A Study of Prevalent Disease Ebola based on the SIR Infect Mathematics Model

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Abstract
Ebola is spreading in three countries in west Africa, we analyzed the three countries’s data statistical since Ebola outbreak, it is concluded that the early spread of the virus obeyed population growth model, based on the geographical location of the three annotations as well as the number of cases, roughly concluded that epidemic distribution of three units in the provinces in west Africa, through the study the growth and characteristics of every province in the past cases through the use of MATLAB fitting out the number of cases and the number of new cases, transmission speed, the number of health, the new number of rehabilitation, the death toll of the relationship between the equation. Data that can be used by preliminary study late new cases, to determine the required distribution, the number of different countries in different parts of the combined with the characteristics of local transportation system will be drug delivery to site, to avoid uneven and dispensing pharmacy not timely phenomenon; At the same time, governments should strengthen the epidemic protection propaganda, strictly implement isolation and resistance increase in the spread of the epidemic.

Keywords: On-demand Distribution; Relation Equation; The Effective Control; Data Fitting

1. Introduction
Infectious Disease Model has been researching for a long time [3]. In 1927, Kermack and McKendrick built the famous SIR Model for spread of disease when they researching the spread regularity of London in 1655 to 1666 and an epidemic of Bombay in 1906. It played an important role of the research of infectious diseases. And then they came up with the “Threshold theory” to distinguish weather a kind of disease is epidemic or not, laying a solid foundation to the Epidemic Dynamics. Herbert and others have researched a infectious disease model with quarantine item. When entering constants, the model will provide SIQS and SIQR model with bilinear and standard occurrence rate, which discuss the stability of Balance Point. Abroad, in 1760 Bernoulli had studied the spread of smallpox problem in mathematical model, 1906 Hamer repeated in order to make the measles popular reasons clearly, structured and analyzed a discrete time model, Below a critical value, the prevalence of malaria will be under control. 1927 Kermack and McKendrick in 1665 ~ 1666 in order to research the epidemic law of bubonic plague in London and epidemic law of the plague in mumbai in 1906, constructed the famous SIR bin model, throughout the year. SIS bin model is put forward in 1932. Analysis of the established mathematical model, put forward the disease is popular “threshold theory”, which laid a foundation for research on mathematics model of infectious diseases. Nearly, over the past 20 years, the international research progress on the dynamics of infectious diseases is extremely rapid, a large number of mathematical models are used to analyze all kinds of infectious diseases, the mathematical model related to contact transmission, vertical transmission, the spread of insect-borne spread in different ways, also consider the incubation period of the disease, to explore prevention and cross infection, age structure, space, factors related to migration and proliferation.

In our Model, we build an epidemic model that considers treatment and quarantine measures. After studying it, we conclude that governments should pay attention to propaganda for the Ebola epidemic, strengthen the treatment measures and implement the policy of isolation.
strictly. Only by that way can the Ebola virus epidemic of three West Africa countries be controlled.

Through the analyzing, the result shows that the drug-dependence has been improved by the bad habits (washing hands in one basin, eating together, etc.), unhealthy (burial) custom and poverty (water-lacking, semi-primitive living habits). It’s a terrible news because unless they change it efficiently, or they would still suffering the higher risk of being infected by other kinds of epidemics.

2. Model Building

This paper tries to build a model with treatment and quarantine in the case that we already have the medicine to cure Ebola virus disease and the vaccine to prevent it. In our model we effectively control the Ebola virus epidemic of Guinea, Liberia and Sierra Leone, and then we successfully stop the Ebola virus epidemic. At the end, we eradicating Ebola virus of West Africa, the three countries of West Africa don’t exist in the list of affected areas of WHO.

The following model is established based on the above analysis,

\[
\begin{align*}
S' &= \Lambda - \beta I_u S - \mu S + \delta (1 - p) \sigma (1 - q) I_u + u_2 I, \\
E' &= \beta I_u - (\mu + \sigma) E, \\
I_u' &= \delta p E - (\mu + \sigma + u_i) I_u, \\
I_i' &= \sigma q I_u + u_i I_u - (\mu + u_i) I_i.
\end{align*}
\]

We assume that all new import enter into S. From the model we can see that when \( t \geq 0 \), all non-negative initial values has non-negative solutions. We use reproducing matrix[4] to get the basic reproduction number:

\[
R_0 = \frac{\beta \Lambda \delta P}{\mu (\mu + \delta) (\mu + \delta + u_i)}. \tag{2}
\]

It is easy to get the disease free equilibrium:

\[
P_0 = (S_0, E_0, I_{u0}, I_{i0}) = \left( \frac{\Lambda}{\mu}, 0, 0, 0 \right) \tag{3}
\]

In regard to system, because when \( t \to \infty \), \( S + E + I_u + I_i \to \frac{\Lambda}{\mu} \), we use this limit equation and set \( I = \frac{\Lambda}{\mu} - S - E - I_u \), so we can discuss the system \( (I') \) only about \( S, E \) and \( I_u \) in the invariant set \( \Omega \), and

\[
\Omega = \left\{ (S, E, I_u) \left| S, E, I_u \geq 0, S + E + I_u \leq \frac{\Lambda}{\mu} \right. \right\}. \tag{4}
\]

In the disease free equilibrium, the Jacobian matrix of the system \( (I') \) is

\[
J = \begin{pmatrix}
-\beta I_u - \mu - u_2 & \delta (1 - p) - u_2 & -\beta S + \sigma (1 - q) - u_1 \\
\beta I_u & -(\mu + \delta) & \beta S \\
0 & \delta \rho & -(\mu + \sigma + u_i)
\end{pmatrix}. \tag{5}
\]

At first we analyses the stability of the disease free equilibrium. Set \( \lambda_1, \lambda_2, \lambda_3 \) as Jacobian matrix’s three eigenvalues at \( P_0 \), we get that \( \lambda_1 = -\mu - u_2 \), and

\[
\lambda_2 + \lambda_3 = -(\mu + \delta) - (\mu + \delta + u_i) < 0,
\]

\[
\lambda_2 \lambda_3 = (\mu + \delta)(\mu + \sigma + u_i)(1 - R_0),
\]

The three eigenvalues are negative, and at this time \( P_0 \) has local asymptotic stability. However, when \( R_0 > 1 \), there will be positive eigenvalues, \( P_0 \) is not stable. So we draw the conclusion that when \( R_0 < 1 \), the disease free equilibrium, \( P_0 \), has global stability.

We set former constant function, \( u_1 \) and \( u_2 \), as a function about treatment of time variable, \( t \). We suppose that \( 0 \leq u_i \leq 1 \) and \( u_i (i = 1, 2) \) is measurable function.

In order to minimize the quantity of patients infected by Ebola virus and minimize the social cost of the countries where the Ebola virus epidemic broke out in a certain time interval, we set our objective function as

\[
J(u_1, u_2) = \int_{t_0}^{t_f} \left[ E(t) + I_u(t) + I_i(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) \right] dt \tag{6}
\]

Because the integrand of the objective function is convex about \( u_i (i = 1, 2) \) and the right parts of the system(0) is bounded, we can see that \( u_1 \) and \( u_2 \) which optimize the objective function exist, and making that

\[
\Gamma = \left\{ (u_1, u_2) \in L^1 \left( t_0, t_f \right) | 0 \leq u_i \leq 1, i = 1, 2 \right\}. \tag{7}
\]

For the purpose of getting optimal solution, we use Lagrange multiplier. We make Lagrangian function: \( L(S, E, I_u, I_i) \) of system:

\[
L(S, E, I_u, I_i) = E(t) + I_u(t) + I_i(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) \tag{8}
\]

And we define Hamiltonian function of this problem as H:
\[ H = L(S,E,I_u,I_r) + \sum_{i=1}^{4} \lambda_i f_i = L(S,E,I_u,I_r) + \]
\[ \lambda_1 \left[ \Lambda - \beta I_u S - \mu S + \sigma (1-q) I_u + \delta (1-p) E + u_i(t) I_r \right] + \lambda_2 \left[ \beta I_u S - (\mu + \delta) E \right] + \lambda_3 \left[ \delta p E - (\mu + \delta + u_i(t)) I_u \right] + \lambda_4 \left[ \sigma q I_u + u_i(t) I_u - (\mu + u_i(t)) I_r \right] \]  

(9)

In the function, \( f_i \) (\( i = 1, 2, 3, 4 \)) represents the right parts of system, \( \lambda_i \) (\( i = 1, 2, 3, 4 \)) is adjoint variable.

Since \( 0 \leq u_i \leq 1 \) (\( i = 1, 2, 3, 4 \)), there will be three cases when considering control space \( \Gamma \):

(1) \( u_i^* = 0, \) if \( \frac{\partial H}{\partial u_i} < 0 \);

(2) \( 0 < u_i^* < 1, \) if \( \frac{\partial H}{\partial} = 0 \);

(3) \( u_i^* = 1, \) if \( \frac{\partial H}{\partial} > 0 \).

So it is with \( u_2^*(t) \). So we can see that

\[ u_1^* = \max \left\{ \min \left\{ \frac{(\lambda_3 - \lambda_4) I_u^*}{B_1}, 1, 0 \right\} \right\}, \]

\[ u_2^* = \max \left\{ \min \left\{ \frac{(\lambda_4 - \lambda_3) I_u^*}{B_2}, 1, 0 \right\} \right\}. \]

(10)

It can be seen that \( u_1^* \) and \( u_2^* \) can be calculated. We can use our conclusion to minimize the quantity of patients infected by Ebola virus and minimize the social cost of the countries where the Ebola virus epidemic broke out.

3. Results & Analysis

By assign values to the model according to the actual situation, we can see the trends of \( R_0 < 1 \) and \( R_0 > 1 \):
Only in this way, the Ebola virus epidemic of three West Africa countries can be controlled under a certain scope, and ultimately we can achieve the goal of eradication of Ebola.

4. Conclusions

Our suggestions are reasonable and objective, fully considered the common situation and the living atmosphere in West Africa. The solution is a prediction of the whole area, which has got a good result fitting well the information that are already known. However, the only problem troubles most is the lack of data, and the credibility about data which we have got. Something accord that in some days there are less total amount of people dead in Ebola than the day before, which confused us much in some time point. About the solution, there are still something have not been so reasonable in West Africa, and in spite of that, the model works well as if nothing happened.

References


