ISSN 2348 – 7968

Effects of Papoulis-Gerchberg Iterative Scheme with Real Orthogonal Transform for Signal Reconstruction

Tsung-Ming Lo

Department of Computer and Communication Engineering, China University of Technology.

Hukou City, Hsinchu County, Taiwan, R.O.C
e-mail: c36ltm@cute.edu.tw

Abstract

The iterative algorithm of Papoulis-Gerchberg is simple and efficient solution to signal reconstruction problem. However, the PG algorithm is usually slowly convergent, and a band-limited signal practically is difficult to be obtained. In this work, we present and discuss an innovative approach for restoring lost data with a specific preprocess for meeting boundary conditions of real orthogonal transforms in an iteration methodology. The simulation indicates that a better mean square error of the reconstruction and a faster convergence rate can be obtained if the proposed preprocess concept is adapted.

Keywords: *Papoulis-Gerchberg (PG), band-limited, mean square error (MSE), lost data reconstruction, real orthogonal transforms (ROT).*

1. Introduction

Research on the data loss reconstruction in digital signal processing has been mounting obviously the last two decades [1]-[5]. In various applications of signal transmission and processing, there is always a possibility of loss of data. The missing data have to be interpolated somehow if the quality of the speech signal is not to be sacrificed. Among all, the PG algorithm is famous for iterative transformations between time and frequency domains to recover lost data [4]. It has been proven that the PG algorithm can be solved by a set of linear equations [6]; however, to have a fast computation for a PG algorithm, a fast Hartley transform is proposed to replace a fast Fourier transform in the PG algorithm [7]. Besides, some iterative techniques such as Jacobi's method and Gauss-Seidel method have also been proposed in the time domain to reconstruct the lost data and the iterative convergence rate obtained is faster than that with the original PG algorithm [8]. Because the real orthogonal transform is periodicity-oriented, if the signal is not a band-limited signal in the transform domain, reconstructed results of lost data may be in a range out of expectation. This problem always occurs in the reconstruction of lost data with iterative algorithm [6]-[8].

In general, the band-limited signal is difficult to be obtained even if a sampling rate is higher than two times the bandwidth of the continuous signal. Because of the periodic properties of an orthogonal transform, a signal bandwidth becomes unlimited if the data length of the signal is not an integer multiple of period of the original signal. The above methods also require an original signal to be a band-limited signal in orthogonal transform domain. Since the original signal can be approximated as band-limited signal, which uses the preprocess algorithms called a boundary-matching signal [4], [9], we will apply the concept of boundary matching to tackle the problem of enhancing lost data reconstruction.

2. The Iterative Scheme for Signal Reconstruction

The Shannon-Whittacker-Kotel’nikov sampling theorem [10] ensures that a finite energy band-limited signal uniformly being sampled at or above its Nyquist rate can be uniquely determined by its sample values. In other words, sampled at the Nyquist rate is uniquely determined by its sample, which is independent of every other sample. The sample values from an oversampled band-limited signal are dependent. As a result, if one or more data are lost, the missing data can be recovered from the remaining known values. However, it has also be proven that the lost data reconstruction in a band-limited signal can be determined by solving a set of linear equations based on the discrete time Fourier transform domain [6]. Using this particular set of the linear equations to reconstruct the signal will often yield a result out of the expectation. In this paper, we utilize the PG algorithm to obtain a new set of linear equations based on real orthogonal transform domain. In general, it is not efficient to solve the set of linear equations by calculating an inverse matrix if the size of matrix is too large. Several iterative methods are utilized to approximate the final result progressively in the time domain [8], and we will utilize these methods to solve the proposed linear equations to increase the rate of convergence.

2.1 Papoulis-Gerchberg algorithm

The conventional PG algorithm [11] is an iterative procedure in which one alternates between object domain and frequency domain while constraining the signal to its known values and to a finite bandwidth. In [9], the PG algorithm for a band-limited signal x(n) have to adjusted as the following procedure:

Step 1: Set *g(n)* as

$$g(n)=\begin{cases} x(n) & \text{if } x(n) \quad \text{is known} \\ 0 & \text{if } x(n) \quad \text{is lost} \end{cases} \square$$

Note that this is equivalent to set the unknown sample values to zero.

Step 2: Perform discrete Fourier transform (DFT) for *g(n)* as

$$G(k)=\frac{1}{N}\sum_{n=0}^{N-1} g(n)e^{-j\frac{2\pi kn}{N}}$$

Step 3: Truncate G(k) into band-limited spectrum, defined as F(k),

$$F(k)=\begin{cases} G(k) & \text{if } 0 \le k \le k_0 \quad \text{and} \quad N-k_0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

Where N is the length of the x(n) and k0 is the bandwidth of the original signal, x(n).

Step 4: Perform inverse discrete Fourier transform (IDFT) for *F(k)* as

$$f(n)=\sum_{k=0}^{N-1} F(k)e^{j\frac{2\pi kn}{N}}$$

Step 5: Update the known data f(n) to obtain new g(n),

$$g(n)=\begin{cases} x(n) & \text{if } x(n) \quad \text{is known} \\ f(n) & \text{if } x(n) \quad \text{is lost} \end{cases} \square$$

And go to Step 2.

2.2 The iteration method of Papoulis-Gerchberg

With [9], [12], we let *A* denote a set of index of the lost data and the length is N_A. That is, given a set of data $\{x(n) \mid n \notin A\}$, the unknown set of the lost data $\{x(n) \mid n \in A\}$ should be determined. Let gi denote the N_A–dimensional vector and the indices of the lost sample are arranged in the increasing order of index at the i-th iteration. Then the PG algorithm can be expressed as

$$g_{i+1}(m)=\sum_{n=0}^{N-1} g_i(n)(\frac{2k_0+1}{N})Sind(m,n). \quad (2.1)$$

Where

$$Sind(m,n)=\frac{\sin[(2k_0+1)\pi(m-n)/N]}{(2k_0+1)\sin[\pi(m-n)/N]}.$$

The term $(2k_0+1)/N$ in equation (2.1) can be looked as the oversampling rate and it denoted as r. Therefore the vector g_{i+1} at the *(i+1)th* iteration can be obtained as

$$g_{i+1}=h+S\,g_i, \quad (2.2)$$

where ***S*** is a principle submatrix of the Toeplitz matrix ***M*** with elements $\{rSind(m,n) \mid (m,n)\in A\}$, and ***h*** is a vector with elements as

$$h(m)=r\sum_{\substack{n=0\\ n\notin A}}^{N-1} g_i(n)Sind(m,n).$$

From equation (2.2), the g_{i+1} can be solved by use of the evaluation of the power series and is obtained as

$$g=\lim_{i\to\infty} g_i=[I-S]^{-1}h. \quad (2.3)$$

2.3 The iteration method of Jacobi

With [8], let

$$\boldsymbol{D}=diag(\boldsymbol{I}\text{-}\boldsymbol{S}) \text{ and } \boldsymbol{A}=\boldsymbol{D}\text{-}(\boldsymbol{I}\text{-}\boldsymbol{S}) \quad (2.4)$$

where *diag(**I-S**)* is expressed as a diagonal of the ***(I-S)*** in an attempt to form a diagonal matrix. Substituting equation (2.4) into equation (2.3), we have

$$(\boldsymbol{D}\text{-}\boldsymbol{A})g=h \quad (2.5)$$

and above equation can be iterated as

$$g_{i+1}=D^{-1}Ag_i+D^{-1}h$$

2.4 The iteration method of Gauss-Seidel

With [8], let

$$\boldsymbol{A}=\boldsymbol{L}+\boldsymbol{U} \quad (2.6)$$

where ***L*** and ***U*** are the strictly lower and upper matrix of matrices ***A***, respectively. Substituting equation (2.6) into equation (2.5), we have

$$(D-L-U)g=h$$

and the above equation can be iterated as

$$g_{i+1}=(D-L)^{-1}Ug_i+(D-L)^{-1}h$$

3. Boundary Condition of the Real Orthogonal Transform

The real orthogonal transform, discrete sine transform (DST), discrete cosine transform (DCT) and generalized discrete ***W*** transform (GWT), are generally used to analyze a periodic signal $x_p(n)$, which is expanded by a signal *x(n)*. Therefore, if the difference between two ends in signal *x(n)* is too large, a large amount of components in high frequency should be induced. However, it is unfortunate that most of the signals fail to satisfy boundary conditions, and as a result, the signal should be transformed into a new signal that matches some boundary conditions such that the transformed signal approximates a band-limited signal.

In order to retain the properties of band-limited signal, Wang proposed a method to map an original digital signal with another type of new signal that satisfies some boundary conditions and approximates a band-limited signal to reduce the amount of undesired high frequencies for the sampled signal [13]. Base on the concept of boundary matched, when a real orthogonal transform is used for lost data reconstruction, it is always implied that $x^{(z)}(0) = x^{(z)}(N)$, where superscript z means the (z)'s derivation of $x(n)$. In general, the sampled signal is not easily to meet this condition. However, we can process the original signal into a signal with matching the boundary condition in the time domain. And then, the accuracy of lost data reconstruction by using the ROT with PG algorithm can be greatly improved. Here, the critical boundary conditions are set, and the corresponding mapping for the input data is also developed. Due to the periodic property of ROT, we can obtain the boundary condition of a signal in ROT domain as following.

Due to the periodic property of ROT, we may require that the signal $x(n)$ must meet following boundary conditions

$$x^{(z)}(0) = x^{(z)}(N) = 0 \qquad for \quad z = 0,1,\cdots \tag{3.1}$$

where z and N expresses the (z)'s derivatives and the length of original sequence respectively.

3.1 The boundary matching with DST

Let the length N sequence as $x(n)$, $n = 0,1,\ldots, N-1$, and the N-point forward / inverse type I DSTs (DST-I / IDST-I) [14] can be defined as follows:

$$X(k) = \frac{2}{N}\sum_{n=0}^{N-1} x(n)\sin(\frac{kn\pi}{N}) \qquad k = 0,1,\ldots, N-1 \tag{3.2}$$

$$x(n) = \sum_{k=0}^{N-1} X(k)\sin(\frac{kn\pi}{N}) \qquad n = 0,1,\ldots, N-1 \tag{3.3}$$

Taking the $(2p)$'s derivative of (3.3) with respect to index n:

$$x^{(2p)}(n) = (-1)^{(p)}\sum_{k=0}^{N-1}(\frac{k\pi}{N})^{(2p)} X(k)\sin(\frac{kn\pi}{N}).$$

Therefore, the $(2p)$'s boundary conditions for the discrete reconstruction using the DST-I can be set as:

$$x^{(2p)}(0) = x^{(2p)}(N) = 0 \qquad for \quad p = 0,1,\cdots \tag{3.4}$$

Then, a polynomial of $(2p+1)$'s order $t_{2p+1}(n)$ is proposed to meet the $(2p)$'s boundary conditions of (2.1) in order to improve the performance of discrete reconstruction. However, if the original signal $x(n)$ does not satisfy the boundary conditions in (3.4), a new signal that matches the boundary conditions, say $y(n)$, can be generated from $x(n)$. Consider $y(n)$ as:

$$y(n) = x(n) - t_{2p+1}(n)$$

where $t_{2p+1}(n)$ is a $(2p+1)$ order polynomial sequence, i.e.

$$t_{2p+1}(n) = \sum_{m=0}^{2p+1} a_m\, n^m \quad .$$

For trivial case, p=0, derive the coefficients as the following.

$$a_1 = x(0) \quad , \quad a_2 = \frac{x(N)-x(0)}{N}.$$

3.2 The boundary matching with DCT

Let the length $N+1$ sequence as $x(n)$, $n = 0,1,\ldots,N$, and the $N+1$-point forward / inverse type I DCTs (DCT-I / IDCT-I) [14] can be defined as follows:

$$X(k) = \frac{2}{N}C_k\sum_{n=0}^{N} C_n x(n)\cos(\frac{kn\pi}{N}) \qquad k = 0,1,\ldots, N \tag{3.5}$$

$$x(n) = \sum_{k=0}^{N} X(k)\cos(\frac{kn\pi}{N}) \qquad n = 0,1,\ldots, N \tag{3.6}$$

where

$$C_j = \begin{cases} 1/2 & for \quad j = 0 \quad or \quad N \\ 1 & ortherwise \end{cases}.$$

Taking the $(2p+1)$'s derivative of (3.6) with respect to index n:

$$x^{(2p+1)}(n) = (-1)^{(p+1)}\sum_{k=0}^{N}(\frac{k\pi}{N})^{(2p+1)} X(k)\sin(\frac{kn\pi}{N}). \tag{3.7}$$

From (3.7), the boundary conditions of the DCT-I become

$$x^{(2p+1)}(0) = x^{(2p+1)}(N) = 0 \qquad for \quad p = 0,1,\cdots. \tag{3.8}$$

Similarly, in the method with DCT-I where $(2p+2)$ order polynomial $t_{2p+2}(n)$ is assumed, the boundary-matching signal, $y(n)$, becomes

$$y(n) = x(n) - t_{2p+2}(n),$$

where

$$t_{2p+2}(n) = \sum_{m=0}^{2p+2} a_m n^m.$$

For trivial case, p=0, derive the coefficients as the following.

$$a_1 = x'(0) \quad , \quad a_2 = \frac{x'(N)-x'(0)}{2N}.$$

3.3 The Generalized Discrete W Transform (GWT)

Let $x(n)$, n=0,1,…,N, be a length $N+1$ of discrete time signal. GWT [15] pair is defined as:

$$X(k) = \frac{2}{N}\sum_{n=0}^{N} C_n x(n) \sin(\frac{\pi}{4} + (k+\alpha)(\frac{2n\pi}{N} - \pi)) \quad k = 0,1,\cdots,N-1$$

$$x(n) = \sum_{k=o}^{N-1} X(k) \sin(\frac{\pi}{4} + (k+\alpha)(\frac{2n\pi}{N} - \pi)) \quad n = 0,1,\cdots,N$$

where

$$\alpha = \frac{1}{\pi}\tan^{-1}(\frac{x(N)-x(0)}{x(N)+x(0)})$$

and

$$C_n = \begin{cases} \frac{1}{2} & for \quad n=0 \quad or \quad N \\ 1 & otherwise \end{cases}.$$

Similarly, the boundary conditions of the GWT are found as:

$$x^{2p}(0) = (-1)^p \sin(\frac{\pi}{4} - \alpha\pi)\sum_{k=o}^{N-1}(-1)^k (k+\alpha)^{2p} X(k) \tag{3.9a}$$

$$x^{2p}(N) = (-1)^p \sin(\frac{\pi}{4} + \alpha\pi)\sum_{k=o}^{N-1}(-1)^k (k+\alpha)^{2p} X(k) \tag{3.9b}$$

and

$$x^{(2p+1)}(0) = (-1)^p \cos(\frac{\pi}{4} - \alpha\pi)\sum_{k=o}^{N-1}(-1)^k (k+\alpha)^{2p+1} X(k) \tag{3.10a}$$

$$x^{(2p+1)}(N) = (-1)^p \cos(\frac{\pi}{4} + \alpha\pi)\sum_{k=o}^{N-1}(-1)^k (k+\alpha)^{2p+1} X(k) \tag{3.10b}$$

From (3.9) and (3.10), the boundary conditions can be derived as:

$$x^{(2p)}(N) = \beta\, x^{(2p)}(0). \qquad p = 0,1,\ldots,.$$

and

$$x^{(2p+1)}(N) = \frac{1}{\beta}\, x^{(2p+1)}(0) \qquad p = 0,1,\ldots,.$$

where

$$\beta = \frac{\sin(\frac{\pi}{4}+\alpha\pi)}{\sin(\frac{\pi}{4}-\alpha\pi)} = \frac{\cos(\frac{\pi}{4}-\alpha\pi)}{\cos(\frac{\pi}{4}+\alpha\pi)} = \frac{x(N)}{x(0)} \tag{3.11}$$

or

$$\alpha = \frac{1}{\pi}\tan^{-1}(\frac{x(N)-x(0)}{x(N)+x(0)}). \tag{3.12}$$

The GWT is dependent on signal *x(n)* and satisfies boundary conditions naturally if α is chosen by Eq.(3.12). When α=0, α=1/4, and α=-1/4, it is interesting that the GWT can be simplified to be discrete Hartley transform (DHT), DST and DCT respectively. As a result, GWT can be implemented by the use of fast algorithms of these special transforms in an attempt to reduce the complexity of the computation.

4. The Proposed Efficient Reconstruction Algorithm

Based on the prior investigation of boundary matching with ROT, by inserting a subtraction by a polynomial in the PG algorithm, using boundary-matched concept, a significant performance and speed up of its convergence has been achieved. It is possible that the lost data may becomes non-band-limited even the original signal is band-limited. Here, we propose a novel approaches to restore the lost sample by mapping the original signal into boundary-matching signal in the time domain, and the entire algorithm can be summarized as follows:

Step 1: Perform the preprocess for the signal *x(n)*, except the lost data as:

$$g(n) = \begin{cases} x(n) - t_{z+1}(n) & for\ n \notin A \\ 0 & for\ n \in A \end{cases}.$$

where

$$t_{z+1}(n) = \sum_{m=0}^{z+1} a_m n^m, \begin{cases} z = 2p & for \quad DST,\ p = 0,1,\ldots \\ z = 2p+1 & for \quad DCT,\ p = 0,1,\ldots \end{cases}.$$

Step 2: Calculate the matrix **S** and (**I-S**).

Step 3: Take the matrices D, A, L and U from (I-S).

Step 4: Set the iterative matrix, denoting itermat, as:

$$itermat \;= \begin{cases} S & for\ Populis-Gerchberg\ \ method \\ D^{-1}A & for\ Jacobi's \qquad method \\ (D-L)^{-1}U & for\ Gauss-Seidel \qquad method \end{cases}.$$

And the iterative vector, denoting *itervec*, as:

$$itervec = \begin{cases} h & for\ Populis-Gerchberg\ \ method \\ D^{-1}h & for\ Jacobi's \qquad method \\ (D-L)^{-1}h & for\ Gauss-Seidel \qquad method \end{cases}.$$

Step 5: Calculate

$$g_{i+1} = itermat \times g_i + itervec.$$

Step 6: If the change of the reconstructed data between *(i)th* and *(i+1)th* iteration is not clear, then go to step7; otherwise, go to Step 5.

Step 7: Perform the postprocess to obtain the reconstructed signal as:

$$x_r(n) = g(n) + t_{z+1}(n) \quad .$$

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 1 Issue 6, August 2014.
www.ijiset.com
ISSN 2348 – 7968

5. Simulation of Examples

The simulation results of the example below are presented in both methodologies: first the proposed algorithm and secondly other approach using the iterative method that can increase the speed of the convergence rapidly.

5.1 Simulation signal

For example, the signal *x(n)* is

$$x(n) = 0.5\sin(n\pi/8) + 0.5\sin(n\pi/4) + 0.5\cos(n\pi/6) + 0.5\cos(n\pi/5)$$

Assume that the signal length equals 29, the number of lost data is set to 10 and the relative index set, ***A***, is given as ***A***={4,6,9,11,13,15,18,20,21,24}

In this paper, we choose the lowpass band all as *π/2*, it is the worst case that we do not have anything about the actual band of the signal.

5.2 The criterion of Mean Square Error

Assuming $g_i(n)$ expresses the reconstructed signal at the *i-th* iteration, the *MSE* between the original and reconstructed signals at the *i-th* iteration, *MSEi*, can be given as

$$MSE_i = \frac{1}{N}\sum_{n=0}^{N-1}\left|x(n) - g_i(n)\right|^2 .$$

Derived from the Parseval's theorem, the Papoulis-Gerchberg algorithm is proved to yield a reduction in *MSE* at each iteration [11].

5.3 Simulation results

In terms of *MSE*, the performances of the various algorithms, applied to restore the missing data, are summarized in Tables 1-2 and Figs. 1-3. As shown in Table 1 (explained by Figs. 1 and 2), the proposed approaches using preprocess with boundary-matched concept always can obtain a nice results. In addition, from Table 2 (explained by Fig.3), we find the Jacobi's and Gauss-Seidel methods rapidly increase the speed of the convergence and the iterative convergence rate is faster than that with the original PG algorithm, especially the Gauss-Seidel method.

Table 1: The *MSE* with and without preprocess method for the PG algorithm

Iteration	1	10	30	100
DST	0.0611	0.0060	0.0017	0.0015
DCT	0.0538	0.0046	0.0011	0.0010
DSTP*	0.0774	0.0038	7.20×10^{-5}	2.17×10^{-5}
DCTP*	2.1134	0.0641	4.14×10^{-4}	2.34×10^{-6}
GWT**	0.0524	0.0043	4.09×10^{-4}	2.72×10^{-4}

* DSTP and DCTP express the preprocessed DST and DCT respectively.
** The GWT naturally satisfies the boundary conditions and no preprocess is needed.

Table 2: The *MSE* of original PG, Jacobi's, and Gauss-Sedibel methods

Iteration	1	10	30	100
GWT	0.0524	0.0043	4.09×10^{-4}	2.72×10^{-4}
GWT_J*	0.0288	0.0073	2.74×10^{-4}	2.72×10^{-4}
GWT_G*	0.0130	2.89×10^{-4}	2.72×10^{-4}	2.72×10^{-4}

*The subscribe indices J and G in Table 2 stand for the Jacobi's method and Gauss-Seidel method respectively.

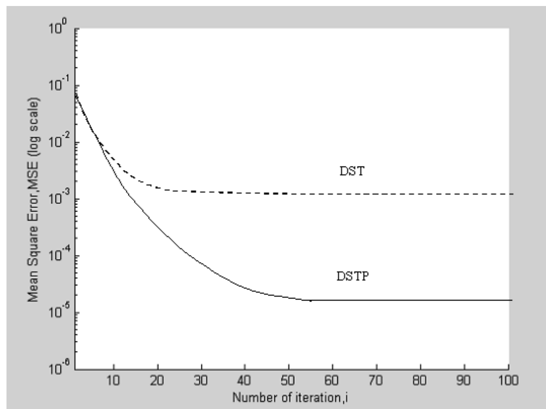

Fig. 1 The results of PG algorithm with and without preprocess for DST.

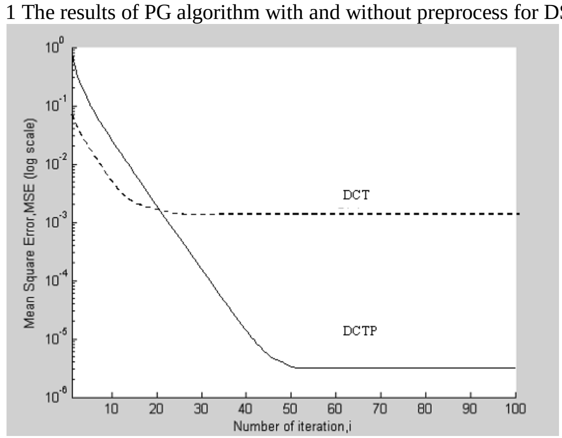

Fig. 2 The results of PG algorithm with and without preprocess for DCT.

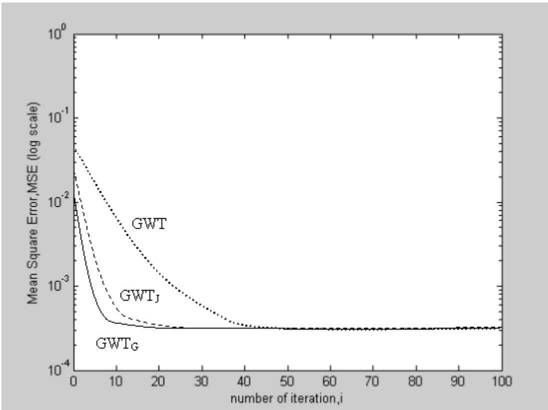

Fig. 3 The results of PG algorithm, Jacobi's and Gauss-Seidel methods with GWT.

6. Conclusions

An innovative and efficient reconstruction algorithm is proposed in this paper. By using the preprocessing and postprocessing technique of boundary-matching signal, a measureable enhancement of real orthogonal transform can be achieved. Our results indicate that the new method yields a better performance than the original method does. In addition, we also perform a number of different iteration methods in an attempt to rapidly increase the speed of the convergence, especially the Gauss-Seidel method. The simulation results indicated that the lost data are nearly reconstructed perfectly and the mean square error can be minimized to 10^{-6}.

The experimental results also indicate that the signal bandwidth is reduced significantly by adapting the proposed preprocessing method, and our iterative procedure also increases the speed of convergence rapidly, which then converges to a very good result especially in the DCT method.

References

[1] F. A. Marvasti, P. M. Clarkson, M. V. Dokic, Ut Goenchanart and Chuande Liu, "Reconstruction of Speech Signals with Lost Samples", *IEEE Transactions on Signal Processing.*, vol. 40, no. 12, 1992, pp. 2897-2903.

[2] A.K. Brodzik, "Signal Extrapolation in the Real Zak Space ", *IEEE Transations on Signal Processing.*, 2002, vol. 50, no. 8, pp. 1857-1964.

[3] B. G. Salomon and H. Ur, "Accelerated Iterative Band-Limited Extrapolation Algorithms", *IEEE Signal Processing Letters.*, vol. 11, no. 11, 2004, pp. 871-874.

[4] Tsung-Ming Lo, Kuang-Shyr Wu, "An Accelerated Iterative Convergence Scheme for PG Algorithm," *ISCA 24th International Conference on Computers and Their Applications (CATA 2009),* Holiday Inn Downtown-Superdome , New Orleans, Louisiana, USA, Session 3B, pp. 87~91.

[5] Marcelino, S.; Assuncao, P.; Soares, S., "Lost block reconstruction in depth maps using color image contours", *Picture Coding Symposium (PCS, 2012),* pp. 253-256.

[6] R. J. Marks, II, "Restoring lost samples from an oversampled band-limited signal", *IEEE Trans. Acoustic. Speech, and Signal Process.*, vol. ASSP-31, no. 3, 1983, pp.752-755.

[7] C. Y. Hsu and Y. F. Liou, "Fast discrete extrapolation via the fast Hartley transform", *IEEE Trans. Circuits and Syst.–II,* vol. 40, no. 8, 1993, pp.502-504.

[8] Paulo Jorge S. G. Ferreira, "Noniterative and fast iterative methods for interpolation and extrapolation", *IEEE Trans. Acoustic. Signal Process.*, vol. 42, no. 11, 1994, pp. 3278-3282.

[9] C. Y. Hsu and C. M. Lo, "Restoring lost samples with boundary matched", *Intelligent Signal Processing and Communication Systems, ISPACS 2005*. Proceedings of 2005 International Symposium, pp.565-568.

[10] R. J. Marks II, *Advances in Shannon Sampling*. Berlin, Vienna, New York: Springer-Verlag, 1993.

[11] A. Papoulis, *Signal Analysis.* New York: McGraw-Hill, 1977.

[12] Chau-Yun Hsu and Tsung-Ming Lo, "Signal Reconstruction with Boundary-matching via Iterative Algorithm," *IEICE Trans. on Fundamentals of Electronics, Communications and Computer Sciences*, vol. E89-A, no.11, 2006, pp.3283~3289.

[13] Z. Wang, J.J. Soltis and W.C. Miller, "Improved approach to interpolation using the FFT", *Elect. Lett.*, vol. 28, no. 25, 1992, pp.2320-2322.

[14] Y. Wu, Z. Zhu, "A neural net approach to DCT-I, DST-I, and DFT", *Aerospace and Electronics Conference, 1994. NAECON 1994.*, Proceedings of the IEEE 1994 National, vol.1, pp.119 – 122.

[15] Z. Wang, G. A. Jullien and W. C. Miller, "The generalized discrete W transform and its application to interpolation", *Signals, Systems and Computers, 1992 Conference.*, vol.1, pp. 241-245.

Tsung-MingLo received the M.E. degree in 1998and the D.E. degree in2006, both from Tatung University, Taiwan. From 1989 to 2007, he was a scientist working on the researches and developments in electronic warfare systems in Chung-Shan Institute of Science and Technology. He is currently an Assistant Professor in China University of Technology. His research interests include digital signal processing and wireless communication. Dr. Lo is a member of IEICE.