure. In this paper changes in the parameters of the DC motor are taken into account as interval uncertainty, and the robustness of the controller is analyzed. We assume that the values of the physical constants of the DC motor may change in an interval. The proposed method benefits from design specifications given in Monje – Vinagre et al.(2004) tuning method and C. Yeroglu and N. Tan(2011) method that proposed a robust $PI^{\lambda}D^{\mu}$ controller tuning algorithm for first-order and FOPDT system with parametric uncertainty structure. Preliminary study of this section has been presented in the C. Yeroglu, N. Tan (2011).The transfer function of the DC motor has the form (3). A dc motor system with parametric uncertainty can be represented as

$$G_2(s) = \frac{[\overline{k}, \underline{k}]}{s([\overline{\tau}, \underline{\tau}]s + 1)} \quad (15)$$

Where $\underline{k}, \underline{\tau}$ are lower limits and $\overline{k}, \overline{\tau}$ are upper limits of the parameters, respectively. The fractional-order controller is designed to obtain the desired performances for the given interval system. Now since a first order plant system is required for using the C. Yeroglu, N. Tan tuning method, a MATLAB file called "getfod.m" is used to approximate the second order system (15) by first order plant system (12). Now we can design robust $PI^{\lambda}D^{\mu}$ controller to control dc motor with parametric uncertainty structure.

6. SIMULATION RESULTS

6.1 Performance of DC motor System with robust $PI^{\lambda}D^{\mu}$ Controller

Consider the dc motor system (17).We assume that the values of the physical constants of the DC motor change in an interval. The dc motor system with parametric uncertainty has the following form:

$$G_1(s) = \frac{k}{s(\tau s + 1)} \quad (16)$$

Where

$k \in [\underline{k}, \overline{k}] = [0.06, 0.1]$, $\tau \in [\underline{\tau}, \overline{\tau}] = [0.04, 0.06]$.

Now the "getfod.m" MATLAB file is used to approximate a second order system by first order plant system. The approximated FOPDT system obtained is given as:

$$G_1(s) = \frac{k}{\tau s + 1} e^{-Ls} \quad (17)$$

Where

$k \in [\underline{k}, \overline{k}] = [600, 1000]$, ,

$L \in [\underline{L}, \overline{L}] = [0.04, 0.06]$,

$\tau = 10000$.

Design specifications for the system are given as follows: Gain crossover frequency $\omega_{cg} = 0.1 rad/s$, phase margin $\phi_{pm} = 90^0$, robustness to variation in the gain of the plant must be fulfilled, desired noise attenuation $A = -20dB$ for the $\omega \geq \omega_t = 10 rad/s$ and desired value of sensitivity function $B = -20dB$ for the frequency $\omega \leq \omega_s = 0.01 rad/s$.

The robust $PI^{\lambda}D^{\mu}$ controller to control $G_1(s)$ is obtained as

$$C_1(s) = 50 + \frac{0.51}{s^{0.4859}} + 3.13 s^{0.2746} \quad (18)$$

Step responses of $C_1(s)G_1(s)$ are shown in Fig. 4.

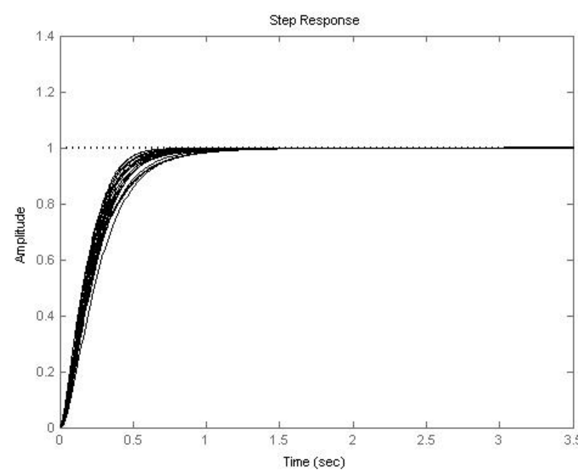

Fig. 4 Step responses of the $C_1(s)G_1(s)$

6.2 Comparison with other $PI^{\lambda}D^{\mu}$ Controllers

In this section, we compare robust $PI^{\lambda}D^{\mu}$ controller with other $PI^{\lambda}D^{\mu}$ controllers. For comparison, we consider designed fractional controller in A.Rajasekhar et al. (2011) that is based on Artificial Bee Colony (ABC) and modified version of ABC namely L-ABC algorithm. The ABC and L-ABC algorithms are implemented as in A.Rajasekhar et al. (2011). The step responses of the system controlled by ABC -$PI^{\lambda}D^{\mu}$ controller, L-ABC-$PI^{\lambda}D^{\mu}$ controller and robust $PI^{\lambda}D^{\mu}$ controller are shown in Figs. 5. The best controller parameters and the performance indices are shown in Table 1.

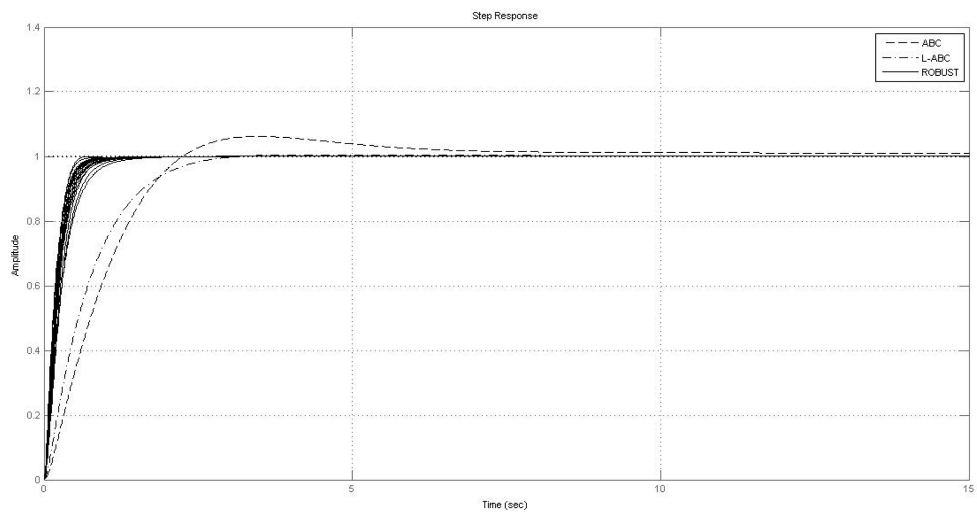

Fig. 5 Step responses of the ABC - $PI^{\lambda}D^{\mu}$, L-ABC- $PI^{\lambda}D^{\mu}$ and robust $PI^{\lambda}D^{\mu}$ controller

Table 1: Best controller parameters and performance indices of different $PI^{\lambda}D^{\mu}$ controllers

Controller	K_p	k_i	k_d	μ	λ	E_{ss}	T_r	T_s	M_p(%)
ROBUST- $PI^{\lambda}D^{\mu}$	50	0.51	3.13	0.2746	0.4859	0.004	**0.295~0.6426**	**0.4624~1.2096**	**0**
L-ABC- $PI^{\lambda}D^{\mu}$	0.497	15	0.9936	0.107	0.0379	**0.0018**	1.5461	2.4392	0.6398
ABC - $PI^{\lambda}D^{\mu}$	0.162	12	0.616	0.172	0.212	0.246	1.1917	6.454	30.081

With observe Fig. 5 and Table 1, we derive that proposed robust $PI^{\lambda}D^{\mu}$ controller has better performance than the ABC-$PI^{\lambda}D^{\mu}$ and L-ABC-$PI^{\lambda}D^{\mu}$ controller. This figure shows that the settling time and rise time and overshoot from the robust $PI^{\lambda}D^{\mu}$ controller at worst is better from ABC- $PI^{\lambda}D^{\mu}$ and L-ABC- $PI^{\lambda}D^{\mu}$ controller while the steady state error from the proposed controller is almost the same as the L-ABC-$PI^{\lambda}D^{\mu}$ controller. It can be seen that the step response of the dc motor system with parametric uncertainty controlled by robust $PI^{\lambda}D^{\mu}$ controller has very good performance.

7. Conclusions

In this paper we proposed a robust $PI^{\lambda}D^{\mu}$ controller to control DC motor systems with parameter uncertainty structure. The proposed method benefits from design specifications given in Monje – Vinagre et al. tuning method and C. Yeroglu and N. Tan method that proposed a robust $PI^{\lambda}D^{\mu}$ controller tuning algorithm for first-order and FOPDT system with parametric uncertainty structure. The simulation results illustrate that the proposed robust $PI^{\lambda}D^{\mu}$ controller for DC motor systems has best control performance than other optimal $PI^{\lambda}D^{\mu}$ controllers that so far is presented, and also, for model uncertainties, the robust $PI^{\lambda}D^{\mu}$ controller is more robust. The proposed technique may apply as an efficient method to design robust optimal practical fractional-order controllers for practical systems to reject external disturbances.

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