An Energy Efficient Methodology For Cognitive Radio Networks

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Abstract—We propose continuous power allocation strategies for secondary users (SUs) based on sensing the primary user (PU) channels in a multiband cognitive radio (CR) network. Unlike the conventional sensing-based spectrum sharing, where there are two transmit power levels corresponding to whether the PU is sensed present or not, in the proposed strategy, the power levels are continuous functions of the sensing statistics, and optimized with respect to the achievable rate of the SU. The continuous power allocation function is parameterized by some channel parameters of the PU and SU, and we treat the cases of perfect and quantized channel state information (CSI) separately, where the former provides an upper bound on the achievable rate with full channel information; and the latter constitutes an efficient practical power allocation method for the SU with statistic/partial channel information. The power control process consists of two phases: in the first phase, the SU listens to the multiple bands licensed to the PU and obtains the sensing statistics, e.g., the received signal energies on these bands; in the second phase, the SU adjusts its transmit power levels on these bands based on the sensing results. Optimal power allocation schemes are derived to maximize the achievable rate at the SU under several possible combinations of the peak/average transmit power constraints at the SU and the peak/average interference power constraints at the PU. Simulation results demonstrate that the proposed strategies can significantly improve the achievable throughput of the SU compared to the conventional methods. Index Terms—Cognitive radio (CR), power allocation, multi- band spectrum sensing, underlay, opportunistic spectrum access, channel quantization.

I. INTRODUCTION

As a promising solution to the spectrum scarcity problem, the cognitive radio (CR) system has received much attention lately [1], where the secondary user (SU) can access the primary bands without degrading the quality of service (QoS) of the primary user (PU). Currently, there exist three main spectrum access approaches for CR: i) Underlay or the so-called spectrum sharing scheme, where the SU is allowed to coexist with the PU as long as the QoS of the PU is protected [2]-[4]; ii) Opportunistic spectrum access, where the SU can only access the primary bands that are detected to be idle [5]-[7]; and iii) the combination of the above two, i.e., sensing-based spectrum sharing, where the SU first senses the frequency spectrum to determine the status of the PU and then chooses its transmit power based on the decision [8]-[12]. According to both theoretical analysis and simulations, sensing-based spectrum sharing achieves the maximum secondary rate under a given interference constraint to the PU [13]. This approach consists of two phases: sensing and data transmission. During the sensing slot, the SU performs spectrum sensing and determines whether the PU is absent or not. During data transmission, the SU
accesses the primary band with a high transmit power when the PU is determined to be absent and with a low transmit power otherwise, in order to control the interference caused to the PU. This approach was first proposed in [11] for a single band channel case, for which the optimal sensing time and power levels were obtained. The work was extended in [13] to the multiband case under the average transmit power and average interference power constraints. Clearly, all three spectrum access approaches essentially adopt either constant or binary power allocation. However, such strategies are by no means optimal, and by allowing the transmit power to be continuous, the achievable rate can be significantly increased. In this paper, we propose a continuous power allocation framework based on the SU’s sensing statistics in CR networks which provides an upper bound on the achievable rate of the SU. Such a continuous power control function is parameterized by the channel gains of the primary and secondary links. Since in practice, the channels gains are feedback to the transmitter through some low-rate control channels, to arrive at a practical power allocation scheme, we quantize the channel gains and design the power control functions based on the quantized channel gains. The conventional constant or binary power allocations are special cases of the proposed strategies. The power allocation process is composed of two slots, namely sensing slot and transmission slot. In the first slot, some sensing statistics about the PU are collected based on which the transmit power is determined; in the second slot, the SU transmits data using the power level obtained in the sensing slot. Using the convex optimization tools, under several possible combinations of the peak/average transmit power constraints at the SU and the peak/average interference power constraints at the PU, the optimal power allocation functions are derived to maximize the average achievable rate at the SU. The remainder of this paper is organized as follows. Section II describes the system model and summarizes the conventional power allocation strategies. The proposed continuous power allocation schemes under both perfect and quantized CSI are developed in Section III and Section IV, respectively. Simulation results are given in Section V and conclusions are drawn in Section VI.

![System model for the SU under sensing-based spectrum access.](image)

**II. SYSTEM MODEL AND BACKGROUND**

**A. System Model**

Consider a CR network with a pair of primary transmitter and receiver, and a pair of secondary transmitter and receiver which opportunistically access the primary multiband channel under a given constraint of interference level to the PU as depicted in Fig.1. Similar to [9], [13], we assume that the number of multiband channels is M, and the channels are orthogonal narrowbands. Let $\gamma_{1,j}$, $\gamma_{2,j}$, $h_j$, and $g_j$
denote the instantaneous channel power gains (i.e., squared magnitudes of the complex channel gains) of channel \( j \) from the primary transmitter (PU-Tx) to the secondary transmitter (SU-Tx), from PU-Tx to the secondary receiver (SU-Rx), from SU-Tx to the primary receiver (PU-Rx) and from SU-Tx to SU-Rx, respectively. The frame structure is based on the conventional two-phase model, namely sensing slot with duration \( \tau \) and transmission slot with duration \( T - \tau \), as shown in Fig. 2. During the sensing slot, the SU-Tx listens to all the \( M \) narrowbands and obtains their accumulated energies. In the conventional schemes, spectrum sensing is performed in this slot and the decisions on the status (active/idle) of the channels are made. When transmitting, the SU-Tx accesses the primary multiband with the optimal powers decided by the accumulated energies in order to meet the interference constraint at the PU-Rx. We assume that PU is either present or absent throughout the entire sensing phase. The \( i \)-th received signal sample at channel \( j \), \( r_{i,j} \), is modeled as

\[
r_{i,j} = \begin{cases} 
    n_{i,j}, & \text{if } H_{0,j} \text{ is true} \\
    \sqrt{\gamma_{1,j}} s_{i,j} + n_{i,j}, & \text{if } H_{1,j} \text{ is true}
\end{cases}
\tag{1}
\]

where the hypothesis \( H_{0,j} \) and \( H_{1,j} \) corresponds to idle and busy channel \( j \) respectively; \( s_{i,j} \) is the \( i \)-th symbol transmitted from PU-Tx in channel \( j \) which is assumed to follow a circularly symmetric complex Gaussian distribution with zero mean and variance \( P_{s,j} \), i.e., \( s_{i,j} \sim \mathcal{N}(0, P_{s,j}) \); and \( n_{i,j} \sim \mathcal{N}(0, N_0) \) is the additive noise. Assume that \( s_{i,j} \) and \( n_{i,j} \) are independent of each other.

![Frame Structure of the Cognitive Radio Network](image)

The detection statistic \( x_j \) for channel \( j \) based on the accumulated received signal energy can be written as

\[
x_j = \sum_{i=1}^{f_s} |r_{i,j}|^2
\tag{2}
\]

where \( f_s \) is the sampling frequency at SU-Tx. Then the probability density functions (pdf) of \( x_j \) conditioned on \( H_{0,j} \) and \( H_{1,j} \) are given by [14]

\[
f_0(x_j) \triangleq f(x_j|H_{0,j}) = \frac{x_j^{\tau f_s - 1} e^{-\frac{x_j}{\tau f_s}}}{\Gamma(\tau f_s)(N_0)^{\tau f_s}},
\tag{3}
\]

and

\[
f_1(x_j) \triangleq f(x_j|H_{1,j}) = \frac{x_j^{\tau f_s - 1} e^{-\frac{x_j}{\tau f_s}}}{\Gamma(\tau f_s)(N_0 + \gamma_{1,j} P_{s,j})^{\tau f_s}},
\tag{4}
\]
respectively, where \( \Gamma(.,.) \) denotes the gamma function.

**B. Conventional Sensing-based Power Allocation Strategies**

In the conventional sensing-based power allocation schemes, using energy detection as the decision rule, the SU-Tx compares \( x_j \) to a threshold \( \theta_j \) to decide whether channel \( j \) is occupied by the PU or not. Thus the decision rule is given by

\[
x_j \begin{cases} 
H_1,j & \text{if } x_j \geq \theta_j \\
H_0,j & \text{otherwise}
\end{cases}
\]

\( \theta_j \), and the probabilities of false alarm and detection

for channel \( j \) can be obtained as

\[
q_{f,j} = \int_{\theta_j}^{\infty} f_0(x_j) dx_j = \frac{\Gamma(\tau f_s, \frac{\theta_j}{N_0})}{\Gamma(\tau f_s)}, \quad (5)
\]

and

\[
q_{d,j} = \int_{0}^{\theta_j} f_1(x_j) dx_j = \frac{\Gamma(\tau f_s, \frac{\theta_j}{N_0 \gamma_{s,j} P_{s,j}})}{\Gamma(\tau f_s)}, \quad (6)
\]

respectively, where \( \Gamma(.,.) \) denotes the upper incomplete gamma function. Sensing-based spectrum sharing: The SU-Tx adapts its transmit power based on the decision made during the sensing slot. If channel \( j \) is detected to be absent, the SU-Tx will transmit with high power \( P_0,j \), otherwise, with low power \( P_1,j \). Thus the average achievable rate of the SU-Tx is given by

\[
R_{old} = \frac{T}{\tau} \sum_{j=1}^{M} \left\{ p_{0,j} (1 - q_{f,j}) \log_2 \left( 1 + \frac{g_j P_0,j}{N_0} \right) + p_{1,j} \right\} \log_2 \left( 1 + \frac{g_j P_1,j}{N_0 + \gamma_{s,j} P_{s,j}} \right)
\]

where \( p_{0,j} \) and \( p_{1,j} = 1 - p_{0,j} \) are the idle and busy probabilities of channel \( j \) respectively. Then, the problem of maximizing the average achievable rate of SU-Tx under the power constraints can be formulated as
where denotes a feasible set of power constraints to satisfy the QoS of PU-Tx [6]-[9]. Opportunistic spectrum access: Different from the sensing-based spectrum sharing approach, when transmitting, if channel j is detected to be busy, the SU-Tx will not use this channel \( P_1 j = 0 \). A pre-defined threshold on the detection probability \( q_{th d,j} \) is chosen to protect the PU. The optimization problem can be formulated similar to (8) by substituting \( P_1 j = 0 \) and adding \( q_{d,j} \geq q_{th d,j} \). Underlay: The SU can coexist with the PU under the condition of meeting the QoS of the PU without sensing. Thus the problem can be formulated similar to (8) by substituting \( \tau = 0 \) and \( P_0 j = P_1 j \).

### III. SENSING-BASED CONTINUOUS POWER ALLOCATION - PERFECT CSI CASE

From the discussions in Section II-B, we conclude that, the underlay approach adopts a constant transmit power, whereas both the opportunistic spectrum access and the sensing-based spectrum sharing employ two different powers corresponding to the two types of channel status (busy/idle). In this paper, we propose to adapt the transmit power continuously with respect to the spectrum sensing variable \( x_j \) in (2). In this section, we derive the optimal continuous power allocation strategies under the assumption of perfect CSI, i.e., the channel parameters \( \gamma_1,j,\gamma_2,j,g_j \) and \( h_j \) in Fig.1 are assumed to be ergodic, stationary and known at the SU. Define the transmit power for channel j as a function of the received signal energy \( x_j \), i.e., \( P(x_j) \), and obviously it satisfies the non-negative constraint as

\[
P(x_j) \geq 0, \forall x_j.
\]  

(9)

Then the conventional power allocation rules become the special cases. Specifically, for the sensing-based spectrum sharing or opportunistic spectrum access, it has the following form

\[
P(x_j) = \begin{cases} 
P_0^j, & x_j < \theta_j, \\
0 \text{ or } P_1^j, & x_j \geq \theta_j,
\end{cases}
\]  

(10)

whereas for the underlay approach, \( P(x_j) \) is a constant.

**A. Achievable Rate, Power Constraints and Problem Formulation**

The instantaneous rates of the SU given \( x_j \) for the cases of idle and busy channel j, are given by

\[
R_0(x_j) = \log_2 \left( 1 + \frac{P(x_j)g_j}{N_0} \right),
\]  

(11)

\[
R_1(x_j) = \log_2 \left( 1 + \frac{P(x_j)g_j}{N_0 + \gamma_2,jP_s,j} \right),
\]  

(12)
respectively. Then the average throughput of the SU under the continuous sensing-based power allocation can be written as

$$R = \frac{T}{\tau} \sum_{j=1}^{M} \int_{0}^{\infty} [p_{0,j}R_{0}(x_{j}) f_{0}(x_{j}) + p_{1,j}R_{1}(x_{j}) f_{1}(x_{j})] dx_{j}. \quad (13)$$

In practice, due to the nonlinearity of the power amplifiers, the peak transmit power has to be constrained. Let $\hat{P}_{j}$ be the maximum peak transmit power for channel $j$. Then we have

$$P(x_{j}) \leq \hat{P}_{j}, \forall x_{j}, j. \quad (14)$$

In order to meet the long-term power budget of the SU, an average transmit power constraint should also be considered, which can be written as

$$\frac{T}{\tau} \sum_{j=1}^{M} \int_{0}^{\infty} P(x_{j}) [p_{0,j}f_{0}(x_{j}) + p_{1,j}f_{1}(x_{j})] dx_{j} \leq \bar{P}. \quad (15)$$

Furthermore, to protect the instantaneous QoS of the PU, peak interference power has to be constrained as

$$h_{j}P(x_{j}) \leq \hat{I}_{j}, \forall x_{j}, j, \quad (16)$$

where $\hat{I}_{j}$ is the peak interference power level in channel $j$ that is tolerable by the PU. To protect the long-term QoS of the PU, an average interference power constraint should be imposed. Under the proposed continuous power allocation, the interference is caused when the PU is present but the SU still transmits with power $P(x_{j})$. Setting the maximum average interference power as $\bar{I}_{j}$, the average interference power constraint can be formulated as

$$\frac{T}{\tau} \int_{0}^{\infty} p_{1,j}h_{j}P(x_{j}) f_{1}(x_{j}) dx_{j} \leq \bar{I}_{j}, \forall j. \quad (17)$$

Finally, the problem of maximizing the average achievable rate of the SU-Tx under the power constraints can be formulated as

$$\max_{\tau \{P(x_{j})\} \in F'} R \quad \text{s.t.} \quad (9), \quad 0 \leq \tau \leq T, \quad (18)$$

where $F'$ is the feasible set specified by a particular combination of the power constraints (14)-(17). Since different types of transmit and interference power constraints may apply at the same time for SU and PU due to the practical requirements, we consider all possible combinations of the power constraints similar
to [2], [10]. The following lemma is instrumental to solving (18). Lemma 1: Problem (18) is concave with respect to the transmit power \( P(x_j) \) under any combination of constraints of (14)-(17).

\[
\text{Proof. The proof is trivial since } \frac{\partial^2 R}{\partial P(x_j)} < 0,
\]

and the constraints (14)-(17) are all convex over \( P(x_j) \), thus under any combination of the constraints, problem (18) is concave over \( P(x_j) \).

Note that in general the rate \( R \) in (18) is a highly nonlinear and non-convex function of \( \tau \) and therefore there is no efficient way of optimizing over \( \tau \). Following [6], [13], we will simply use a one-dimensional exhaustive search within the interval \([0, T]\) to find the optimal \( \tau \). Thus in the following, we focus on finding the optimal power allocations.

**B. Average Transmit Power and Average Interference Power Constraints**

Consider the constraints of average transmit power at SU and average interference power at PU, thus the constraints become (15) and (17). First we write the Lagrangian \( \mathcal{L}(P(x_j), \lambda, \mu) \) for problem (18) under the constraints (15) and (17) as

\[
\mathcal{L}(P(x_j), \lambda, \mu) = R + \lambda \left( P - \frac{T - \tau}{T} \sum_{j=1}^{M} \int_{0}^{\infty} P(x_j) \left[ p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j) \right] dx_j \right),
\]

\[
+ \sum_{j=1}^{M} \mu_j \left( I_j - \frac{T - \tau}{T} \int_{0}^{\infty} p_{1,j} h_j P(x_j) f_1(x_j) dx_j \right) \tag{19}
\]

where \( \lambda, \mu \geq 0 \) are dual variables corresponding to (15) and (17). Define the Lagrange dual function \( g(\lambda, \mu) \) corresponding to problem (18). Then we can build the dual optimization problem as

\[
\min_{\lambda \geq 0, \mu \geq 0} g(\lambda, \mu) = \sup_{P(x_j) \geq 0} \mathcal{L}(P(x_j), \lambda, \mu). \tag{20}
\]

It follows from Lemma 1 that, the optimal value of problem (20) is equal to that of problem (18). Thus we can solve the dual optimization problem (20) instead of solving (18). In (20), we have to obtain the supremum of \( \mathcal{L}(P(x_j), \lambda, \mu) \). To find the optimal \( P(x_j) \) for each \( x_j \), we take the derivative of \( \mathcal{L}(P(x_j), \lambda, \mu) \) with respect to \( P(x_j) \), which can be obtained as
By setting the above equation to 0 and applying the constraint (9), the optimal power allocation $P(x_j)$ for the given Lagrange multipliers $\lambda$ and $\mu$ is given by

$$P(x_j) = \left[ \frac{A_{j,0} + \sqrt{\Delta_{j,0}}}{2} \right]$$

where $[x]^+ \triangleq \max(0, x)$, and

$$A_{j,0} = \frac{\log_2(e) [p_{0,j}f_0(x_j) + p_{1,j}f_1(x_j)]}{\frac{\lambda [p_{0,j}f_0(x_j) + p_{1,j}f_1(x_j)] + \mu_j p_{1,j}h_j f_1(x_j)}{2N_0 + \gamma_{2,j} P_{s,j}} - \frac{\log_2(e) j,0}{g_j}}$$

$$\Delta_{j,0} = A_{j,0}^2 + \frac{4}{g_j} \left\{ \frac{N_0 (N_0 + \gamma_{2,j} P_{s,j})}{g_j} + \frac{\log_2(e) [p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)] (N_0 + \gamma_{2,j} P_{s,j}) + p_{1,j} f(x_j) N_0}{\lambda [p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)] + \mu_j p_{1,j} h_j f_1(x_j)} \right\}.$$ 

Proposition 1: $P(x_j)$ is a non-increasing function with respect to $x_j$.

Proof. From (3), we have

$$f_1(x_j) = e^{-\frac{p_{0,j} f_0(x_j)^2}{N_0} \frac{N_0}{N_0 + \gamma_{1,j} P_{s,j}}} \left( \frac{N_0}{N_0 + \gamma_{1,j} P_{s,j}} \right)^{\tau f_s},$$

and obviously it is an increasing function over $x_j$. Through some simple manipulations, the monotonicity of $A_{j,0}$ is equivalent to the monotonicity of the following term

$$C_j = \frac{1 + \frac{p_{1,j} f_1(x_j)}{p_{0,j} f_0(x_j)}}{1 + \frac{\mu_j h_j (p_{1,j} f_1(x_j))}{p_{0,j} f_0(x_j)}}.$$ 

Using (25) and (26), it follows that $A_{j,0}$ is a decreasing function of $x_j$. Similarly, we can also prove that $j,0$ is a decreasing function of $x_j$. Thus from (22), we can conclude that $P(x_j)$ is a non-increasing function with respect to $x_j$. 

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Remark: An interpretation of Proposition 1 is that, with a smaller \( x_j \), the probability that channel \( j \) is busy is smaller, thus the SU can transmit at higher power to effectively use the primary band. On the other hand, with a larger \( x_j \), lower transmit power should be used to avoid harmful interference to the PU. Next the optimal values of the Lagrange multipliers \( \lambda \) and \( \mu \) need to be found to obtain the optimal power allocation strategy \( P(x_j) \). Subgradient based methods are used here to find the optimal solution, e.g., the ellipsoid method [17] or the gradient descent method[18]. In particular, for the gradient descent method, the Lagrange multipliers are updated according to the following

\[
\lambda_{\text{new}} = \lambda_{\text{old}} + t_1 c,
\]
\[
\mu_{\text{new}} = \mu_{\text{old}} + t_2 v,
\]

where \( t_1 > 0 \) and \( t_2 > 0 \) are step-size parameters and the subgradients \( c \) and \( v \) are given by the following proposition. Proposition 2: The subgradient of the Lagrange dual function \( g(\lambda, \mu) \) is \([c, v]\), where

\[
e - \hat{P} - \frac{T - \tau}{T} \sum_{j=1}^{M} \int_{0}^{\infty} P'(x_j) [p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)] dx_j,
\]
\[
v_j = I_j - \frac{T - \tau}{T} \int_{0}^{\infty} p_{1,j} h_j P'(x_j) f_1(x_j) dx_j,
\]

and \( P(x_j) \) is the optimal power allocation for fixed \( \lambda \) and \( \mu \), given by (22)-(24).

Proof. Denote \( \tilde{\lambda} \geq 0 \) and \( \tilde{\mu} \geq 0 \) as any feasible values of the dual function \( g(\lambda, \mu) \), and \( \tilde{P}(x_j) \) as the corresponding optimal power allocation. It then suffices to show that

\[
g(\tilde{\lambda}, \tilde{\mu}) \geq g(\lambda, \mu) + ([\tilde{\lambda}, \tilde{\mu}] - [\lambda, \mu]) [c, v]^T
\]
holds for any \( \tilde{\lambda} \) and \( \tilde{\mu} \). We have

\[
g(\tilde{\lambda}, \tilde{\mu}) = \sup_{P(x_j) \geq 0} L(P(x_j), \tilde{\lambda}, \tilde{\mu}) = L(\tilde{P}(x_j), \tilde{\lambda}, \tilde{\mu})
\]
\[
\geq L(P(x_j), \lambda, \mu) - [\tilde{\lambda}, \tilde{\mu}] - [\lambda, \mu]) [c, v]^T
\]
\[
- g(\lambda, \mu) + ([\tilde{\lambda}, \tilde{\mu}] - [\lambda, \mu]) [c, v]^T.
\]

C. Peak Transmit Power and Peak Interference Power Constraints

In this case, the constraints become (14) and (16), which can be combined as \( P(x_j) \leq \min\{\hat{P}_j, \hat{I}_j/\hat{h}_j\} \). Therefore, the rate is maximized by transmitting at the maximum instantaneous power given by

\[
P(x_j) = \begin{cases} 
\hat{P}_j, & \text{if } h_j \leq \hat{I}_j/\hat{P}_j, \\
\hat{I}_j/\hat{h}_j, & \text{otherwise.}
\end{cases}
\]

From (30), we can see that, under the constraints of peak transmit power and peak interference power, the transmit power is independent of the received energy and is the same as that of the conventional power allocation strategies. When \( \hat{h}_j \) is below a given threshold, the SU-Tx can transmit with its maximum power \( \hat{P}_j \), which meets the interference power constraint at PU-Rx. This implies that deep fading
between SU-Tx and PU-Rx is good for both protecting the primary transmission and optimizing the secondary transmission. Substituting (30) into (18), the optimal sensing time is \( \tau = 0 \) which means that spectrum sensing is not necessary in this case.

**D. Peak Transmit Power and Average Interference Power Constraints**

In this case, the constraints become (14) and (17). The optimal power allocation is given by the following lemma. Lemma 2: The optimal solution to (18) subject to the power constraints (14) and (17) is

\[
P(x_j) = \begin{cases} \hat{P}_j, & \frac{T - \tau}{T} p_{1,j} h_j \hat{P}_j \leq \tilde{I}_j, \\ f\left(\frac{A_{j,1} + \sqrt{\Delta_{j,1}}}{2}, 0, \hat{P}_j\right), & \text{otherwise,} \end{cases}
\]

where the function \( f(x, [a, b]) \) is defined as

\[
f(x, [a, b]) = \begin{cases} a, & x < a, \\ x, & a \leq x \leq b, \\ b, & x > b, \end{cases}
\]

and

\[
A_{j,1} = \frac{\log_2(e) p_{0,j} f_0(x_j)}{\mu_j h_j p_{1,j} f_1(x_j)} + \frac{\log_2(e)}{\mu_j} - \frac{2N_0 + \gamma_{2,j} P_{s,j}}{g_j},
\]

\[
\Delta_{j,1} = A_{j,1}^2 + 4 \left\{ \frac{\log_2(e) p_{0,j} f_0(x_j)(N_0 + \gamma_{2,j} P_{s,j})}{\mu_j h_j p_{1,j} f_1(x_j)} \right\} \
+ \frac{\log_2(e) N_0}{\mu_j} - \frac{N_0(N_0 + \gamma_{2,j} P_{s,j})}{g_j},
\]

and \( \mu_j \) is the nonnegative dual variable associated with the constraint (17).

**Proof.** To cases have to be considered in order to prove this lemma

**Case 1:** (17) in this problem is satisfied with strict inequality, i.e., for any

\[
P(x_j) \in [0, \hat{P}_j],
\]

(17) holds inequality. Thus in this case, we get

\[
\frac{T - \tau}{T} p_{1,j} h_j \hat{P}_j < \tilde{I}_j,
\]

and in order to maximize the objective function, we should set \( P(x_j) = \hat{P}_j \). Case 2: (17) in this problem is satisfied with equality. In this case, first consider the objective function with the constraint (17). Following the same steps as in Section III-B, the corresponding solution is

\[
P(x_j) = \frac{\hat{A}_{j,1}^{\tau} + \sqrt{\Delta_{j,1}}}{2}. \text{ Together with the constraint } 0 \leq P(x_j) \leq \hat{P}_j, \text{ the solution becomes}
\]

\[
P(x_j) = f\left(\frac{\hat{A}_{j,1}^{\tau} + \sqrt{\Delta_{j,1}}}{2}, 0, \hat{P}_j\right)
\]
Subgradient based methods are also used here to obtain the optimal $\mu_j$. Since $f_1(x_j)/f_0(x_j)$ is an increasing function over $x_j$ as shown in (25), it follows that in this case, the power allocation function $P(x_j)$ is also a non-increasing function over $x_j$. It is seen from (31) that, when $\bar{I}_j$ is large enough, the SU can always transmit with its peak power. Otherwise, when the received signal energy $x_j$ is small, the SU will use its peak power, and when $x_j$ is large, it will use the power $\frac{A_{j,1} + \sqrt{\Delta_{j,1}}}{2}$.

E. Average Transmit Power and Peak Interference Power Constraints

In this case, the constraints become (15) and (16). The optimal solution to this problem can be obtained in a similar way as in the proof of Lemma 2, which can be expressed as

$$ P(x_j) = \begin{cases} \frac{f_j}{h_j}, & \text{if } f(\frac{A_{j,2} + \sqrt{\Delta_{j,2}}}{2}, [0, \frac{f_j}{h_j}]), \text{ otherwise,} \end{cases} $$

where

$$ A_{j,2} = \frac{\log_2(e)}{\lambda} - \frac{2N_0 + \gamma_{2,j}P_{s,j}}{g_j}, $$

$$ \Delta_{j,2} = A_{j,2}^2 + \frac{4}{g_j} \left\{ \frac{\log_2(e)\gamma_{2,j}P_{s,j}p_{0,j}f_0(x_j)}{\lambda[p_{0,j}f_0(x_j) + p_{1,j}f_1(x_j)]} + \frac{\log_2(e)N_0}{g_j} \right\}, $$

and $\lambda$ is the nonnegative dual variable associated with the constraint (16). Thus to solve this problem, it holds, we have $P(x_j) = \frac{f_j}{h_j}$ for all $x_j$ for all $x_j$. Otherwise, the subgradient based method can be used to get the optimal $\lambda$ and the corresponding $P(x_j)$. Moreover, it follows from (25) and (35) that, $P(x_j)$ is a non-increasing function over $x_j$. Finally, in Table I, we summarize the algorithm that computes the optimal sensing time and continuous power allocation function for sensing-based multiband spectrum sharing with perfect CSI.

IV. CONTINUOUS SENSING-BASED POWER ALLOCATION - QUANTIZED CSI CASE

Note that the continuous power allocation function $P(x_j)$ obtained in Section III is parameterized by the primary and secondary channel power gains $\gamma_{1,j}, \gamma_{2,j}, g_j$ and $h_j$ shown

TABLE I

Algorithm 1: Optimal sensing time and continuous power allocation function for sensing-based multiband spectrum sharing with perfect CSI. Under peak & peak constraints, get optimal $P(x_j)$ using (30) and $\tau = 0$. Under other constraints, for each $\tau$ in $[0,T]$, do. 1) Initialize $\lambda$ and $\mu$. 2) Repeat: - For each channel $j$, compute $P(x_j)$ using (22), (31) or (35). - Update $\lambda$ and $\mu$ using (27). 3) Until $\lambda$ and $\mu$ converge. End for. Optimal sensing time and power allocation function: $\tau^* = \arg \max \tau \ R(\tau, P(x_j))$, $P^*(x_j) = P(x_j)|_{\tau = \tau^*}$. 

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in Fig. 1. In practice, \( \gamma_{1,j} \) can be estimated locally at SU-Tx. However, \( \gamma_{2,j} \) and \( g_j \) are estimated at SU-Rx, and \( h_j \) is estimated at PU-Rx. These three parameters need to be feedback to SU-Tx through some low-rate control channels. Thus they have to be quantized for practical implementations. In this section, we consider the case of quantized CSI and obtain a finite set of continuous power allocation functions corresponding to the set of quantized CSI values.

### A. Proposed Continuous Power Allocation Strategy with Quantized CSI

The quantizer design is the same for different channel \( j \). Therefore, in this section, we will drop the channel index subscript \( j \) from all parameters for notational clarity. We assume that the PU-Rx employs a quantizer \( Q_h \) for \( h \), that partitions its possible values into \( Q_h \) subsets \( H_1, \ldots, H_{Q_h} \); and the SU-Rx employs a two-dimensional quantizer \( Q_g \) for \( (g, \gamma_2) \), that partitions its possible values into \( Q_g \) subsets \( G_1, \ldots, G_{Q_g} \). Denote \( Q_h(h) \) and \( Q_g(g, \gamma_2) \) as the corresponding indices of the quantized channels. Then we need to design a total number of \( Q_h Q_g \) continuous power allocation functions \( P_{k}(x) \), \( k = 1, \ldots, Q_h, k = 1, \ldots, Q_g \), so that corresponding to the channel values \( (h, g, \gamma_2) \), we will use the power allocation function \( P_{Q_h(h), Q_g(g, \gamma_2)}(x) \). Note that, the special case \( Q_h = 1 \) corresponds to when there is no feedback link between PU-Rx and SU-Tx, and only some statistical information of \( h \) is available at SU-Tx.

### B. Achievable Rate, Power Constraints and Problem Formulation

The instantaneous rates of the SU given \( (h, g, \gamma_2, x) \) when the PU is absent and present, are given by

\[
R_0(h, g, \gamma_2, x) = \log_2 \left( 1 + \frac{P_{Q_h(h), Q_g(g, \gamma_2)}(x) g}{N_0} \right),
\]

and

\[
R_1(h, g, \gamma_2, x) = \log_2 \left( 1 + \frac{P_{Q_h(h), Q_g(g, \gamma_2)}(x) g}{N_0 + \gamma_2 P_s} \right),
\]

respectively. Note that from (3) and (4), the conditional pdfs of \( x \), \( f_0(x) \) and \( f_1(x) \), are independent of the channel parameters \( (h, g, \gamma_2) \). Similar to (13), the average throughput of the SU under the continuous power allocation strategy with quantized CSI can be written as

\[
R_{\text{quan}} = \sum_{\ell=1}^{Q_h} \sum_{k=1}^{Q_g} \mathbb{E}_{G_k} \left\{ P_0 \int_0^\infty \log_2 \left( 1 + \frac{P_{\ell, k}(x) g}{N_0} \right) f_0(x) dx + P_1 \int_0^\infty \log_2 \left( 1 + \frac{P_{\ell, k}(x) g}{N_0 + \gamma_2 P_s} \right) f_1(x) dx \right\} p(\ell, k),
\]

where \( \mathbb{E}_{G_k} \) denotes the expectation over \( (g, \gamma_2) \in G_k \), and

\[
p(\ell, k) \triangleq \Pr(h \in H_{\ell}, (g, \gamma_2) \in G_k).
\]

Then the constraints of the peak/average transmit power at the SU and peak/average interference power at the PU can be correspondingly formulated as
Finally, the problem of finding the quantizers $Q_h$ and $Q_g$, as well as a power allocation strategy $\{P_{k(x)}\}$ to maximize the average throughput of the SU under the power constraints, can be formulated as

$$\min_{\tau, \{P_{k(x)}, Q_h, Q_g\} \in F''} -R_{\text{quan}}$$

$$\text{s.t.} \quad 0 \leq \tau \leq T, \quad P_{\ell,k}(x) \geq 0, \; \forall \ell, k, x,$$

where $F''$ is the feasible set specified by a particular combination of the power constraints (41)-(44). As before, one-dimensional exhaustive search within the interval $[0,T]$ is performed to find the optimal $\tau$.

C. Average Transmit Power and Average Interference Power Constraints

The constraints become (42) and (44) in this case. Then the Lloyd’s algorithm can be employed to solve the optimization problem which is an algorithm for grouping data points into a given number of categories. Specifically, starting from a feasible solution as the initial value, we repeat the following two steps until convergence:

1) determine the power allocation functions $\{P_{k(x)}\}$ given the quantizers $Q_h$ and $Q_g$;

2) determine the quantizers $Q_h$ and $Q_g$ given the set of power allocation functions $\{P_{k(x)}\}$.

Quantizer design: First, we illustrate that the design of the optimal quantizers $Q_h$ and $Q_g$ is equivalent to a vector quantizer design with a modified distortion measure [16].

Incorporating the power constraints by using the Lagrange multipliers, we define the following distortion measure for optimizing the rate

$$R_{h,g,\gamma_2}(P_{\ell,k}(x)) = p_0 \int_0^\infty \log_2 \left( 1 + \frac{P_{\ell,k}(x)g}{N_0} \right) f_0(x) dx$$

$$+ p_1 \int_0^\infty \log_2 \left( 1 + \frac{P_{\ell,k}(x)g}{N_0 + \gamma_2 P_s} \right) f_1(x) dx$$

$$- \mu p_1 \int_0^\infty hP_{\ell,k}(x)f_1(x) dx$$

$$- \lambda \int_0^\infty P_{\ell,k}(x)[p_0f_0(x) + p_1f_1(x)] dx.$$

The optimization problem in (45) is equivalent to selecting $Q_h$, $Q_g$ and $\{P_{k(x)}\}$ to maximize the average distortion $R_{\text{quan}}$ given by
The optimal quantizers \( Q_h \) and \( Q_g \) are then determined by the farthest neighbor rule as

\[
R_{\text{quan}} = \frac{T - T}{T} \sum_{\ell=1}^{Q_h} \sum_{k=1}^{Q_g} \mathbb{E}_{H_\ell,G_k} \{ R_{h,\gamma_2}(P_{\ell,k}(x)) \} p(\ell, k),
\]

where \( \mathbb{E}_{H_\ell,G_k} \) denotes the expectation over \( h \in H_\ell, (g, \gamma_2) \in G_k \).

By the definitions of distortion measure and farthest neighbor rule, a point of \( h \) is partitioned to \( H \) if \( P, k(x), k=1,...,Q_g \) make the distortion measure largest. It is seen from (48) that the design of the two quantizers are coupled; thus we need to iterate by fixing one and optimizing the other. The following lemma is instrumental to obtaining the optimal quantizers of \( h \).

**Lemma 3:** For \( h_1 < h_2 < h_3 \), if \( h_1 \in H_{\ell_1}, h_2 \in H_{\ell_2} \) and \( \ell_1 \neq \ell_2 \), we have \( h_3 \notin H_{\ell_1} \).

**Proof.** First, define the following function

\[
f(h, \ell) = \sum_{k=1}^{Q_g} \mathbb{E}_{G_k} \{ R_{h,\gamma_2}(P_{\ell,k}(x)) \} = a_{\ell} + b_{\ell} h,
\]

where

\[
a_{\ell} = \sum_{k=1}^{Q_g} \mathbb{E}_{G_k} \left\{ p_0 \int_0^x \log_2 \left( 1 + \frac{P_{\ell,k}(x)q_0}{N_0} \right) f_0(x) dx \right. \\
+ p_1 \int_0^x \log_2 \left( 1 + \frac{P_{\ell,k}(x)q_2}{N_0+\gamma_2 P_x} \right) f_1(x) dx - \lambda \int_0^x P_{\ell,k}(x) p_0 f_0(x) dx \\
+ p_1 f_1(x) d\lambda \{ \int_0^x -\mu P_{\ell,k}(x) f_1(x) dx \}. \\
\]

From \( h_1 \in H_{\ell_1}, h_2 \in H_{\ell_2} \) and (48), we can get that

\[
f(h_1, \ell_1) - f(h_1, \ell_2) > 0 \quad \text{and} \quad f(h_2, \ell_1) - f(h_2, \ell_2) < 0.
\]

It follows from (49) that, \( f(h, \ell) \) and \( f(h, \ell_1) - f(h, \ell_2) \) are
TABLE II

<table>
<thead>
<tr>
<th>Quantizer design for $h$ given ${G_k}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Initialize the set $\Theta = {1, ..., Q_h}$.</td>
</tr>
<tr>
<td>Set $i = \arg \max_{\ell \in \Theta} f(0, \ell)$, $\Theta \leftarrow \Theta \setminus i$.</td>
</tr>
<tr>
<td>2: For $l = 1 : Q_h - 1$, do.</td>
</tr>
<tr>
<td>3: Calculate $h_k$ satisfying $f(h_k, i) - f(h_k, k) = 0, k \in \Theta$.</td>
</tr>
<tr>
<td>Set $\eta_i = \min_{k \in \Theta} h_k$. Assign $H_i = [\eta_{i-1}, \eta_i]$.</td>
</tr>
<tr>
<td>Set $i = \arg \min_{k \in \Theta} h_k$, $\Theta \leftarrow \Theta \setminus i$.</td>
</tr>
<tr>
<td>4: End for.</td>
</tr>
<tr>
<td>5: Set the last element in $\Theta$ as $i$, $H_i = [\eta_{Q_h-1}, \eta_{Q_h})$.</td>
</tr>
</tbody>
</table>

both monotonic functions over $h$. Thus, for any $h_3 > h_2$, $f(h_3, \ell_1) - f(h_3, \ell_2) < 0$. Hence $h_3 \notin H_{\ell_1}$. □

From Lemma 3, using the proof by contradiction, we can conclude that the range of $h$ should be divided into $Q_h$ continuous intervals. Define $Q_h + 1$ thresholds $\eta_0, \eta_1, ..., \eta_{Q_h}$ to form $Q_h$ intervals, where $\eta_0 = 0$, $\eta_{Q_h} = +\infty$. Then $H_i$ corresponds to one of the intervals $[\eta_{i-1}, \eta_i], i = 1, ..., Q_h$. Based on Lemma 3, we calculate $\eta_i$ sequentially and assign $\{H_i\}$ as follows. Initialize the set $\Theta = \{1, ..., Q_h\}$ as the number of unassigned set. The first interval is assigned to the set that makes $f(h, \ell)$ the largest at point $h = 0$, i.e., $[\eta_0, \eta_1) = H_i$, where $i = \arg \max_{\ell \in \Theta} f(0, \ell)$. Exclude the assigned set from $\Theta$, i.e., $\Theta \leftarrow \Theta \setminus i$. Then in each iteration $l$, we calculate $h_k$ that satisfies $f(h_k, i) - f(h_k, k) = 0, k \in \Theta$ and decide the junctional point between last assigned set and all the unassigned set (the solution exists and is unique since $f(h, i) - f(h, k)$ is linear in $h$). We choose the smallest of $h_k$ as the $i$th threshold $\eta_i$ and assign the current interval to the set that makes $h_k$ smallest, i.e., $\eta_i = \min_{k \in \Theta} h_k$ and $[\eta_{i-1}, \eta_i) = H_i$ where $i = \arg \min_{k \in \Theta} h_k$. Exclude the current assigned set $i$ from $\Theta$. The algorithm stops when there is only one element in $\Theta$, and we assign the last interval $[\eta_{Q_h-1}, \eta_{Q_h})$ to this element. The algorithm is summarized in Table II.

Power allocation functions:

In (40), because of the ex- pectation operator, the optimization problem in (45) is quite complex. Thus instead of optimizing the true rate, we optimize the upper bound of the rate $R_{quan}$ in (40) by applying the following Jensen’s inequalities
\[ \mathbb{E}_{G_k} \left\{ \log_2 \left( 1 + \frac{P_{t,k}(x)g}{N_0} \right) \right\} \leq \log_2 \left( 1 + \frac{P_{t,k}(x)\bar{g}_k}{N_0} \right), \]
\[ \mathbb{E}_{G_k} \left\{ \log_2 \left( 1 + \frac{P_{t,k}(x)g}{N_0 + \gamma_2 P_0} \right) \right\} \leq \log_2 \left( 1 + P_{t,k}(x)\bar{\xi}_k \right), \] (50)

where \( \bar{g}_k = \mathbb{E}_{G_k}\{g\} \) and \( \bar{\xi}_k = \mathbb{E}_{G_k}\left\{ \frac{g}{N_0 + \gamma_2 P_0} \right\} \). The optimal power allocation functions are then determined by maximizing the generalized partition centroid as
\[ \max_{P_{t,k}(x) \geq 0} \mathbb{E}_{\mathcal{H},G_k}\{ R_{h,\beta,\gamma_2}(P_{t,k}(x)) \} p(\ell, k). \] (51)

Define \( \bar{H}_\ell = \mathbb{E}_{\mathcal{H},\{h\}} \). From (46) and (50), we obtain the
TABLE III

Algorithm 2: Optimal sensing time, quantizers and power allocation functions with quantized CSI under the average transmit power and average interference power constraints.

1. For each $\tau$ in $[0, T]$, do.
   1. Initialize $\lambda$, $\mu$, $Q_h$ and $Q_g$.
   2. Repeat:
      - Get $\{P_{\ell,k}(x)\}$ using (53); Update $\lambda$ and $\mu$ using (27); Until $\lambda$ and $\mu$ converge.
      - Iterate to update $Q_h$ and $Q_g$ using (48) and Table II by fixing one and optimizing the other.
   3. Until $Q_h$ and $Q_g$ converge.
   4. End for.
   5. Optimal sensing time, quantizers and power allocation functions: $\tau^* = \arg \max_{\tau} R_{\text{qua}}(\tau, Q_h, Q_g, P_{\ell,k}(x))$,

\[
\left( P_{\ell,k}(x), Q_h, Q_g^* \right) = \left( P_{\ell,k}(x), Q_h, Q_g \right)_{\tau=\tau^*}.
\]

upper bound of $\mathbb{E}_{H_{\ell},G_k} \{ R_{h,k} (P_{\ell,k}(x)) \}$ as

\[
R_{H_{\ell},G_k} (P_{\ell,k}(x)) = \int_0^\infty \left\{ p_0 \log_2 \left( 1 + \frac{P_{\ell,k}(x) g_k}{N_0} \right) f_0(x) + p_1 \log_2 \left( 1 + P_{\ell,k}(x) \tilde{\xi}_k \right) f_1(x) \right\} dx - \mu p_1 \int_0^\infty \tilde{h}_\ell P_{\ell,k}(x) f_1(x) dx - \lambda \int_0^\infty P_{\ell,k}(x) [p_0 f_0(x) + p_1 f_1(x)] dx.
\]

To find the optimal $P_{\ell,k}(x)$ for each $x$, we set the derivative of $R_{H_{\ell},G_k} (P_{\ell,k}(x))$ with respect to $P_{\ell,k}(x)$ to 0. Finally, we get the power allocations for given Lagrange multipliers $\lambda$ and $\mu$ as

\[
P_{\ell,k}(x) = \left[ \frac{A_{\ell,k}(x) + \sqrt{\Delta_{\ell,k}(x)}}{2} \right]^+,
\]

where

\[
A_{\ell,k}(x) = \log_2(e) \frac{[p_0 f_0(x) + p_1 f_1(x)]}{\lambda [p_0 f_0(x) + p_1 f_1(x)] + \mu \tilde{h}_\ell p_1 f_1(x)} - \tilde{\xi}_k - \frac{N_0}{g_k},
\]

\[
\Delta_{\ell,k}(x) = A_{\ell,k}(x)^2 + \frac{4}{\tilde{g}_k} \left\{ -\frac{N_0}{\tilde{\xi}_k} + \frac{\log_2(e) [p_0 f_0(x) (N_0 + \gamma_2 P_s) + p_1 f_1(x) N_0]}{\lambda [p_0 f_0(x) + p_1 f_1(x)] + \mu \tilde{h}_\ell p_1 f_1(x)} \right\}.
\]
The closed-form solution derived above as a result of the Jensen’s inequalities in (50) serves as a realizable, nontrivial rate upper bound. Similarly, we can prove that $P_k(x)$ is a non-increasing function over $x$. Subgradient-based methods can again be used here to find the optimal Lagrange multipliers $\lambda$ and $\mu$, where the subgradient

$$
[c, v] \rightleftharpoons c = P - \frac{\gamma_{\text{avg}}}{P}.
$$

$$
\sum_{k=1}^{Q} \sum_{x=1}^{Q_x} \left\{ \int_{0}^{\infty} P_{k,x} f(x) g(x) f_{y}(x)\,dx \right\} p(t, k) \text{ and } v = \frac{1}{T} \sum_{k=1}^{Q} \sum_{x=1}^{Q_x} \int_{0}^{\infty} \mathbb{E}[h] P_{k,x} f_{1}(x)\,dx, 
$$

$p(t, k)$ is the optimal power allocation for fixed $\lambda$ and $\mu$.

Finally, the algorithm that computes the optimal sensing time and continuous power allocation function with quantized CSI under the average transmit power and average interference power constraints can be summarized in Table III.

Fig. 3. Power allocation functions under the average transmit and average interference power constraints

**D. Other Power Constraints and Computational Complexity**

For problems under other power constraints, first, we can build the similar distortion measure as (46), then the quantizers design strategy can be solved by the farthest neighbor rule as (48). The power allocation can be solved by applying Jensen’s inequalities as (50) and the optimization procedure like that in Section III. The details are not given here for brevity. Note that the case of perfect CSI serves to provide performance upper bound on the proposed continuous power control scheme. In practice, the scheme with quantized CSI would be employed. All computations are performed offline and the resulting power control rule is stored in a look-up table for real-time implementation. Thus the computational complexity is not a significant concern.

**V. SIMULATION RESULTS**

In this section, we present simulation results for the proposed continuous power allocation strategies with perfect and quantized CSI in a cognitive radio system. Three narrowband channels ($M = 3$) are assumed, each of 1 MHz bandwidth, and the frame duration is fixed as $T = 100$ ms and the sampling
frequency $f_s = 1\text{MHz}$. The target detection probability for each channel is set to $q_{\text{th}} d_j = 0.9$ in the opportunistic spectrum access scheme. We set $\gamma_{1,j} = N_0 = 0 \text{ dB}$, $p_{0,j} = 0.7$, $P_{s,j} = 10\text{dB}$, $I_j = 0\text{dB}$, $P = 10\text{dB}$, $\gamma_{2,j}$, $h_j$ and $g_j$ as Rayleigh distribution with mean $-3$, $0$ and $0$ dB, and unless otherwise mentioned. Fig. 3 compares the power allocation functions under the conventional strategies and the proposed continuous one with perfect CSI for fixed $h_j = g_j = \gamma_{2,j} = 0\text{dB}$. From the figure we can see that, the function $P(x_j)$ for the proposed strategy is a non-increasing function of the received signal energy which corroborates Proposition 1. When $x_j$ is small, the proposed scheme allocates more power than the conventional ones, and when $x_j$ is large, it allocates less power than the conventional schemes. Fig. 4 plots the quantization region and power allocation functions for the proposed continuous strategy with quantized

![Figure 3: Power Allocation Functions](image)

![Figure 4: Quantization Regions and Power Allocation Functions for $Q_h$ and $Q_g$](image)

![Figure 4: Power Allocation Functions](image)

Fig. 4. Quantization region and power allocation functions for $Q_h = Q_g = 2$. Fig. 4(a) shows that, the range of $h$ is divided into 2 continuous intervals. For $(g, \gamma_{2})$, in the simulation, we find that, the range of $(g, \gamma_{2})$ is divided into 2 continuous spaces separated by
Fig. 4(b) shows that, all the functions \{P, k(x)\} are non-increasing over x, and for given Gk, \( P_1(k(x)) \geq P_2(k(x)) \), \( \forall x \) which can be obtained from (53). Fig. 5 presents the average total secondary achievable rate versus the sensing time. Notice that, in fig. 5-8, for each \((h_j, g_j, \gamma_2, j)\), we compute the rate under the continuous power allocation with perfect CSI, and then average to compare with the case of quantized CSI. It is seen that compared with the conventional schemes, the proposed continuous power allocation strategies provide higher secondary achievable rates. By quantizing the channels using 2-3 bits, the continuous power allocation strategy with quantized CSI achieves rates that are quite close to that is achieved with perfect CSI. Fig. 6 shows the average secondary achievable rate under average transmit and average interference power constraints for \( \bar{I}_j = 0 \) dB. From the figure we can observe that, in the

![Graph showing secondary achievable rate vs. sensing time](image1)

**Fig. 5.** Secondary achievable rate vs. the sensing time under the average transmit and peak interference power constraints.

![Graph showing secondary achievable rate vs. P](image2)

**Fig. 6.** Secondary achievable rate vs. \( P \) under the average transmit and average interference power constraints.

The conventional schemes we considered here assume perfect CSI of all channels, however, from the

\[ g = 0.46. \]
figure, we can observe that, the proposed scheme using 2-3 bits of quantized CSI still outperforms them which proves the effectiveness of the continuous power allocation scheme. Fig. 7 shows the average secondary achievable rate under peak transmit and peak interference power constraints for $\hat{I}_j = 6$dB. It is observed that, the rate increases with respect to $\hat{P}_j$ when it is small, which indicates that $\hat{P}_j$ limits the performance of the network. However, when $\hat{P}_j$ is sufficiently large compared with $\hat{I}_j$, the rate stays stable since it is decided by $\hat{I}_j$. When the mean value of $g_j$ becomes larger or that of $h_j$ becomes smaller, the rate becomes larger which indicates that the fading of the SU channel is harmful, but the fading of the channel between SU-Tx and PU-Tx is beneficial in terms of maximizing the rate of SU channel. Fig. 8 shows the average secondary achievable rate under peak transmit and average interference power constraints for $\hat{P}_j = 3$dB. In the low $\hat{I}_j$ region,
the power allocation is decided by $\bar{I}_j$, the rates of the proposed schemes are better than the conventional ones. When $\bar{I}_j$ becomes larger, the rates tend to be equal where the power is decided by $\hat{P}_j$ and when $\bar{I}_j = +\infty$, $P(x_j) = \hat{P}_j$, the proposed schemes become the same as the conventional ones. Fig. 9 simulates the case that no PU to SU link exists and perfect SU link information is available, where only the statistic information of $h_j$ is available at SU-Tx. And in this case, we only have to study the continuous power allocation strategy with perfect CSI. From the figure we can observe that, when $\bar{P}$ is high, the proposed continuous power allocation scheme achieves much higher rates. However, compared with

![Graph showing average secondary achievable rate vs. $P$ under the average transmit and average interference power constraints with no feedback link from PU to SU.](image)

that in Fig. 6, the additional rate in this case is smaller. Moreover, since less information is obtained in this case, the rates of all the strategies are correspondingly smaller than that in Fig. 6.

VI. CONCLUSIONS

We have proposed sensing-based continuous power allocation strategies for secondary users in a multi-band cognitive radio system. The power allocation is a function of the received signal energy by the secondary user, parameterized by various channel parameters of the primary and secondary links. We have treated both the perfect CSI case and the more practical quantized CSI case. For the different possible combinations of constraints on the peak/average transmit power at SU and the peak/average interference power at PU, the power allocation functions are obtained to maximize the average secondary achievable rate (for perfect CSI) or its upper bound (for quantized CSI). Compared with the state-of-the-art sensing-based spectrum access which employs a binary power allocation strategy, the proposed schemes offer significant rate improvement for the secondary users.

REFERENCES


