ABSTRACT

This paper is a proficient approach for solving the problem of design and analysis for Dynamic and Wear load of Bevel gear, principally to aid the industries and designers. Surrounded by the diverse optimization techniques the approach breaks new era by utilizing Geometrical Programming Technique, because this is one of the proficient and enhanced techniques to resolve non-linear equations of the complex and sensible design problems.

The research deals with the design of bevel gear train for lowest dynamic load and wear load. The load capacity of bevel gears is based on either bending or wears capacity whichever is lesser. The tangential force for passing on utmost power has been found by lowering the dynamic load or wear load as per the necessity. The arbitrary nature of design variables has been given appropriate deliberation and probability of fulfilling constraints equation has also been taken care of.

A clarifying instance of bevel gear train design has been considered. The bevel gears are designed and optimized using Geometric Programming technique, considering the nature of parameters, proper values are given to convince constraints. The problem taken is solved for optimization by lowering dynamic load on the gear as well as to lower the wear load on the gear. The manually computed results are compare for the diverse values achieved from the software implemented.

Keywords: Gear, Bevel Gear, Geometrical programming.

INTRODUCTION

Bevel Gears

To recognize the bevel gear tooth geometry, one might first monitor the case of straight bevel gears. If the generating rack used to obtain the cylindrical gear involutes is curved in a horizontal plane into a circular shape, it results in a crown gear which is used to derive the flank form of bevel pinion and gear. For straight bevel gears the crown gear or generating gear can be placed amid the pinion and gear assembly. Its center is located accurately at the intersection point of the pinion and gear shafts.

As a mental implement the crown gear should consist of a very thin material like aluminum foil. For all three elements it is to be in mesh at the similar time. The pinion is situated at the back surface of the crown gear and meshes with the negative teeth while the ring gear is located at the front surface of the crown gear and meshes with the positive teeth. If such an agreement is achievable then the kinematic coupling circumstances of the bevel gear set are satisfied which means the pinion and the gear can mesh with each other also.

A single outline of the generating gear generates a gear on its one surface and the mating member on its other surface. The outline and lead is instantly which causes a straight lead and an octoid outline on the generated teeth. The octoid essentially is the bevel gear analog of an involute. The octoid provides stable ratio and makes the gears insensible to displacements vertical to the pitch line.

Virtual Number of Teeth

The tooth profiles of bevel gear should be urbanized on a spherical plane. As a true expansion of a spherical surface into a plane is not possible, an estimate has to be made for which the virtual number of teeth is to be resolute. This limitation is defined as the number of teeth which a spur gear would have radius of which is equivalent to the back cone distance and having pitch of the bevel gear. This is called Tredgold’s approximation.

The virtual number of teeth is given by

\[ T_v = \frac{2\pi R_b}{P} \]

\[ = \frac{T}{\cos \Psi} \]

Where \( P \) is circular pitch and \( \Psi \) is pitch cone angle.
Force Analysis of Bevel Gear

Fig 1.2 Components of Tooth Force

For force investigation of pair of mating bevel gear, it is unspecified that the total force $P_R$ acts on the pitch point at the middle of the tooth width. The consequential however actually occurs somewhere between the midpoint and the back end of tooth but the fault due to above hypothesis is marginal.

Here we have represented the forces $F_R$, $F_{Ra}$, $F_Rr$ by $P_R$, pressure angle $\phi$ by $\alpha_R$ and pitch angle $\Upsilon$ correspondingly. The mean tooth force $P_R$ is resolved in three equally perpendicular components. They are tangential force $P_{Rt}$, radial force $P_Rr$ and the axial force $P_a$.

The tangential force is considered as following

$$P_t = (2 \frac{M_t}{d})$$

Where $M_t = \text{Transmitted torque} = 60 \times 10^6 \text{ (KW) } / (2\pi N) \quad \text{(N-mm)}$

$d = \text{Pitch circle diameter (mm)}$

Radial force, $P_r = P_t \tan \alpha_R \cos \Psi$

Axial force, $P_a = P_t \tan \alpha_R \sin \Psi$

If the shaft angle is $90^0$ then the following relation holds good

$$P_a = P_r$$

Dynamic Load on Gear Tooth

The tangential force performing on the bevel gear will be

$$P_t = (2 \frac{M_t}{d})$$

The above significance of tangential constituent therefore depends upon rated power and rated speed. In addition to this, there is a dynamic load acting which can be considered by two dissimilar methods i.e. approximate estimation by means of the velocity factor in the preliminary stages of gear design and accurate calculation by Spott’s equation in ending stages.

The effective load $P_{\text{eff}}$ between two meshing teeth is given below

$$P_{\text{eff}} = C_s P_t / C_v$$

$C_s = \text{Service factor}, C_v = \text{velocity factor}$

Velocity factor for bevel gear is given as here

$$C_v = 5.6 / (5.6 + \sqrt{v})$$

Where $v$ is the pitch line velocity in m/s.

In the last stage of gear design, when gear dimensions are recognized, errors are particular and the quality of gear is resolute, the dynamic load is calculated by equation derived by M.F.Spotts.

Depending upon the resources of the pinion and gear, there are three equations for the dynamic load:

(i) Steel pinion with steel gears

$$P_d = e N_p T_p b r_1 r_2 / 2530 \sqrt{(r_1^2 + r_2^2)}$$

(ii) C.I. pinion with C.I. gears
order to evade failure of gear tooth due to bending judgment with beam strength, or wear strength. In this is given as follows:

\[ P_d = eN_p T_p b \alpha_1 r_2 / 3785 \sqrt{(r_1^2 + r_2^2)} \]

(iii) Steel pinion with C.I. gears

\[ P_d = eN_p T_p b \alpha_1 r_2 / 3260 \sqrt{(r_1^2 + r_2^2)} \]

Where 

\[ P_d = \text{dynamic load (N)} \]
\[ e = \text{sum of error between two meshing teeth (mm)} \]
\[ N_p = \text{speed of pinion (RPM)} \]
\[ r_1, r_2 = \text{Pitch circle radii of pinion and gear (mm)} \]

The dynamic load is understood to be inclined at an angle \( \alpha_n \) to the tangent plane. The dynamic load \( P_d \) acts in the same direction as the resultant force \( P \).

Tangential component of the dynamic load is \( P_d \cos \alpha_n \cos \Psi \).

The effective load is given through

\[ P_e = C_s P_i + P_d \cos \alpha_n \cos \Psi \]

This is the effective load in the tangent direction for judgment with beam strength, or wear strength. In order to evade failure of gear tooth due to bending

\[ P_b > P_e \]

In addition to this dynamic load is also given by the equation as below:

\[ P_e = C_s P_i + 21V (Ceb+P_s) / 21V + (Ceb+P_s) \]

Where \( V \) is the pitch line velocity
\[ e = \text{the gear and pinion errors} \]
\[ b = \text{the face width.} \]

Wear Strength of Bevel Gear

Bevel gear is considered to be alike to a formative pinion and formative gear in a plane perpendicular to the tooth element. So the wear strength of bevel gear is given as follows:

\[ (S_w)_n = b'Qd'K \]

Where \( (S_w)_n \) = Wear strength perpendicular to the tooth element
\[ b' = b/\cos \Psi = \text{Face width along the tooth element} \]

\[ d' = d / \cos \Psi = \text{Pitch circle diameter of the formative pinion} \]

Substituting values in the above equation, we get

\[ (S_w)_n = bQdK / \cos^2 \Psi \]

The component of \( (S_w)_n \) in the rotation is denoted by \( S_w \).

Therefore \( S_w = (S_w)_n \cos \Psi \) Or

\[ S_w = bQdK / \cos \Psi \]

Above equation is known as Buckingham’s equation of wear strength. The wear strength \( S_w \) indicates the maximum tangential force that the tooth can transmit without pitting breakdown. It should be always greater than the effective force between the meshing teeth. The virtual number of teeth on the pinion and gear are \( T'_p \) & \( T'_g \) correspondingly. The gear factor \( Q \) for external bevel gear is through

\[ Q = 2 T'_g / (T'_p + T'_g) \]

The pressure angle in a plane perpendicular to the tooth element is \( \alpha_n \). The factor \( K \) is given as

\[ K = 0.16 \text{ (BHN / 100)}^2 \]

The given equation is applicable for steel gears with 20° normal pressure angles. In order to evade failure of gear tooth due to pitting \( S_w > P_e \)

METHODOLOGY & PROGRAMMING

Geometric Programming is a technique for the solution a category of nonlinear Programming problems. It was introduced by Richard Duffin, Clarence Zener and Elmor Peterson. It is applied to diminish functions which are in the form of Posynomials focus to constrain of the identical type. It differentiates from other optimization methods in the prominence it places upon the comparative magnitudes of the terms of the objective function rather than the variables. As an alternative of finding the optimal values of design variables earliest, geometric Programming initial finds the optimal value of the objective function. This characteristic is particularly beneficial in situations where the optimal value of the objective function may be all that is of interest. In such cases, the computation of the optimum design vectors can be misplaced. Another benefit of geometric Programming is that it
frequently reduces a complex optimization problem to one relating a set of immediate linear algebraic equations. The main drawback of the method is that it needs the objective function and the constraints in the outline of Posynomials.

**Posynomial**

In an engineering design condition, often the objective function (like the total cost) \( f(X) \) is described by the sum of numerous component costs \( U_i(X) \) as

\[
f(X) = U_1 + U_2 + \ldots + U_N
\]

In many cases, the component cost \( U_i \) can be articulated as a power function

\[
U_i = c_i x_1^{a_{1i}} x_2^{a_{2i}} \ldots x_n^{a_{ni}}
\]

Where the coefficients \( c_i \) are positive constants, the exponents \( a_{ij} \) are real constants (positive, zero or negative) and the design parameters \( x_1, x_2, \ldots, x_n \) are taken to be positive variables. The functions like \( f \), because of the positive coefficients variables and real exponents are known as Posynomials. For instance, \( f( x_1, x_2, x_3) = 8 - 5x_1 + 21x_2 + 7x_1^2 - 3x_1x_2 + 5x_3 \) is a Posynomial. If the natural formulation of the optimization difficulty does not lead to Posynomial functions, geometric programming methods can still be used to resolve the problem by replacing the real functions by a set of empirically fitted Posynomials over a large range of the parameters \( x_i \).

**Design & Optimization of Bevel Gear for Minimum Dynamic Load**

**Flow Chart of Optimizing Gear for Lowest Dynamic Load by Geometric Programming method**

**Design of Bevel Gear for Lowest Dynamic Load**

**Dynamic Load between gear teeth**

The determination of the dynamic load between gear teeth is a complicated problem because of diverse assumptions involved in the design and magnitude of the errors are unknown etc. So making an allowance for all this the equation for the dynamic load is described by

\[
P_d = \left[2e \sqrt{Km_e}\right] / t \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.1)
\]

Where

\[
e = \text{combined error of mating of two gears (mm)}
\]

\[
t = \text{time for passage of a tooth through the contact zone}
\]

\[
t = 60 / (T.N) \text{ seconds}
\]

where

\[
T = \text{No. of teeth of gear}
\]

\[
N = \text{speed of gear in RPM}
\]

\[
K = \text{spring constant for the pair of machine gear teeth}
\]

If both gears are made of steel, then \( K = 11500 \) bN/mm

If gear is made of steel and pinion of C.I., then \( K = 8000 \) bN/mm

\[
m_e = \text{Equivalent mass for two gears (Kg)}
\]

Gears are prepared in the form of rim or ring in which teeth are cut with inertia of the sprockets or web enough to neglect.
The spring constant of a pair of gear will vary while a tooth is passing through the contact cone since one, two or sometime more, pair may be contact at dissimilar times. During the central portion of the passage a single pair is in contact which must carry the whole load.

The dynamical system of two gears is considered as the masses \( m_1 \) and \( m_2 \) concentrated at the pitch circles connected by a spring comprising of two teeth. For such a system, the effectual \( m_e \) is given by the equation

\[
1/m_e = 1/m_1 + 1/m_2
\]

The mass \( m_1 \) of the pinion and \( m_2 \) of gear is equal to

\[
m_1 = \pi r_1^2 b \frac{\gamma_1}{2g}
\]

\[
m_2 = \pi r_2^2 b \frac{\gamma_2}{2g}
\]

Where \( b \) = thickness in axial direction (mm)
\( g \) = Acceleration due to gravity
\( = 9806.6 \text{ mm/s}^2 \)

\( \gamma_1, \gamma_2 \) = weight density of pinion and gear respectively in N/mm\(^3\)

Considering steel pinion and steel gear combination and taking \( \gamma_1 = \gamma_2 = 0.0000768 \text{ N/mm}^3 \)

Then

\[
1/m_e = \frac{81270000}{b} \left[ \left( \frac{r_1^2 + r_2^2}{r_1^2 r_2^2} \right) \right]
\]

Substituting all these in Eq. (4.1) we get,

\[
P_d = \frac{\epsilon T_1 b r_1 r_2}{\left( 2530^4 \sqrt{(r_1^2 + r_2^2)} \right)}
\]

**Force Analysis Of Bevel Gear**

The resultant force \( P \) acting on the tooth of a bevel gear is resolved into three components \( P_r, P_a, \) and \( P_a \) as shown in fig 4.1. Where \( P_1 \) = Tangential component (N)

\[
P_1 = \text{Tangential component (N)}
\]

\[
P_r = \text{Radial component (N)}
\]

\[
P_a = \text{Axial or Thrust component (N)}
\]

From triangle ABC

\[
P_r = P \sin \alpha \quad \text{......... (a)}
\]

\[
BC = P \cos \alpha \quad \text{......... (b)}
\]

From triangle BDC

\[
P_a = BC \sin \Psi = P \cos \alpha \sin \Psi \quad \text{......... (c)}
\]

\[
P_1 = BC \cos \Psi = P \cos \alpha \cos \Psi \quad \text{......... (d)}
\]

From eqn. (c) and (d)

\[
P_a = P_1 \tan \Psi \quad \text{......... (3)}
\]

From eqn. (a) and (b)

\[
P_r = P_1 \left\{ \frac{\tan \alpha}{\cos \Psi} \right\} \quad \text{......... (4)}
\]

The cases explained above are the cases of straight bevel gear. Now we will talk about spiral bevel gear. Different forces acting on spiral bevel gear are given as following:

Tangential Force, \( P_1 = (2 M_t/d) \)

Radial force, \( P_r = P_1 \left( \tan \alpha \cos \Psi - \sin \beta \sin \Psi \right) / \cos \beta \)

Where \( \beta \) is the mean spiral angle

Axial force, \( P_a = P_1 \left( \tan \alpha \sin \Psi + \sin \beta \cos \Psi \right) / \cos \beta \)

The tangential component is calculated from the relationship

\[
P_1 = (2 M_t/d)
\]

Where \( M_t \) = Transmitted torque = \( 60 \times 10^6 \text{ (KW)} / (2\pi N) \quad \text{(N-mm)} \)

\( d \) = Pitch circle diameter (mm)

The above equation is utilized to resolve the three components of the resulting tooth force

The above value of tangential component therefore depends upon rated power and rated speed. In addition to this, there is a dynamic load acting which can be calculated by two diverse methods i.e.
approximate estimation by means of the velocity factor in the preliminary stages of gear design and precise calculation by Spott’s equation in final stages.

The effective load $P_e$ between two meshing teeth is given by

$$P_e = C_s P_t / C_v$$

$C_s$ = Service factor, $C_v$ = velocity factor

Velocity factor for bevel gear is given as

$$C_v = 5.6 / (5.6 + \sqrt{v})$$

Where $v$ is the pitch line velocity in m/s.

In the last stage of gear design, when gear dimensions are notorious, errors are specific and the quality of gear is determined, the dynamic load is deliberated by equation derived by M.F.Spotts.

Depending upon the materials of the pinion and gear, there are three equations for the dynamic load are

(i) Steel pinion with steel gears

$$P_d = e N_p T_p b r_1 r_2 / 2530 \sqrt{(r_1^2 + r_2^2)}$$

(ii) C.I. pinion with C.I. gears

$$P_d = e N_p T_p b r_1 r_2 / 3785 \sqrt{(r_1^2 + r_2^2)}$$

(iii) Steel pinion with C.I. gears

$$P_d = e N_p T_p b r_1 r_2 / 3260 \sqrt{(r_1^2 + r_2^2)}$$

Where

- $P_d$ = dynamic load (N)
- $e$ = sum of error between two meshing teeth (mm)
- $N_p$ = speed of pinion (RPM)
- $r_1$, $r_2$ = Pitch circle radii of pinion and gear (mm)

The dynamic load is understood to be inclined at an angle $\alpha_n$ to the tangent plane. The dynamic load $P_d$ acts in the same direction as the resultant force $P$.

Tangential component of the dynamic load is $P_d \cos \alpha_n \cos \Psi$.

The effective load is given by

$$P_e = C_s P_t + P_d \cos \alpha_n \cos \Psi$$

This is the effective load in the tangent direction for evaluation with beam strength, or wear strength. In order to avoid breakdown of gear tooth due to bending

$$P_b > P_e$$

Where $P_b$ = Bending load of the gear and is given by

$$P_b = m_n b \sigma_b Y (1 - b/A_0) / \cos \beta$$

Where

- $m_n$ = Normal module of gear (mm)
- $b$ = Face width of gear (mm)
- $\sigma_b$ = Safe bending stress (N/mm$^2$)
- $Y$ = Lewis form factor based on Virtual number of teeth
- $A_0$ = Back Cone Distance

The load capacity of pair of gear is based on bending capacity. The tangential force $P_t$ for transmitting the maximum power is found by minimized $P_d$ and can be spoken by considering the service factor as well as the factor of security as unity

The equation is given by:

$$P_e = P_t + P_d$$

As $P_t - P_e - P_d$

The problem now is to reduce objective function given by equation

$$P_e = C_s P_t + P_d \cos \alpha_n \cos \Psi$$

Satisfying the constraint equation

$$P_b N_2 \cos \beta / 2 \pi r_2 b \sigma_b Y \cos \Psi (1 - b/A_0) \leq 1$$

Geometric Programming Implementation

If the design parameters are $r_1$ and $r_2$ are taken as random variable because of mechanized tolerance etc. their value will vary about mean value. If these random variables follow normal distributions the mean values and standard deviation are $r_1$, $r_2$ and $\sigma_1$, $\sigma_2$ correspondingly, then new objective function in
equivalent deterministic form is given by equation
\[ P_{dD} = P_e + \left[ \sum_{i=1}^{n_i} \left( \frac{dP_e}{dr} \right)^2 r_i, \sigma^2_n \right]^{1/2} \]  
(4.2)

If the constraint equation \( g_j \) is satisfied with a probability \( P_{j} \) and normal variation for probability \( P_{j} \) is given \( \Phi_j (P_j) \) then the new constraint equation in equivalent deterministic form is given by
\[ g_{jd} = g_j - \Phi_j (P_j) \left[ \sum_{i=1}^{n_i} \left( \frac{dP_e}{dr} \right)^2 r_i, \sigma^2_n \right]^{1/2} \]  
(4.3)

Lastly the problem reduces to minimize the objective function given by equation (4.2) fulfilling constraint equation (4.3)

**Design & Optimization of Bevel Gear for Minimum Wear Load**

**Flow Chart of Optimizing Gear for lowest Wear Load By Geometric Programming method**

**Design of Bevel gear for Lowest Wear Load**

**Wear Strength of Bevel Gears**

Bevel gear is measured to be equal to a formative pinion and formative gear in a plane perpendicular to the tooth element. So wear strength of bevel gear is given as
\[ (S_w)_n = b'Qd'K \]  
(5.1)

Where  \( (S_w)_n \) = Wear strength perpendicular to the tooth element

\( b' = b/\cos \Psi \) = Face width along the tooth element

\( d'_p = d_p/\cos \Psi \) = Pitch circle diameter of the formative pinion

Substituting these values in the above equation, we get
\[ (S_w)_n = bQdK/\cos \Psi \]  
(5.2)

The component of \( (S_w)_n \) in the rotation is denoted by \( S_w \)

Therefore \( S_w = (S_w)_n \cos \Psi \)

Or \( S_w = bQdK/\cos \Psi \)  
(5.3)

Above equation is known as Buckingham’s equation of wear strength. The wear strength \( S_w \) indicates the maximum tangential force that the tooth can pass on without pitting failure. It should be always more than the efficient force between the meshing teeth. The virtual number of teeth on the pinion and gear are \( T'_p \) & \( T'_g \) correspondingly. The gear factor \( Q \) for external bevel gear is given through

\[ Q = \frac{2 T'_g}{T'_p + T'_g} \]  
(5.4)

The pressure angle in a plane perpendicular to the tooth element is \( \alpha_n \). The factor \( K \) is given through
\[ K = 0.16 (BHN/100)^2 \]

The above equation is appropriate for steel gears with 20\(^o\) normal pressure angles. In order to evade breakdown of gear tooth due to pitting
\[ S_w > P_e \]

**Force Analysis of Bevel Gear**

The wear load on bevel gear is understood to be at an angle \( \alpha_n \) to the tangent plane as shown in fig. 4.1 in last chapter

Referring to fig we have
\[ S_r = S \sin \alpha_n \]  
(5a)

\[ BC = S \cos \alpha_n \]  
(5b)

From fig we have
\[ S_a = BC \sin \Psi = S \cos \alpha_n \sin \Psi \]  
(5c)
From eqn. (a) and (b) the tangential component of wear strength is given by

\[ S_t = BC \cos \Psi = S \cos \alpha_n \cos \Psi \]  \hspace{1cm} (d)

Consequently the tangential component of the dynamic load is given by

\[ S_t = S \tan \Psi \]  \hspace{1cm} (e)

From eqn. (a) and (b)

\[ S_t = S_t \{\tan \alpha_n / \cos \Psi\} \]  \hspace{1cm} (f)

The tangential component of wear strength is considered from the association

\[ S_t = (2 M_t/d) \]

Where \( M_t \) = transmitted torque (N-mm)

\[ d = \text{pitch circle diameter (mm)} \]

Where \( M_t = 60 \times 10^6 \text{ (KW) / (2\pi N)} \) (N-mm)

The direction thrust components depend upon the hand of the pitch cone angle, on the direction of rotation and on whether the driving or driven gear is under concern.

Consequently tangential component of the dynamic load is given by

\[ S_d \cos \alpha_n \cos \Psi \]

And effective load is given by

\[ S_e = C_s S_t + S_d \cos \alpha_n \cos \Psi \]  \hspace{1cm} (g)

Where \( S_d \) is calculated in a same fashion as \( P_d \).

This is one effective load on the tangential direction for evaluation with beam strength.

In order to evade breakdown of gear tooth due to bending

\[ S_b \geq S_e \]

Where

\[ S_b = \text{Bending load of the gear and is given by} \]

\[ S_b = m_n b \sigma_b Y (1-b/A_0) / \cos \beta \]

Where

\[ m_n = \text{Normal module of gear (mm)} \]

\[ b = \text{Face width of gear (mm)} \]

\[ \sigma_b = \text{Safe bending stress (N/mm}^2) \]

\[ Y = \text{Lewis form factor based on Virtual number of teeth} \]

\[ A_0 = \text{Back Cone Distance} \]

\[ S_b = \frac{2 \pi \tau_2 b \sigma_b Y \cos \Psi (1-b/A_0) / N_2}{d} \]

\[ \text{Or } S_b N_2 / 2 \pi \tau_2 b \sigma_b Y \cos \Psi (1-b/A_0) \leq 1 \]  \hspace{1cm} (i)

The load capacity of pair of gear is based on bending capacity. The tangential force \( S_t \) for passing on the maximum power is found by minimized \( S_t \) and can be spoken by considering the service factor as well as the factor of safety as unity

The equation is given by:

\[ S_e = S_t + S_d \]

As \( S_t = S_e + S_d \)

The problem now is to reduce objective function given by equation

\[ S_e = C_s S_t + S_d \cos \alpha_n \cos \Psi \]  \hspace{1cm} (j)

Satisfying the constraint equation

\[ P_n N_2 / 2 \pi \tau_2 b \sigma_b Y \cos \Psi (1-b/A_0) \leq 1 \]

**Geometric Programming Implementation**

If the design parameters are \( r_1 \) and \( r_2 \) are in use as random variable because of manufacturing tolerance etc. their value will fluctuate about mean value. If these random variables chase normal distributions the mean values and standard deviation are \( r_1, r_2 \) and \( \sigma_1, \sigma_2 \) correspondingly, then new objective function in alike deterministic form is given by equation

\[ S_{ad} = S_e + \left[ \sum_{i=1}^{n} (dS_e/dr_i)^2 \sigma^2_i \right]^{1/2} \]  \hspace{1cm} (5.3.1)

If the constraint equation \( g_j \) is fulfilled with a probability \( S_j \) and normal variation for probability \( S_j \) is given \( \Phi_j \) (\( S_j \)) then the new constraint equation in equivalent deterministic form is given by

\[ g_{ad} = g_j - \Phi_j \sum_{i=1}^{n} (dS_e/dr_i)^2 \sigma^2_i \]  \hspace{1cm} (5.3.2)

Lastly the problem reduces to minimize the objective function given by equation (5.3.1) satisfying constraint equation (5.3.2)
RESULTS AND DISCUSSION

The software is implemented for obtaining the optimum design of bevel gear with lowest dynamic load and wear load as optimization standard. Analysis of the software representation was done with the help of manually considered results.

Analysis of result of design of bevel gear and optimizing it for lowest dynamic load

As shown in flowchart in section 4.1 of chapter 4, the user has to input a variety of parameters i.e. the desired gear ratio, module of gears and pitch cone angle of the gear. The software for design of bevel gear was tested for diverse values of modules of gear keeping all other parameters as constant.

Among all the parameters tested the software is compared for its compatibility with the mathematically solved example. In the given problem, for given module the gear under design concern will have 9 diverse outputs for 9 diverse values of \( w_2 \) which fluctuates between 0.6 and 1.4 for a selected value of weight \( w_2 = 1.0 \), the result computed mathematically is similar as that of what we have got from the output of the computer program within allowable bound of error. Hence it can be safely concluded that the software developed serves its purpose of determining the gear design parameters for exacting material and power transmission with minimization of dynamic load of gear.

But, the gear dimensions computed are not its optimized values. To optimize the computed gear parameters, a constraint equation is utilized to examine the design values. It has been found that not all the gear dimensions computed for a given set of input has optimum values of gear dimensions. Only those values of radius of gear, \( r_2 \) will be optimum values of gear which are greater than or equal to 194.25 mm.

Comparing our program for \( m = 3 \) and gear ratio = 4, only few gear radius, \( r_2 \) for \( w_2 \) equal to 1.1, 1.2, 1.3, 1.4 satisfies the constraint equations and hence they are the optimum value of gear radius satisfying the constraint and hence any of values can be selected for an optimized values of gear radius satisfying all design circumstances in the given problem. It can be accomplished from the results that:

- Designing of gear for lowest dynamic load is free of hardness of gear material.

- Not all the computed radii of gear, \( r_2 \) are optimized values. For optimization it must convince the constraint equation.

- For a predefined weight \( w_2 \) the value of radius \( r_2 \) increases with increase in the value of the module of gear.

- For lower values of module, few values of \( r_2 \) are in optimized choice. And as the values of module increases the range will also increase. It is experiential that module \( m = 4 \) and above the optimized values of \( r_2 \) does not drop below for \( w_2 = 1.0 \). Therefore for higher values of module, we don’t have optimized value of \( r_2 \) for lower values of weight \( w_2 \) i.e. \( w_2 = 0.90 \) or less.

Analysis of result of design of bevel gear and optimizing it for lowest wear load

As shown in flowchart in section 5.1 of chapter 5, the user has to input the diverse parameters i.e. the desired gear ratio, module of gears, pitch cone angle of the gear and BHN. The software for design of bevel gear was tested for diverse values of modules of gear keeping all other parameters as steady.

Surrounded by all the diverse parameters, i.e. gear ratio, pitch angle and BHN value, the software is compared for its compatibility with the manually calculated results. In the given problem, for given module the gear under design contemplation will have 9 different outputs for 9 different values of \( w_2 \) which varies between 0.6 and 1.4.

For a chosen value of weight \( w_2 = 1 \), the result estimated manually is same as that of what we have got from the output of the computer geometric program within permissible limit of error. Hence it can be safely accomplished that the software developed serves its function of determining the gear design parameters for particular material and power transmission with minimization of wear load of gear.

But, the gear dimensions estimated are not its optimized values. To optimize the calculated gear parameters, a constraint equation is used to examine the design values. It has been found that not all the gear dimensions calculated for a given set of input has optimum values of gear dimensions. Only those values of radius of gear, \( r_2 \) will be optimum values of gear which are greater than or equal to 143.57 mm.

Comparing our program for \( m = 3 \), BHN = 300 and gear ratio = 4, only few gear radius \( r_2 \) for \( w_2 \) 1.1, 1.2, 1.3, 1.4 satisfies the constraint equations and hence they are the optimum values of gear fulfilling the
constraint and hence any of values can be selected for an optimized values of gear radius satisfying all design conditions in the given problem. It can be accomplished from the results that:

- Not all the calculated radius of gear \( r_2 \) is optimized values. For optimization it must satisfy the constraint equation.
- For a given weight \( w_2 \) the value of radius \( r_2 \) increases with increase in the value of the module of gear.
- For smaller values of module, few values of \( r_2 \) are in optimized range. And as the values of module increases the range will also increase.
- Designing of gear for minimum dynamic load is dependent of hardness of gear material. For a given module as the BHN number increases the number of radius of gear \( r_2 \) falling in the range in the optimized range decreases.

**Limitations of Software**

Following are the limitations of software:

1. Software is developed for a specific problem of gear design, with a given material.
2. Software is developed for a given power to be transmitted from the gear train so no flexibility of power is there in the software.

**Conclusion**

The software implemented in this project will aid in making judgments concerning the design of bevel gear and optimizing it for lowest dynamic and wear load. The characteristic of the software is that in order to integrate subjectivity, the judgment maker can provide its own design parameters, such as pitch angle of the gear, module, gear ratio, BHN.

The thesis presents a bevel gear design method based on Geometric Programming Technique of optimization which is a conception till now not employed in the bevel gear designing problem. The literature survey helped in knowing about the efforts being carried out in the field of gear designing and Geometric Programming Technique. It recognizes the requirements for, and processes the information about relative significance of all design parameters for given relevance, without which inter – parameter variable comparison could not be accomplished. It productively presents the results of this information concerning the difference in these design parameters of gear in order to determine in the appropriateness for given relevance.

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