Solving Supply Chain Network Gravity Location Model Using LINGO

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Abstract—The manager identifies potential location in each region where the company has decided to locate the plant. It is useful when identifying suitable geographical location within a range. Gravity models are used to find location that minimizes the cost of transporting raw material from the supplier and finished goods to the markets served. This model also assumes that the transportation cost grows linearly with the distance between two points on the plane. In this paper we solve a gravity location problem using LINGO.

Index Terms—geographical, LINGO, potential, transportation.

I. INTRODUCTION

A company might consider investing in the construction of new facilities for a variety of reasons, such as, increasing its production capacity of an existing product, or extending its product range by new product introduction, or entering new markets with the existing and/or new products. Here facility refers to the smallest productive entity that manufactures a single commodity (or, at most a single family of commodities). A plant however, refers to a collection of facilities in the same location and, hence in general will be producing multiple commodities. Construction of a new facility therefore, might mean expansion of an existing plant if it takes place at that site, or otherwise would require opening a new plant. In many investment projects, decisions regarding the location and the size of a new facility to be established are interrelated since capacity acquisition costs are location dependent. A typical example is new facility investments in the international context, where subsidized financing as well as low tax rates are provided by the national governments to attract the multinational companies to locate production plants in their country.  In this case, it is clear that not only the fixed costs that occur due to opening the new facility at a particular site but also the capacity acquisition costs that vary with the size of the new facility are location specific.

A manager identifies potential locations in each region where the company has decided to locate a plant. As a preliminary step, the manager needs to identify the geographic location where potential sites may be considered. Gravity location models can be useful when identifying suitable geographic locations within a region. Gravity models are used to find locations that minimize the cost of transporting raw materials from suppliers and finished goods to the markets served. Next, we discuss a typical scenario in which gravity models can be used.

II. LITERATURE SURVEY

Hasan Selim and Irem Ozkarahan (1), have developed a model to select the optimum numbers, locations and capacity levels of plants and warehouses to deliver products to retailers at the least cost while satisfying desired service level to retailers. A maximal covering approach is used in statement of the service level.

Hadi Mohammad Bidhendi et al (2) have proposed a mixed integer linear programming model and solution algorithm for solving supply chain network design problems in deterministic, multi-commodity, single-period contexts. The strategic level of supply chain planning and tactical level planning of supply chain are aggregated to propose an integrated model. The model integrates location and capacity choices for suppliers, plants and warehouses selection, product range assignment and production flows.

Marc-André Carle, Alain Martel, Nicolas Zufferey (3), have proposed paper proposes an agent-based metaheuristic to solve large-scale multi-period supply chain network design problems. The generic design model formulated covers the entire supply chain, from vendor selection, to production–distribution sites configuration, transportation options and marketing policy choices. The model is based on the mapping of a conceptual supply chain activity graph on potential network locations.

Sanjay Melkote, Mark S. Daskin (4), have presented a mixed integer programming formulation of the facility location/network design problem. Computational experience with problems with up to 40 nodes and 160 candidate links have been reported.

Phuong Ng Thanh Ta, Nathalie Bostel, Olivier Pe’ton (5), have proposed a mixed integer linear program (MILP) for the design and planning of a production–distribution system. The study aims to help strategic and tactical decisions: opening, closing or enlargement of facilities, supplier selection, flows along the supply chain.

Jukka Korpela et al (6), have proposed a framework by which the risks related to a customer supplier relationship, the service requirements by the customers and the strategies of the supplier company can be included in production capacity.
allocation and supply chain design. The framework is demonstrated with a numerical example and it is based on integrating the analytic hierarchy process (AHP) and mixed integer programming (MIP).

Atefeh Baghalian, Shabnam Rezapour, Reza Zanjirani Farahani (7), have developed a stochastic mathematical formulation for designing a network of multiproduct supply chains comprising several capacitated production facilities, distribution centres and retailers in markets under uncertainty.

Ragheb Rahmaniani, Mohammad Saeid Mehrabad, Hojjat Ashouri (8), have proposed an extension of the capacitated facility location problem under uncertainty, where uncertainty may appear in the model’s key parameters such as demands and costs. They have developed the mathematical formulation in order to allow partial satisfaction by introducing penalty costs for unsatisfied demands. In general, this model optimizes location for predefined number of capacitated facilities in such a way that minimizes total expected costs of transportation, construction, and penalty costs of uncovered demands, while relative regret in each scenario must be no greater than a positive number ($p > 0$).

III. THE LINDO STORY

Since 1979, LINDO Systems software has been a favorite of business and educational communities alike. LINDO Systems has dedicated itself to providing powerful, innovative optimization tools that are also flexible and easy to use. LINDO Systems has a long history of pioneering powerful optimization software tools. In 1988, LINGO became LINDO Systems first product to include a full featured modeling language. Users were able to utilize the modeling language to concisely express models using summations and subscripted variables. In 1993, LINGO added a large scale nonlinear solver. It was unique in that the user did not have to specify which solver to use. LINGO would analyze the model and would engage the appropriate linear or nonlinear solver. Also unique to the LINGO’s nonlinear solver was the support of general and binary integer restrictions. With the addition of the nonlinear solver, LINGO essentially replaced GINO as LINDO Systems premier product for nonlinear optimization.

GINO made its debut in 1984 and was the first ever nonlinear solver available on the PC Platform. In 1994, LINGO became the first modeling language software to be included in a popular management science text. In 1995, The first Windows release of LINGO was shipped. Today, LINDO Systems continues to develop faster, more powerful versions.

IV. WHAT IS LINGO?

LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems concisely, solve them, and analyze the solution. Optimization helps you find the answer that yields the best result; attains the highest profit, output, or happiness; or achieves the lowest cost, waste, or discomfort. Often these problems involve making the most efficient use of your resources—including money, time, machinery, staff, inventory, and more. Optimization problems are often classified as linear or nonlinear, depending on whether the relationships in the problem are linear with respect to the variables.

LINGO includes a set of built-in solvers to tackle a wide variety of problems. Unlike many modeling packages, all of the LINGO solvers are directly linked to the modeling environment. This seamless integration allows LINGO to pass the problem to the appropriate solver directly in memory rather than through more sluggish intermediate files. This direct link also minimizes compatibility problems between the modeling language component and the solver components.

Local search solvers are generally designed to search only until they have identified a local optimum. If the model is non-convex, other local optima may exist that yield significantly better solutions. Rather than stopping after the first local optimum is found, the Global solver will search until the global optimum is confirmed. The Global solver converts the original non-convex, nonlinear problem into several convex, linear subproblems. Then, it uses the branch-and-bound technique to exhaustively search over these subproblems for the global solution. The Nonlinear and Global license options are required to utilize the global optimization capabilities.

V. THE MODELING FRAMEWORK

In this section, the provided mathematical formulation by Sunil Chopra, Peter Meindl and D V Kalra (9) for logistics network design problems is considered. Our presented model for deterministic SCND problems is largely inspired from this work.

Gravity models assume that both the markets and supply sources can be located as grid points on a plane. All distances are calculated as the geometric distance between two points on the plane. These models also assume that the transportation costs grows linearly with the quantity shipped. We discuss a gravity model for locating a single facility that receives raw material from the supply sources and ships finished product to markets. The basic input to the model are as follows: $x_n, y_n = $ coordinate location of either a market or supply source $n$ $F_n = $ cost of shipping one unit for one mile between the facility and either market or supply source $n$ $D_n = $ quantity to be shipped between facility and market or supply source $n$

If $(x, y)$ is the location selected for the facility, the distance $d_n$ between the facility at location $(x, y)$ and the supply source or market $n$ is given by

$$d_n = \sqrt{(x - x_n)^2 + (y - y_n)^2}$$

(1)

the total transportation cost (TC) is given by

$$TC = \sum_{n=1}^{k} D_n F_n$$

(2)

The optimal solution is one that minimizes the total TC in equ (2).

VI. NUMERICAL EXAMPLE

The following example was taken from the book by Sunil Chopra, Peter Meindl and D V Kalra (9).

Consider for example, Steel Appliances (SA), a
manufacturer of high quality refrigerators and cooking ranges. SA has one assembly factory located near Denver, from which it has supplied the entire United States. Demand has grown rapidly and the CEO of SA has decided to set up another factory to serve its eastern markets. The supply chain manager is asked to find a suitable location for the new factory. Three parts plants located in Buffalo, Memphis and St Louis will supply parts to the new factory, which will serve markets in Atlanta, Boston, Jacksonville, Philadelphia and New York. The coordinate location, the demand in each market, the required supply from each parts plant and the shipping cost for each supply source or market are shown in Table below.

Table 1: Input Data

<table>
<thead>
<tr>
<th>Sources/Markets</th>
<th>Transportation Cost $/Ton Mile (F_r)</th>
<th>Quantity in Tons (D_r)</th>
<th>Coordinates x, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Sources</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buffalo</td>
<td>0.90</td>
<td>500</td>
<td>700 1200</td>
</tr>
<tr>
<td>Memphis</td>
<td>0.95</td>
<td>300</td>
<td>250 600</td>
</tr>
<tr>
<td>St. Louis</td>
<td>0.85</td>
<td>700</td>
<td>225 825</td>
</tr>
<tr>
<td>Markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atlanta</td>
<td>1.5</td>
<td>225</td>
<td>600 500</td>
</tr>
<tr>
<td>Boston</td>
<td>1.5</td>
<td>150</td>
<td>1050 1200</td>
</tr>
<tr>
<td>Jacksonville</td>
<td>1.5</td>
<td>250</td>
<td>800 300</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>1.5</td>
<td>175</td>
<td>925 975</td>
</tr>
<tr>
<td>New York</td>
<td>1.5</td>
<td>300</td>
<td>1000 1080</td>
</tr>
</tbody>
</table>

VII. LINGO PROGRAM

MODEL:
SETS:
SOURCES_MARKETS: TRANSPORTATION_COST, QUANTITY, X,Y,D;
ENDSETS

DATA:
SOURCES_MARKETS = BUFFALO, MEMPHIS, ST_LOUIS, ATLANTA, BOSTON, JACKSONVILLE, PHILADELPHIA, NEW_YORK;
TRANSPORTATION_COST = 0.9, 0.95, 0.85, 1.5, 1.5, 1.5, 1.5, 1.5;
QUANTITY = 500, 300, 700, 225, 150, 250, 175, 300;
X = 700, 250, 225, 600, 1050, 800, 925, 1000;
Y = 1200, 600, 825, 500, 1200, 300, 975, 1080;
ENDDATA
MIN = TC;
TC = @SUM(SOURCES_MARKETS: ((XX-X)^2 + (YY-Y)^2)^0.5)*QUANTITY*TRANSPORTATION_COST);
END

VIII. COMPUTATIONAL EFFICIENCY

An intel CORE i5 processor with 4GB RAM was used to process the model. Branch and Bound solver was used.

A. Numerical Problem size
Total variables: 11
Nonlinear variables: 2
Integer variables: 0
Total constraints: 2
Nonlinear constraints: 1
Total nonzeros: 4

B. Run Time
The problem was solved in less than 1 second.
Fig 2: Result

IX. CONCLUSION

The manager thus identifies the coordinates $(x, y) = (681, 882)$ as the location of the factory that minimizes total cost TC. From a map, these coordinates are close to the border of North Carolina and Virginia. The precise coordinates provided by the gravity model may not correspond to a feasible location. The manager should look for desirable sites close to the optimal coordinates that have the required infrastructure and the appropriate worker skills available. Thus in this paper we have successfully used the LINGO program to solve the problem.

REFERENCES


