

Design and Implementation of Controllers in an Inverted Pendulum

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Abstract

Control of the Inverted Pendulum system in both simulated and physical laboratory is possible as the stability of the system can be attained. Finding or calculating the vectors of gains K using pole placement method and performing the closed loop simulations for the non-linear simulink model with a full state feedback controller of gains calculated makes the implementation possible. The controller is implemented by using the gain calculated on the laboratory physical inverted pendulum system. The stability of the simulated and physical laboratory inverted pendulum system was attained. The variations in time taken for the system to attain the upright position may be due to high K value of gains which make the system approach the stability.

Key Words: Controllers, Design, Implementation, Inverted Pendulum, Stability

Introduction

The inverted pendulum system is a standard problem in the area of control systems. They are often useful to demonstrate concepts in linear control such as the stabilization of unstable systems by varying the pendulum position and using the gains. Since the system is inherently nonlinear, it has also been useful in illustrating some of the ideas in nonlinear control [1]. This work consists of two parts of experimental procedures. Part 1 deals with finding or calculating the vectors of gains K using pole placement method and performing the closed loop simulations for the non-linear using simulink model with a full state feedback controller of gains calculated while Part 2 is implementing the controllers by using the gain calculated on the laboratory physical inverted pendulum system and compares their stability.

The main aims and objectives of this work is to linearized a non-linear Simulink inverted pendulum model, calculate the vector of gains K using pole placement techniques, to perform

closed-loop simulations of the non – linear Simulink pendulum model with a full state feedback controller calculated and to implement the controllers on a physical laboratory inverted pendulum system.

Methodology

Part 1

The distance from the rod pivot center to the weight center value was changed as variable y_w = 35 in initialization file **pendulum_parameters.m** by launching matlab window command and implement the change. The **pendulum_lin_gains_students.mdl** file was also opened in Simulink and linearised the model using the following Matlab command. $[A \ B \ C \ D] = \text{linmod}(\text{'pendulum_lin_gains_students'}, [0;0;0;0], 0)$.

The vector of gains K for the linearised model was calculated using a pole placement method. The poles chosen for this system are: [-1.25, -1.5, -1.6 and -1.8 Hz]. These values results were obtained from the inherent dynamics of the system including limitations of the actuators, (DC motor) were later reported. The vector variable poles were defined by implemet Poles = [-1.25;-1.5;-1.6;-1.8]*2*pi. Then the vector of gains was also calculated using $K = \text{place}(A, B, \text{Poles})$, theta_zero was changed 0 from 0.02 in the matlab command window.

The closed-loop of the system was performed using value of K under Gain was entered in the block options and chosen Matrix ($K*u$) as the method of multiplication. Then, the file **pendulum_closed_loop.mdl** was used as a template in Simulink Gain block. This file was included two custom blocks for input signal generation (a sequence of steps and a sequence of ramps). Current gain value of the block called cart_position_reference was chosen to achieve steady-state gain of the closed-loop system equal one for $y_w = 0.35m$, it should be noted that for reference equal 4000 counts steady state output (i.e. cart position) is also 4000 counts. The simulation was performed for the following closed-loop simulations:

- a) Stabilization of pendulum for initial pendulum angle (variable **theta_zero**) equal 0.02 radians. Cart position reference (i.e. input to the block called **cart_position_reference** in **pendulum_closed_loop.mdl**) is zero.
- b) Stabilization of pendulum while cart position reference is a sequence of steps (output of the block called Trajectory 1 – Steps) and initial pendulum angle (variable **theta_zero**) is zero.

- c) Stabilization of pendulum while cart position reference is a sequence of ramps (output of the block called Trajectory 2 – Ramps) and initial pendulum angle (variable **theta_zero**) is zero.

Part 2

The weight position on the pendulum rod was adjusted using ruler such that distance from rod pivot centre to weight centre is equal to the value your controller had been designed for (i.e. $y_w = 0.35m$).

The power to the control box was turned OFF, ECP executive program was entered and background displayed. The control box was powered ON; on the displayed the file **invpend210SGD.cfg** was located and selected at local disk C of the system. It should be noted that the hardware is the brain and software is the mind behind the program on the system.

The first mass carriage was positioned at the center of travel and the pendulum rod was also oriented downward and motionless. The reading of **Encoder 1** was observed and reset on the system. Setup was entered and Setup Control Algorithm was also chose. Sampling time T_s was at 0.008840s and it can be reset if not.

The Algorithms **Invpend210sgdPS.alg** was selected via Edit Algorithm and Control algorithm editor, the values of $K_1 = -0.0265$, $K_2 = -0.0113$, $K_3 = 2.3710$ and $K_4 = 0.4181$ of the state feedback controller was loaded in the file menu of the editor and saved. It should be noted here that this process provides self-erecting functionality and implement a high performance control as the inverted pendulum once erected. The editor file was exit without any changes to the algorithm. The pendulum was displaced using ruler to approximate 20° and withdrawn quickly, if not (i.e. the controller is not active) the pendulum rod and carriage will be repositioned and the process will be repeated again. The real-time algorithm kicked the pendulum at properly phased movement of its oscillation cycle and it increased the amplitude of its swung. The pendulum approached the inverted position and algorithm switched to a control law that captured the pendulum in an inverted position, if not the process as to be repeated.

The Data Acquisition was setup via data menu and these items were selected Command Position, Encoder 1 and Variable Q10. Also the data sample period was at 2 meaning the data will be collected every second servo cycle. It should be noted here that in every case $2 * 0.00884 = 0.0177$ seconds. Also Variable Q10 was assigned in the real-time algorithm to same as Encoder 4 with an offset of 8192 counts (1/2 revolution) so that, origin was in the inverted position.

The trajectory 1 was entered via Command menu and step and setup was also selected. The step size was at 2000, dwell time = 4000ms and repetitions = 2. The execute was selected with Normal Data sampling and execute Trajectory 1 only was checked and ran the trajectory. Step move of 2000 counts a dwell of 4 seconds and return step move were noticed respectively. It was noted that base motion of the cart was reversed into the initial part of the step response non minimum phase behaviour of the controlled system. The base moved near its command position and subsequently returned. It was also noted that the pendulum rod moved initially in a direction ‘pointing forward’ the new set point as the base accelerated in that direction. It latter moved in the direction as the base decelerated and maintained near zero (vertical) orientation during regulation at the new position. All these actions were automatically executed by closed loop control.

The data was uploaded on the plotting menu and setup plot were also selected. Encoder 1 and Commanded Position were plotted on the left axis variableQ10 was also plotted on the right axis and time on the x-axis respectively.

For executing ramp trajectory the following steps were taken, the trajectory 1 was re-entered through command menu and ramp was selected, the following parameters were verified Distance = 2000 counts Velocity = 2000 counts/sec, Dwell time = 3000ms and the number of repetitions = 2.

Results and Discussion

Part 1

The simulation of an inverted pendulum for both step and ramp response at $y_w = 0.35m$ were presented in the graphs below.

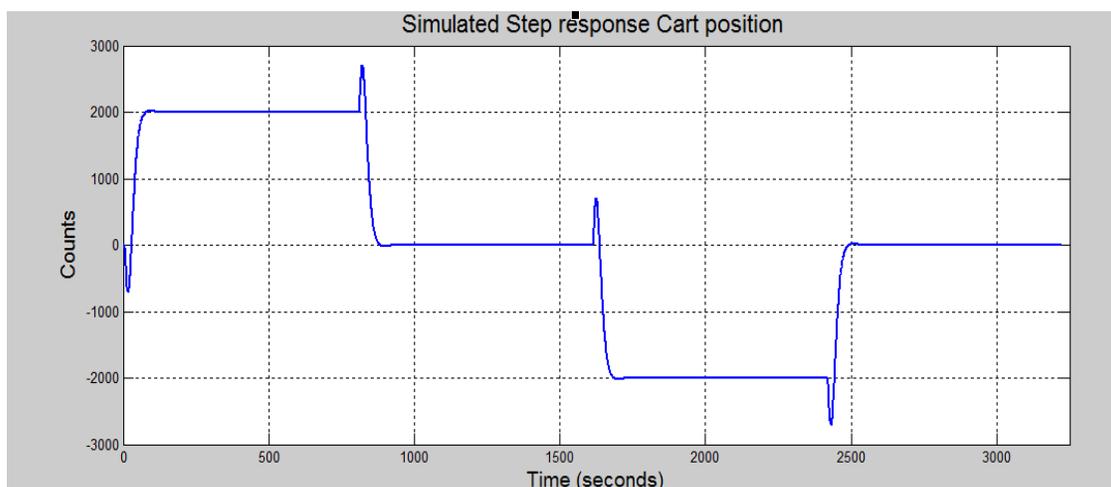


Figure 1: Simulated step response Cart position

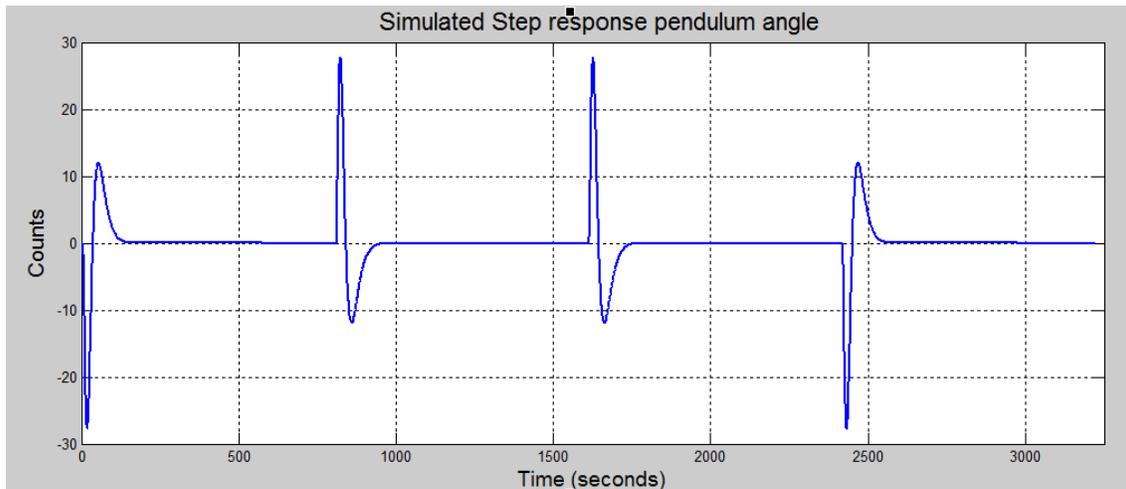


Figure 2: Simulated step response pendulum angle

Figure 1 and 2 shows the simulated step response of an inverted pendulum system. The non-minimum phase behaviour of the system is noticed. The swings-up occurred gradually, reaches initial maximum at relatively 100 second, responding to the bounded oscillations of the cart. The swing-up continues as the cart decelerates. Up to 2500 seconds the swing up controller is in control. Then, the state feedback transition algorithm takes over completely for the rest of the time, stabilizing the pendulum in the inverted position and homing the cart to the reference point. It should be noted that the swing up and stability of this system is around 2600 seconds.

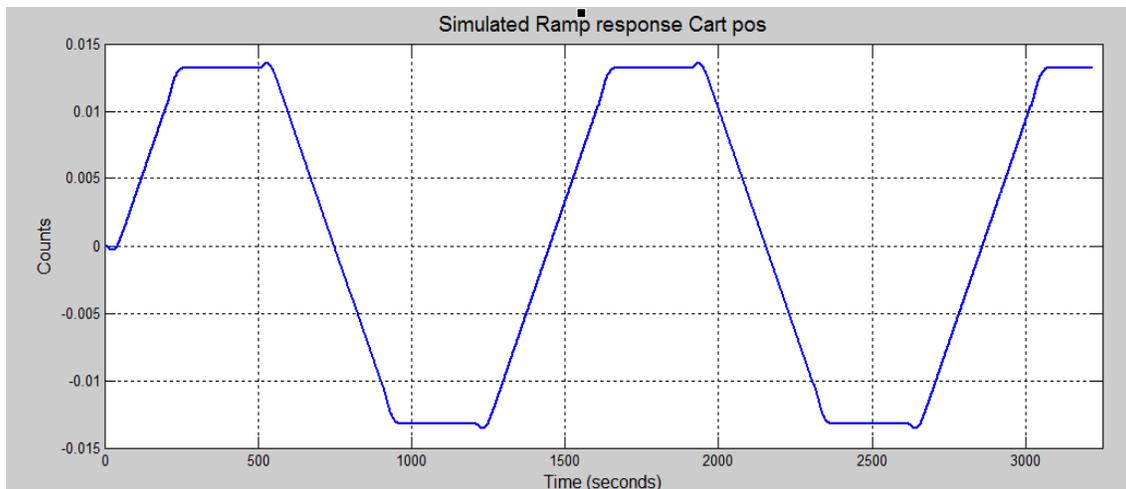


Figure 3: Simulated ramp response cart position

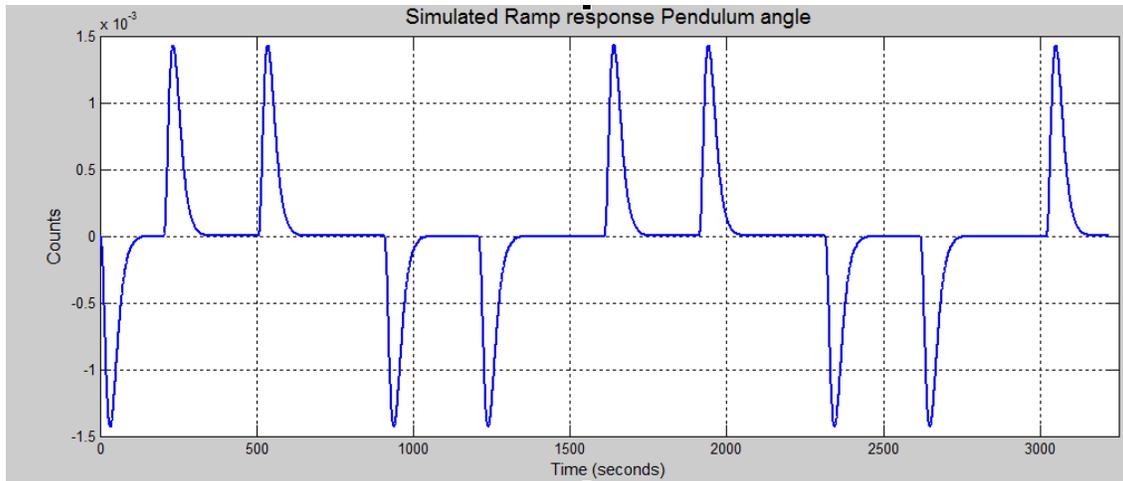


Figure 4: Simulated ramp response pendulum angle

Figure 3 and 4 shows the simulated ramp response of an inverted pendulum system. The non-minimum phase behaviour of the system is noticed. The swings-up occurred gradually, reaches initial maximum at relatively 250 second, responding to the bounded oscillations of the cart. The swing-up continues as the cart decelerates and accelerates. Up to 2500 seconds the swing up controller is in control. Then, the state feedback transition algorithm takes over completely for the rest of the time, stabilizing the pendulum in the inverted position and homing the cart to the reference point.

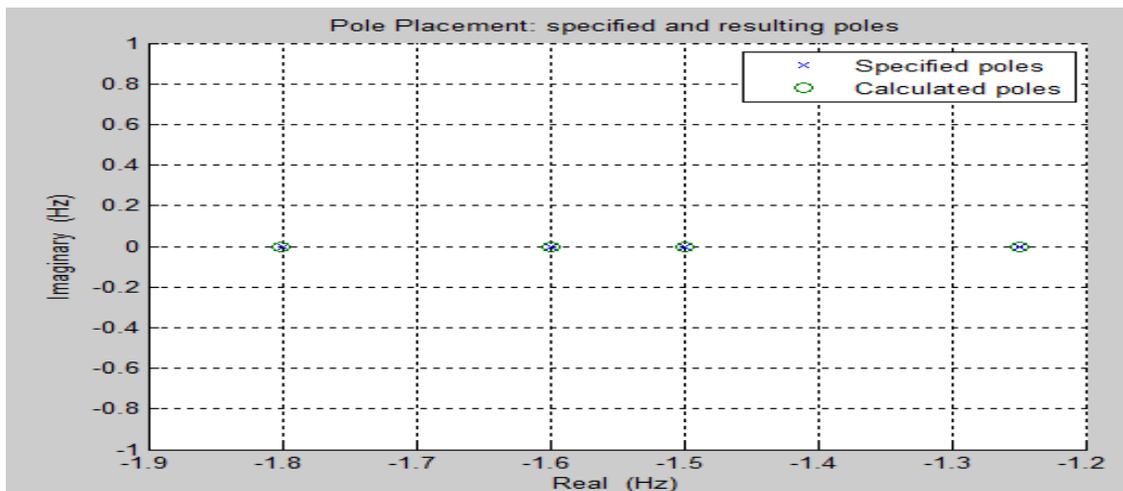


Figure 5: Pole placement specified and resulting poles

Figure 5 shows the stability of the simulated inverted pendulum with the specified and calculated poles values at -1.2, -1.5, -1.6 and -1.8 respectively. Since, the inverted pendulum is linear, the poles all lie inside the Left Hand Plane (LHP) and the system is said to be stable.

Part 2

Data were collected from experimental runs where each control scheme swings up the pendulum from an initially downwards position to an upright position and balances the pendulum around the unstable equilibrium point.

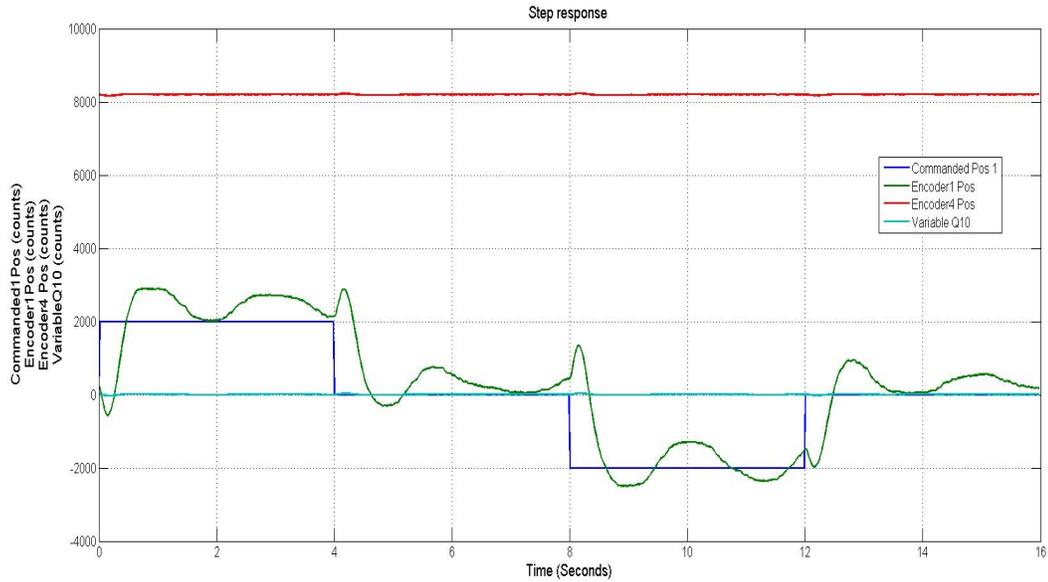


Figure 6: Step response for the pendulum angle and cart position

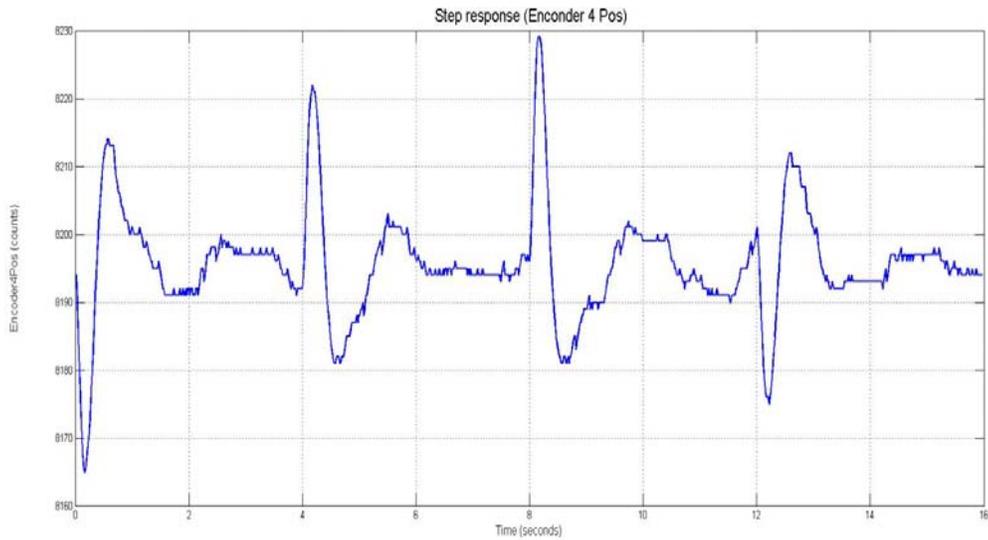


Figure 7: Step response for Encoder 4 position

Figure 6 shows the plot of Commanded 1 pos, Encoder 1, Encoder 4 and Variable Q10 (counts) against Time (seconds) during the closed-loop step response. Figure 7 shows the

Encoder 4 against Time separately, because of the scaling problem encountered during the plotting.

From the graph at the initial stage, the base motion is reversed, which shows non- minimum phase behaviour of the system (i.e. below zero). The swing of the pendulum rod (Encoder 1) increase gradually to its set direction moves above commanded position reaches it maximum position in approximately 1 second at 2700 counts as the cart accelerate in that direction. It tends to be stable at 2, 4,7,14 and 16 seconds respectively. It can also be shown here that it moves in opposite direction as the cart decelerates to maintain a vertical orientation during the new set position and steady state error is noticed from the graph. The Encoder 4 in figure 7 shows that in every 1 second it has one complete oscillation and has it maximum and minimum counts between 8229 and -8175.

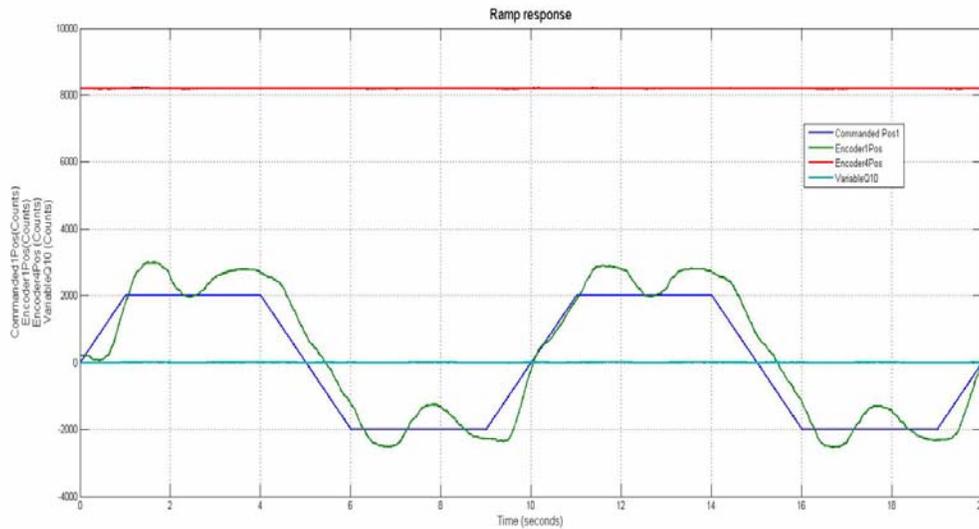


Figure 8: Ramp response for pendulum angle and cart position

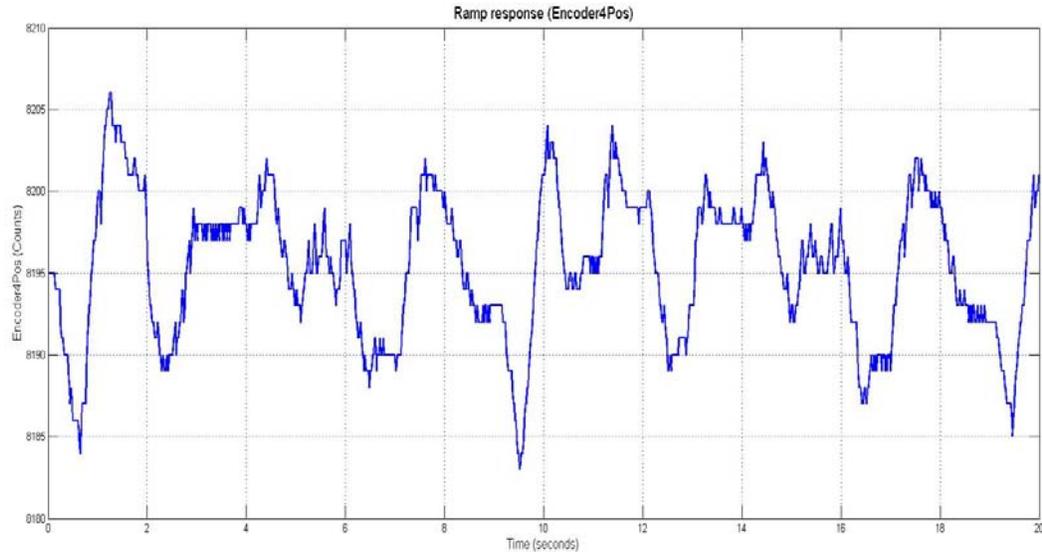


Figure 9: Ramp response for Encoder 4 position

Figure 8 shows the plot of Commanded 1 pos, Encoder 1, Encoder 4 and Variable Q10 (counts) against Time (seconds) during the close-loop step response.

At the initial stage from the graph, there is non-minimum phase behaviour of the system. The swing of the pendulum rod (Encoder 1) increase gradually to its set direction moves above commanded position reaches it maximum position in 1.8seconds at 2700 counts as accelerate in that direction. It tends to be stable at 3.5 and 13 seconds as shown in figure 8 and 9 respectively. In figure 9, Encoder 4 graph shows to complete one oscillation in every 1 second and has the maximum and minimum counts 8306 and -8176 respectively.

Comparison between the simulated and practical inverted pendulum

The results for the simulated and practical inverted pendulum systems in both step and ramp response were compared. The systems were both appeared to be stable and non – minimum phase behaviour was noticed. Although, there is no direct correlation between the time taken to swing the pendulum to its upright position for both simulated and physical systems due to the magnitude of gains. If a gain is too high, make the pendulum approach the upright position with too high a velocity and thus, the stabilizing controller will unable to balance the pendulum. On the other hand, a gain with too low values may not provide enough energy to the pendulum so that it can reach the upright position. Also, the reliability of the controller in performing the task varies depending on the gain selected. The variations in the oscillation of

the physical inverted pendulum can be caused by Coloumb friction, pinion backlash, motor dead-zone and magnetic hysteresis and mechanical imperfection.

Conclusion

The results presented in part 1 and 2 verify that the system designed and implemented in these experiments was successful. The K values (i.e. $K_1 = -0.0265$, $K_2 = -0.0113$, $K_3 = 2.3710$ and $K_4 = 0.4181$) obtained using poles placement method when the $y_w = 0.35\text{m}$ in the simulated inverted pendulum system and these K values were implemented in the physical laboratory inverted pendulum. The non – minimum phase behaviour of both simulated and laboratory inverted pendulum systems were noticed. Although, the results obtained show no direct correlation between time - taken to swing the pendulum to its upright position for the two systems. These variations may be due to high K values which make the system approach the upright position with too high velocity. The physical inverted pendulum shows high robustness for any external disturbance as the cart is homing back and front during the measurement as stability attained.

References

- [1] Control of an Inverted Pendulum [Online]. Available:
http://www.ece.ucsb.edu/~roy/student_projects/Johnny_Lam_report_238.pdf.
[Accessed: 19- Dec- 2014].
- [2] M. Bugeja, (2000) Non-Linear Swing-Up And Stabilizing Control Of An Inverted Pendulum System
- [3] U. Bogumil, Lecture note Control and Instrumentation De Montfort University Leicester, United Kingdom, 2013
- [4] U. Bogumil, Laboratory sheet Control and Instrumentation De Montfort University Leicester, United Kingdom, 2013