

Bi-level FH-CDMA scheme for Communication Systems over Rayleigh and Rician Channels

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Abstract

This paper proposes a “two-level” frequency hopping code-division multiple-access (FH-CDMA) scheme for digital wireless communication systems for the fading channels like Rayleigh and Rician channel. This new approach provides better flexibility in the selection of modulation codes and FH patterns for different users. This scheme can be modified to support more possible users without increasing the number of FH patterns by making the partitioned modulated codes. The spectral efficiency and performance of the scheme are analyzed in this paper. The simulation results show that this two-level FH-CDMA scheme can support higher data rate and greater Spectral efficiency than Goodman’s FSK-FH-CDMA scheme at certain conditions.

Keywords: Code division multiple access, frequency hopping, fading channel, spectral efficiency.

1. Introduction

FH-CDMA provides frequency diversity and it reduces the multipath fading and interference [1], [2]. The FH-CDMA gives better result than direct-sequence CDMA [1], [4] in some ways like reduced near-far problem, better resistance to multiple access interference (MAI) and multipath interference, less stringent power control., The FH-CDMA system allows multiple users to share the same transmission channel simultaneously by assigning a unique FH pattern to each user [6], [5]. “One-hit” FH patterns is designed in order to minimize MAI [6], [8] which occurs if many users use the same carrier frequency in a particular time slot. Our proposed scheme provides more data rate than Goodman’s MFSK-FH-CDMA.

The bi-level scheme is more flexible for selecting modulation codes and FH-patterns because each element of the modulation codes needs to be conveyed by an element of the FH-pattern. So it can be used in different operating requirements. In this paper a partitioning method on modulation codes is proposed where the modulation codes with lower cross-correlation values are grouped together. So multiple user can take the same FH-pattern due to the partitioned two-level FH-CDMA, there by the number of possible users increases. The performance analysis of this

scheme is demonstrated for AWGN channel as well as Rayleigh and Rician channels. The results of our system are compared with Goodman’s MFSK-FH-CDMA on the basis of data rate and spectral efficiency in the next sections of this paper where the new system gives better efficiency than the existing one.

2. New Bi-Level FH CDMA Scheme

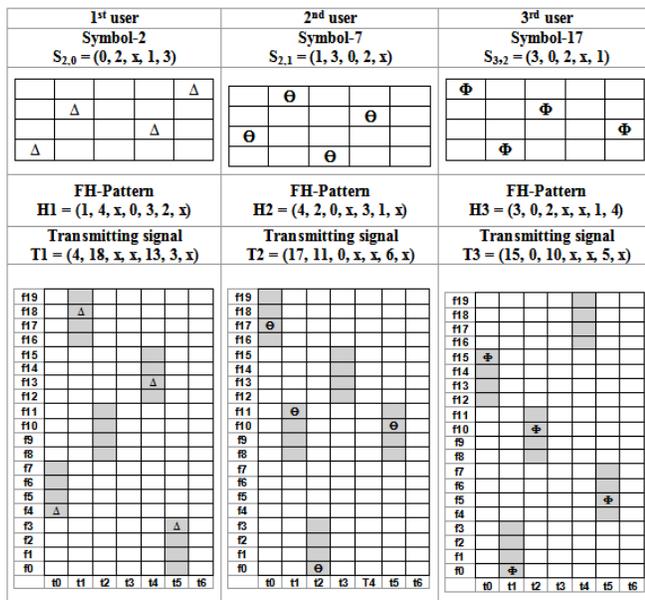
In this new bi-level FH-CDMA scheme, the available transmission bandwidth is divided into M_h number of frequency bands with M_m number of carrier frequencies in each band. So total $(M_m \cdot M_h)$ numbers of carrier frequencies are found. In the first level (modulation level), a number of serial data bits is grouped together and represented by a symbol. Each symbol is, in turn, represented by a modulation code of dimension $M_m \times L_m$ and weight (i.e., number of elements) w_m , where M_m is the number of frequencies, L_m is the number of time slots (i.e., code length). The number of data bits that can be represented by a symbol depends on the number of available modulation codes. If there are Φ_m available modulation codes, each symbol can represent up to $\lfloor \log_2 \Phi_m \rfloor$ data bits, where $\lfloor \cdot \rfloor$ is the floor function.

In the second level (FH level), each user is assigned a unique FH pattern of dimension $(M_h \times L_h)$ and weight (i.e., number of elements) w_h , where M_h is the number of frequencies, L_h is the number of time slots (i.e., pattern length). The elements in the modulation codes and FH patterns determine the carrier frequencies of the final FH-CDMA signals. While an element of a modulation code defines the carrier frequency used in a frequency band in a given time slot, an element of the FH pattern determines which frequency band (out of M_h bands) to use. In our scheme, we can choose any families of $(M_m \times L_m, w_m, \lambda_{a,m}, \lambda_{c,m})$ modulation codes and $(M_h \times L_h, w_h, \lambda_{a,h}, \lambda_{c,h})$ FH patterns as long as $w_h \geq L_m$, where $\lambda_{a,m}$ ($\lambda_{a,h}$) and $\lambda_{c,m}$

($\lambda_{c,h}$) denote the maximum autocorrelation sidelobes and cross-correlation values of the modulation codes (FH patterns), respectively.

TABLE – 1
25 PRIME SEQUENCES OF DIMENSIONS (4*5,4,0,1)

	Gr – 0	Gr – 1	Gr – 2	Gr – 3	Gr – 4
i_2	$i_1 = 0$	$i_1 = 1$	$i_1 = 2$	$i_1 = 3$	$i_1 = 4$
0	0000x	0123x	02x13	031x2	0x321
1	1111x	123x0	1302x	1x203	10x32
2	2222x	23x01	2x130	203x1	210x3
3	3333x	3x012	302x1	3x120	3210x
4	xxxxx	x0123	x1302	1203x	x3210



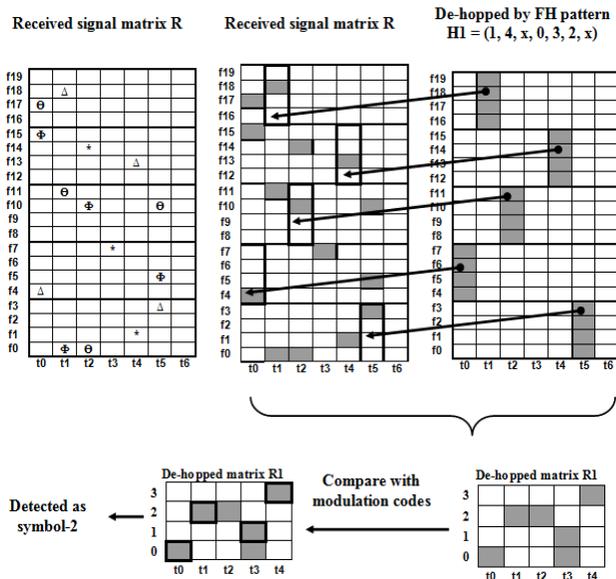
To illustrate the main concept of our two-level FH-CDMA scheme, we here use prime sequences [8] as the modulation codes; other codes, such as the RS sequences [6], quadratic congruence codes (QCCs) [10], and multilevel prime codes (MPCs) [11], can also be used. The prime sequences are constructed in Galois field GF(p) of a prime number p. Each prime sequence of weight $w_m = p$ is denoted by $S_{i1,i2} = (S_{i1,i2,0}, S_{i1,i2,1}, \dots, S_{i1,i2,1}, \dots, S_{i1,i2,p-1})$, where the lth element $s_{i1,i2,l} = i2 \oplus (i1 \odot l)$ represents the frequency used in the lth position (i.e., time slot) of $S_{i1,i2}$, $\{i1, i2, l\} \in GF(p)$, “ \oplus ” denotes a modulo-p addition, and “ \odot ” denotes a modulo-p multiplication. Since these prime sequences are used as the modulation codes, each element of $S_{i1,i2}$ determines which carrier frequency of a frequency band in a given time slot to use. If the number of available carrier frequencies is restricted or the sequence weight needs to be varied in order to achieve certain scheme performance, we can always adjust

the sequence weight to be $w_m < p$ by dropping the largest $(p - w_m)$ elements in $S_{i1,i2}$. As a result, the construction algorithm gives $\Phi_m = p^2 - p + w_m$ prime sequences of weight $w_m \leq p$ and length $L_m = p$ with $\lambda_{c,m} = 1$ (i.e., symbol interference). For example, with $p = 5$ and $w_m = 4$, Table I shows twenty-four ($M_m \times L_m, w_m, \lambda_{a,m}, \lambda_{c,m}$) = (4 × 5, 4, 0, 1) prime sequences, where “x” denotes the drop of the fifth element in order to have a code weight of four. Using these prime sequences as the modulation codes, we can support at most twenty-four symbols and each symbol represents $\lfloor \log_2 24 \rfloor = 4$ data bits.

As mentioned earlier, we can choose any FH patterns for the second level of our two-level FH-CDMA scheme as long as $w_h \geq L_m$. To illustrate this, we choose the ($M_h \times L_h, w_h, \lambda_{a,h}, \lambda_{c,h}$) = (5 × 7, 5, 0, 1) prime sequences as the one-hit FH patterns and the top sixteen ($M_m \times L_m, w_m, \lambda_{a,m}, \lambda_{c,m}$) = (4 × 5, 4, 0, 1) prime sequences in Table I as the modulation codes. Fig. 1 shows the encoding process of three simultaneous users. If the data symbols of these three users at one time instant are “2”, “7”, and “17”, then we pick $S1 = S2,0 = (0, 2, x, 1, 3)$, $S2 = S2,1 = (1, 3, 0, 2, x)$, and $S3 = S1,1 = (3, 0, 2, x, 1)$ as the modulation codes, respectively. Let the one-hit FH patterns of these three users be $H1 = (1, 4, x, 0, 3, 2, x)$, $H2 = (4, 2, 0, x, 3, 1, x)$, and $H3 = (3, 0, 2, x, x, 1, 4)$. The carrier frequency used in each frequency band in a time slot is determined by superimposing (element-by-element) all $w_m = 4$ elements of S_k on top of the first w_m non-“x” elements of H_k , and the “x”-elements of S_i produce empty frequency bands in the final two-level FHCDMA signal, where $k = \{1, 2, 3\}$. The shaded columns in the transmitting signals, T_k , of Fig. 1 represent the frequency bands specified by the corresponding FH patterns, H_k , for $k = \{1, 2, 3\}$. In summary, the two-level FH-CDMA signal can be represented by $T_k = (T_{k,0}, T_{k,1}, \dots, T_{k,i}, \dots, T_{k,Lh-1}) = S_k \Delta (M_m H_k)$, where $T_{k,i}$ represents the carrier frequency used in the ith time slot and Δ denotes the superimpose operation. For example, the two-level FH-CDMA signal of the first user is found to be $T1 = (0+1*4, 2+4*4, x+4*x, x+0*x, 1+3*4, 3+2*x, x+x*4) = (4, 18, x, x, 13, 3, x)$ after superimposition. Similarly, the other two simultaneous users have $T2 = (17, 7, 0, x, x, 6, x)$ and $T3 = (15, 0, 10, x, x, 5, x)$.

In a receiver, the received two-level FH-CDMA signals of all users and effects of MAI, fading, and noise (i.e., hits, deletions, and false alarms) are hardlimited, dehopped, and finally decoded in order to recover the transmitted data symbols. Fig. 2 illustrates the decoding and detection processes of user 1. The received signal R is first hardlimited and then dehopped by user 1’s FH pattern H1 to give a dehopped signal R1 of dimension 4 × 5. The role

of the dehopping process simply brings the frequency bands in each time slot of R back to the baseband, according to the frequency bands specified by H1. The elements of R1 are compared with the elements of all modulation codes in use. The modulation code (e.g., S3,0, with its elements shown as circles in Fig. 2) with the minimum distance from the shaded slots of R1 is chosen as the recovered symbol.



3. Performance Analysis

In FH-CDMA systems, MAI depends on the crosscorrelation values of FH patterns. For our two-level FHCDMA scheme, the cross-correlation values of the modulation codes impose additional (symbol) interference and need to be considered. Assume that one-hit FH patterns of dimension $M_h \times L_h$ are used and the transmission band is divided into $M_M M_h$ frequencies, in which M_M frequencies are used to carry the modulation codes of weight w_m . The probability that a frequency of an interferer hits with one of the w_m frequencies of the desired user is given by

$$q = \frac{w_m^2}{M_m M_h L_h} \tag{1}$$

Assume that there are K simultaneous users, the probability that the dehopped signal contains n entries in an undesired row is given by [13]

$$P(n) = \binom{w_m}{n} \sum_{i=0}^n (-1)^i \binom{n}{i} \left[1 - q + \frac{(n-i)q}{w_m} \right]^{K-1} \tag{2}$$

Over AWGN, and Rayleigh and Rician fading channels, false alarms and deletions may introduce detection errors to the received FH-CDMA signals. A false-alarm probability, p_f , is the probability that a tone is detected in a receiver when none has actually been transmitted. A deletion probability, p_d , is the probability that a receiver missed a transmission tone. For these three types of channels, the false-alarm probability is generally given by [13]

$$p_f = \exp(-\beta_0^2/2) \tag{3}$$

For an AWGN channel, the deletion probability is given by [16]

$$p_d = 1 - Q\left(\sqrt{2(\overline{E_b}/N_o) \cdot (k_b/w_m)}, \beta_0\right) \tag{4}$$

where β_0 denotes the actual threshold divided by the rootmean-squared receiver noise, k_b is the number of bits per symbol, $\overline{E_b}/N_0$ is the average bit-to-noise density ratio, $Q(a,b) = \int_a^\infty \exp[-(x^2 + b^2)/2] I_0(bx) dx$ is Marcum's Q- function, and $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order. To minimize the error probability, the optimal β_0 of an AWGN channel should be a function of the signal-to-noise ratio (SNR), $(\overline{E_b}/N_o) \cdot (k_b/w_m)$ and can be more accurately written as [13]

$$\beta_0 = \sqrt{2 + \frac{(\overline{E_b}/N_o) \cdot (k_b/w_m)}{2}} \tag{5}$$

rather than an inaccurate constant value (i.e., $\beta_0 = 3$, used in [6], [8], [11]).

For a Rayleigh fading channel, the deletion probability is given by [13]

$$p_d = 1 - \exp\left\{ \frac{-\beta_0^2}{2 + 2(\overline{E_b}/N_o) \cdot (k_b/w_m)} \right\} \tag{6}$$

Similarly, the optimal β_0 of a Rayleigh fading channel can be more accurately written as [13]

$$\beta_0 = \sqrt{2 + \frac{2}{(\overline{E_b}/N_o) \cdot (k_b/w_m)} \times \sqrt{\log[1 + (\overline{E_b}/N_o) \cdot (k_b/w_m)]}} \tag{7}$$

Finally, for a Rician fading channel, the deletion probability is given by [13]

$$p_d = \left[1 - Q\left(\sqrt{\frac{2\rho(\overline{E_b}/N_o) \cdot (k_b/w_m)}{1 + \rho + (\overline{E_b}/N_o) \cdot (k_b/w_m)}}, \beta_1\right) \right]$$

(8)

where the Rician factor ρ is given as the ratio of the power in specular components to the power in multipath components [15]. Similarly, β_0 and β_1 can be more accurately written as [14, eqn. (7-4-14)]

$$\beta_0 = \sqrt{2 + \frac{(\overline{E_b/N_o}) \cdot (k_b/w_m)}{2}} \quad (9)$$

$$\beta_1 = \frac{\beta_0}{\sqrt{1 + (\overline{E_b/N_o}) \cdot (k_b/w_m)/(1 + \rho)}} \quad (10)$$

Including the noise or fading effect, the probability that the dehopped signal contains n entries in an undesired row is given by [6], [8], [16]

$$P_s(n) = \sum_{j=0}^n \sum_{r=0}^{\min[n-j, w_m-n]} \left[P(n-j) \binom{n-j}{r} \times p_d^r (1-p_d)^{n-j-r} \right. \\ \left. \binom{w_m-n+j}{r+j} \times p_f^{r+j} (1-p_f)^{w_m-n-r} \right] \\ + \sum_{j=1}^{w_m-n} \sum_{r=j}^{\min[n+j, w_m-n]} \left[P(n+j) \binom{n+j}{r} \times p_d^r (1-p_d)^{n+j-r} \right. \\ \left. \binom{w_m-n-j}{r-j} \times p_f^{r-j} (1-p_f)^{w_m-n-r} \right] \quad (11)$$

In addition, an error may occur in this scheme even when the undesired rows have less-entries than the desired rows. It is because the nonzero cross-correlation values of the modulation codes add extra undesired entries. To account for this, let A_i^z denote the conditional probability of the number of hits being increased from z to $z+i$, where $i \in [1, \lambda_{c,m}]$. To account for the effect of $\lambda_{c,m} \neq 0$, we derive a new probability of having a peak of z as

$$P'_s(z) = A_{\lambda_{c,m}}^z P_s(z - \lambda_{c,m}) + A_{\lambda_{c,m}-1}^z \times P_s(z - (\lambda_{c,m} - 1)) \\ + \dots + A_1^z P_s(z - 1) + \left(1 - \sum_{t=1}^{\lambda_{c,m}} A_t^{z+t} \right) P_s(z) \quad (12)$$

where $A_t^{z+t} = 0$ when $z+t > w_m$.

If there are $2^{k_b}-1$ incorrect rows, the probability that n is the maximum number of entries and that exactly t unwanted rows contain n entries is given by [6], [8], [16]

$$P_r(n, t) = \binom{2^{k_b}-1}{t} [P'_s(n)]^t \left[\sum_{m=0}^{n-1} P'_s(m) \right]^{2^{k_b}-1-t} \quad (13)$$

Over a noisy or fading channel, the probability of having an entry in a desired row is $1-p_d$. Therefore, the probability that there exist n entries in a desired row is given by

$$P_c(n) = \binom{w_m}{n} (1-p_d)^n (p_d)^{w_m-n} \quad (14)$$

The desired symbol is detected wherever the maximum number of entries in the t incorrect rows is less than n . As the receiver decides which symbol (out of 2^{k_b} symbols) is recovered by searching for the modulation code with the largest matching entries, the bit error probability (BEP) is finally given by [6], [8], [16]

$$P_b(K) = \frac{2^{k_b}}{2(2^{k_b}-1)} \times \left\{ 1 - \sum_{n=1}^w \left[P_c(n) \sum_{t=0}^{2^{k_b}-1} \frac{1}{t+1} P_r(n, t) \right] \right\} \quad (15)$$

4. Performance and SE Comparison

In this section, we compare the performances of the new two-level FH-CDMA and Goodman's MFSK/FH-CDMA schemes under the condition of same transmission parameters: $M_g = M_m$, $M_h = L_g$, $L_g = L_h$, and $w_g = w_m$, where M_g , L_g , and w_g are the number of frequencies, number of time slots, and weight of FH patterns utilized by Goodman's MFSK/FHCDMA scheme, respectively. As illustrated in [16], the prime sequences may give at most two hits in Goodman's MFSK/FHCDMA scheme under a symbol-asynchronous assumption. The main difference is that Goodman's MFSK/FH-CDMA scheme supports M_g modulation symbols (represented by the orthogonal frequencies), while the two-level FH-CDMA scheme supports $p^2 - p + w_m$ symbols with the symbolinterference level $\lambda_{c,m} = 1$ if the prime sequences in Section II are used as the modulation codes. This symbol interference is accounted for by the probability term $P'_s(z)$ in (12).

BEPs for AWGN, Rayleigh and Rician channels at different test cases are shown in following figures which are obtained from simulation.

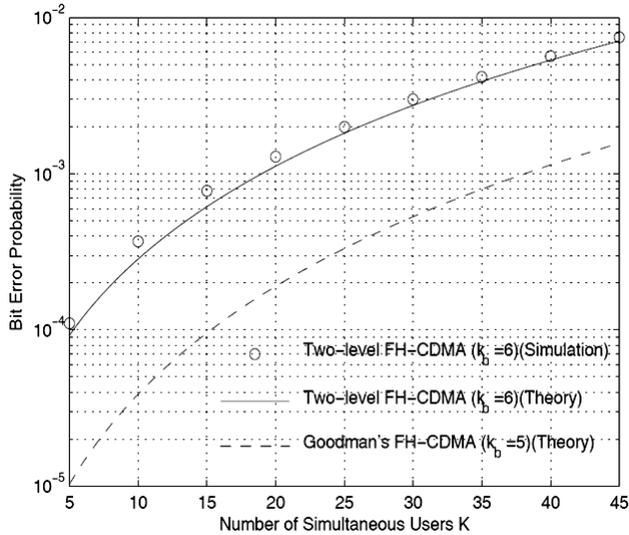


Fig-3: BEPs of the Bi-level FH-CDMA and Goodman's FH-CDMA schemes versus the no. of K simultaneous users over Rayleigh-channel.

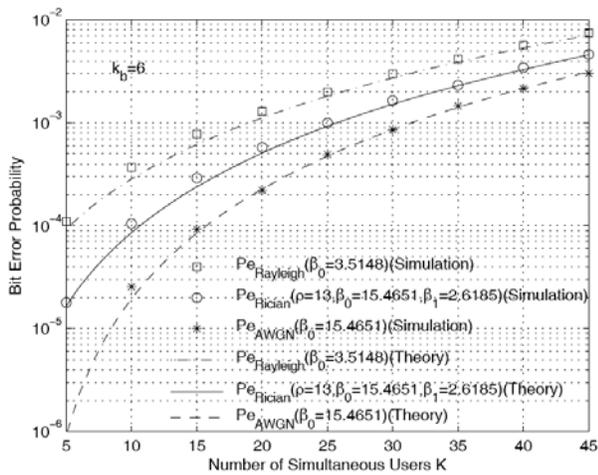


Fig-4: BEPs of the Bi-level FH-CDMA scheme versus the no. of K simultaneous users over AWGN, Rayleigh and Rician channels.

Table 2: SE comparison of both schemes $P_e = 10^{-2}, 10^{-3}$ based on the parameters from figure – 4

Bit error probability	$P_e = 10^{-2}$	$P_e = 10^{-3}$
Goodman's HF-CDMA ($k_b = 2$)	K=144 SE=13.93%	K=56 SE=5.42%
Bi-level FH-CDMA ($k_b = 3$)	K=126 SE=18.29%	K=53 SE=7.69%

The spectral efficiency of the system can be calculated as

$$SE = \frac{k_b K}{ML} \tag{16}$$

5. Conclusion

In this paper, we proposed a new two-level FH-CDMA scheme. The prime/FH-CDMA and RS/FH-CDMA schemes were special cases of our scheme. The performance analyses showed that the two-level FH-CDMA scheme provided a trade-off between performance and data rate. The partitioned two-level FH-CDMA scheme increased the number of possible users and exhibited higher data rate and greater SE than Goodman's MFSK/FH-CDMA scheme. In summary, the new scheme offered more flexibility in the design of FH-CDMA systems to meet different operating requirements.

5. References

- [1] J.-Z. Wang and L. B. Milstein, "CDMA overlay situations for microcellular mobile communications," IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 603-614, Feb./Mar./Apr. 1995.
- [2] G. Kaleh, "Frequency-diversity spread-spectrum communication system to counter bandlimited Gaussian interference," IEEE Trans. Commun., vol. 44, no. 7, pp. 886-893, July 1996.
- [3] Y. R. Tsai and J. F. Chang, "Using frequency hopping spread spectrum technique to combat multipath interference in a multi-accessing environment," IEEE Trans. Veh. Technol., vol. 43, no. 2, pp. 211-222, May 1994.
- [4] J.-Z. Wang and J. Chen, "Performance of wideband CDMA systems with complex spreading and imperfect channel estimation," IEEE J. Sel. Areas Commun., vol. 19, no. 1, pp. 152-163, Jan. 2001.
- [5] G. Einarsson, "Address assignment for a time-frequency, coded, spreadspectrum system," Bell Syst. Tech. J., vol. 59, no. 7, pp. 1241-1255, Sep. 1980.
- [6] S. B. Wicker and V. K. Bhargava (eds.), Reed-Solomon Codes and Their Applications. Wiley-IEEE Press, 1999.
- [7] D. J. Goodman, P. S. Henry, and V. K. Prabhu, "Frequency-hopping multilevel FSK for mobile radio," Bell Syst. Tech. J., vol. 59, no. 7, pp. 1257-1275, Sep. 1980.
- [8] C.-Y. Chang, C.-C. Wang, G.-C. Yang, M.-F. Lin, Y.-S. Liu, and W. C. Kwong, "Frequency-hopping CDMA wireless communication systems using prime codes," in Proc. IEEE 63rd Veh. Technol. Conf., pp. 1753- 1757, May 2006.
- [9] G.-C. Yang and W. C. Kwong, Prime Codes with Applications to CDMA Optical and Wireless Networks. Norwood, MA: Artech House, 2002.

- [10] E. L. Titlebaum and L. H. Sibul, "Time-frequency hop signals—part II: coding based upon quadratic congruences," *IEEE Trans. Aero. Electron. Syst.*, vol. 17, no. 4, pp. 494-500, July 1981.
- [11] C.-H. Hsieh, G.-C. Yang, C.-Y. Chang, and W.C. Kwong, "Multilevel prime codes for optical CDMA systems," *J. Opt. Commun. Netw.*, vol. 1, no. 7, pp. 600-607, Dec. 2009.
- [12] M.-F. Lin, G.-C. Yang, C.-Y. Chang, Y.-S. Liu, and W. C. Kwong, "Frequency-hopping CDMA with Reed-Solomon code sequences in wireless communications," *IEEE Trans. Commun.*, vol. 55, no. 11, pp. 2052-2055, Nov. 2007.
- [13] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. McGraw-Hill, 1996.
- [14] G.-C. Yang, S.-Y. Lin, and W. C. Kwong, "MFSK/FH-SSMA wireless systems with double media services over fading channels," *IEEE Trans. Veh. Technol.*, vol. 49, no. 3, pp. 900-910, May 2000.
- [15] U. Svasti-Xuto, Q. Wang, and V. K. Bhargava, "Capacity of an FHSSMA system in different fading environments," *IEEE Trans. Veh. Technol.*, vol. 47, no. 1, pp. 75-83, Feb. 1998.
- [16] C.-Y. Chang, H.-T. Chen, G.-C. Yang, and W. C. Kwong, "Spectral efficiency study of QC-CHPCs in multirate optical CDMA system," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 9, pp. 118-128, Dec. 2007.
- [17] T. Mabuchi, R. Kohno, and H. Imai, "Multiuser detection scheme based on canceling cochannel interference for MFSK/FH-SSMA system," *IEEE J. Sel. Areas Commun.*, vol. 12, no. 4, pp. 593-604, May 1994.
- [18] T.-C. Lin, C.-C. Hsu, C.-Y. Chang, G.-C. Yang, and W. C. Kwong, "Study of MFSK/FH-CDMA wireless communication systems without symbol-synchronous assumption," in *Proc. IEEE Sarnoff Symp.*, pp. 1-5, Apr. 2007.