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A Suzuki-Type Common Fixed Point Theorem for generalized (ψ, φ)- weak contractions

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Abstract

Common fixed point results are presented for generalized (ψ, φ) - weak contractive mappings with constants in complete metric spaces. Our results extend previous results of Chugh (2014), Đorić (2009), Zhang and Song (2009), as well as of Kikkawa and Suzuki (2008), Rhoades (2001), Nadler (1969) and others.

***MSC*:** 54H25 and 47H10

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1. Introduction

Let (X, d) be a complete metric space and T be a self map on X. Then T is called a contraction if there exists $r \in [0, 1)$ such that

$$d(Tx, Ty) \le r\, d(x, y) \quad \text{for all } x, y \in X.$$

We know that if X is complete, then every contraction has a fixed point (Banach contraction mapping principle).

The following general common fixed point theorem is due to Sastry and Naidu [18].

Theorem 1.1. Let X be a complete metric space and $S, T: X \to X$. Assume there exists $r \in [0, 1)$ such that for every $x, y \in X$,

$$d(Sx, Ty) \le r \max\left\{ d(x, y), d(x, Sx), d(y, Ty), \frac{d(x, Ty) + d(y, Sx)}{2} \right\}. \tag{1.1.}$$

Then S and T have a unique common fixed point.

Very recently, Suzuki [22] introduced a weaker notation of contraction and obtained the following theorem, which is a new type of generalizations of the classical Banach contraction principle.

Theorem 1.2. Define a non-increasing function θ from $[0, 1)$ onto $\left(\frac{1}{2}, 1\right]$ by

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \le r \le \frac{1}{2}(\sqrt{5}-1) \\ \frac{1-r}{r^2} & \text{if } \frac{1}{2}(\sqrt{5}-1) \le r \le \frac{1}{\sqrt{2}} \\ \frac{1}{1+r} & \text{if } \frac{1}{\sqrt{2}} \le r < 1. \end{cases}$$

Let X be a complete metric space and let $T: X \to X$. Assume there exists $r \in [0, 1)$ such that for each $x, y \in X$,

$$\theta(r)\, d(x, Tx) \le d(x, y) \text{ implies } d(Tx, Ty) \le r\, d(x, y).$$

Then T has a unique fixed point.

This type of generalization of contraction mapping has been a very active field of research during last five years. Suzuki contractive condition has been dealt with in a number of papers [4], [5], [7], [9-15] and [19-23]. A mapping $T: X \to X$ is said to be φ – weak contractive if there exists a map $\varphi: [0, +\infty) \to [0, +\infty)$ with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ such that

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) \quad \text{for all } x, y \in X.$$

The concept of φ−weak contractive mappings was defined by Daffer and Kaneko [3] in 1995.

Rhoades [16] proved the following fixed point theorem for φ – weak contractive single-valued map generalizing the Banach contraction principle.

Theorem 1.3. Let (X, d) be a complete metric space and let $T: X \to X$ be a map such that

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)), \quad \text{for all } x, y \in X,$$

where $\varphi: [0, +\infty) \to [0, +\infty)$ is a continuous and non-decreasing function with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$. Then T has a unique fixed point.

Also, two maps $S, T: X \to X$ are called generalized φ – weak contraction if there exists a map $\varphi: [0, +\infty) \to [0, +\infty)$ with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ such that

$$d(Sx, Ty) \leq M(x, y) - \varphi(M(x, y)),$$

for all $x, y \in X$, where

$$M(x,y) = \max\left\{d(x, y), d(x, Sx), d(y, Ty), \frac{d(x, Ty) + d(y, Sx)}{2}\right\}.$$

In 2014, Chugh [2] proved the following result.

Theorem 1.4. Let (X, d) be a complete metric space and let $S, T : X \to X$. Assume that there exists $r \in [0, 1)$ such that for every $x, y \in X$,

$$\min\{d(x, Sx), d(y, Ty)\} \leq (1+ r)\, d(x, y)$$

implies

$$d(Sx, Ty) \leq d(x, y) - \varphi\, d(x, y),$$

Then there exists $z \in X$ such that $z \in Sz \cap Tz$.

In 2009, Zhang and Song [24] proved the following theorem for generalized φ- weak contraction (see also [17]) which is defined for two mappings and gave conditions for the existence of a common fixed point.

Theorem 1.5. Let (X, d) be a complete metric space and let $S, T: X \to X$ be two mappings such that for all $x, y \in X$

$$d(Sx, Ty) \leq M(x, y) - \varphi(M(x, y)),$$

where

$\varphi: [0, +\infty) \to [0, +\infty)$ is lower semi-continuous function with $\varphi(t) = 0$ if and only if $t = 0$.

Then there exists the unique point $z \in X$ such that $z = Tz = Sz$.

Dutta and Choudhary [8] gave the following theorem by introducing a new generalization of contraction principle.

Theorem 1.6. Let (X, d) be a complete metric space and let $T: X \to X$ be a map satisfying the inequality

$$\psi\, d(Tx, Ty) \leq \psi\, d(x, y) - \varphi(d(x, y)),$$

where $\psi, \varphi: [0, +\infty) \to [0, +\infty)$ are both continuous and monotone non-decreasing functions with $\varphi(t) = \psi(t) = 0$ if and only if $t = 0$. Then T has a unique fixed point.

Đorić [6] used generalized (ψ, φ) weak contraction which is defined for two maps and gave conditions for the existence of a common fixed point as follows.

Theorem 1.7. Let (X, d) be a complete metric space and Let $S, T: X \to X$ be two selfmaps such that for all $x, y \in X$

$$\psi\, d(Sx, Ty) \leq \psi(M(x, y)) - \varphi(M(x, y)),$$

where

(i) $\psi: [0, +\infty) \to [0, +\infty)$ is continuous and monotone nondecreasing function with $\psi(t) = 0$ if and only if $t = 0$.

(ii) φ: $[0, +\infty) \to [0, +\infty)$ is lower semi-continuous function with $\varphi(t) = 0$ if and only if $t = 0$.

Then there exists the unique point $z \in X$ such that

$z = Tz = Sz$.

In this paper, we established a common fixed point theorem which is generalization of Theorem 1.7. The idea is in line with Theorem 1.3 where a generalization of Theorem 1.1 has been established by use of Suzuki contractive condition.

2. Main Results

In this paper, the following theorem is our main result.

Theorem 2.1. Let (X, d) be a complete metric space and let S and T be maps on X. Assume that for each $x, y \in X$,

$$\frac{1}{2}\min\{d(x, Sx), d(y, Ty)\} \le d(x, y) \text{ implies}$$

$$\psi(d(Sx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)), \quad (2.1)$$

where ψ and φ are defined as above. Then S and T have a common fixed point.

Proof: Take $x_0 \in X$. Putting $x_1 = Tx_0$ and $x_2 = Sx_1$, then let $x_3 = Tx_2$ and $x_4 = Sx_3$.

Inductively, Choose a sequence $\{x_n\}$ in X such that

$$x_{2n+1} = Tx_{2n} \text{ and } x_{2n+2} = Sx_{2n+1}$$

for all $n \ge 0$.

As
$$\frac{1}{2} d(x_{n-1}, x_n) \le d(x_{n-1}, x_n). \quad (2.2)$$

Now if n is odd and suppose

$$d(x_{n-1}, x_n) \le d(x_n, x_{n+1}). \quad (2.3)$$

Then by (2.2) and (2.3)

$$\frac{1}{2}\min\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} \le d(x_{n-1}, x_n).$$

And this implies (2.1), that is, we have

$$\psi(d(Sx_n, Tx_{n-1})) \le \psi(M(x_n, x_{n-1})) - \varphi\, M(x_n, x_{n-1}).$$

Suppose if $d(x_n, x_{n+1}) \le d(x_{n-1}, x_n)$. (2.4)

And by (2.2), $\frac{1}{2} d(x_{n-1}, x_n) \le d(x_{n-1}, x_n)$.

So
$$\frac{1}{2} d(x_n, x_{n+1}) \le d(x_{n-1}, x_n). \quad (2.5)$$

Then by (2.4) and (2.5)

$$\frac{1}{2}\min\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} \le d(x_{n-1}, x_n).$$ And this implies (2.1), that is, we have

$$\psi(d(Sx_n, Tx_{n-1})) \le \psi(M(x_n, x_{n-1})) - \varphi\, M(x_n, x_{n-1}).$$

It follows from property of the function φ that if n is an odd,

$$\psi(d(x_{n+1}, x_n)) = \psi(d(Sx_n, Tx_{n-1}))$$

$$\begin{aligned}
\psi(d(Sx_n, Tx_{n-1})) &\le \psi(M(x_n, x_{n-1})) - \varphi\, M(x_n, x_{n-1}) \\
&= \psi\left(\max\left\{\begin{matrix} d(x_n, x_{n-1}), d(x_n, Sx_n), \\ d(x_{n-1}, Tx_{n-1}), \dfrac{d(x_n, Tx_{n-1}) + d(x_{n-1}, Sx_n)}{2} \end{matrix}\right\}\right) \\
&\quad -\varphi\left(\max\left\{\begin{matrix} d(x_n, x_{n-1}), d(x_n, Sx_n), \\ d(x_{n-1}, Tx_{n-1}), \dfrac{d(x_n, Tx_{n-1}) + d(x_{n-1}, Sx_n)}{2} \end{matrix}\right\}\right) \\
&= \psi\left(\max\left\{\begin{matrix} d(x_n, x_{n-1}), d(x_n, x_{n+1}), \\ d(x_{n-1}, x_n), \dfrac{d(x_n, x_n) + d(x_{n-1}, x_{n+1})}{2} \end{matrix}\right\}\right) \\
&\quad -\varphi\left(\max\left\{\begin{matrix} d(x_n, x_{n-1}), d(x_n, x_{n+1}), \\ d(x_{n-1}, x_n), \dfrac{d(x_n, x_n) + d(x_{n-1}, x_{n+1})}{2} \end{matrix}\right\}\right).
\end{aligned}$$

$$\psi(d(x_{n+1}, x_n)) \le \begin{pmatrix} \psi\left(\max\left\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\right\}\right) \\ -\varphi\left(\max\left\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\right\}\right) \end{pmatrix}$$

$$\psi(d(x_{n+1}, x_n)) \le \psi(d(x_n, x_{n-1})) - \varphi\left(d(x_n, x_{n-1})\right)$$
$$\le \psi(d(x_n, x_{n-1}))$$

i.e. $\psi(d(x_{n+1}, x_n)) \le \psi(d(x_n, x_{n-1}))$.

So by the property of ψ, we have

$$d(x_{n+1}, x_n) \le d(x_n, x_{n-1}).$$

Similarly if n is even, we obtain

$$d(x_{n+1}, x_n) \le d(x_n, x_{n-1}).$$

Therefore, for all $n \ge 0$, $d(x_{n+1}, x_n) \le d(x_n, x_{n-1})$ and so $\left\{d(x_{n+1}, x_n)\right\}$ is monotonic non-increasing and bounded below, so their exists $r \ge 0$ such that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = r = \lim_{n\to\infty} d(x_n, x_{n-1}) \quad (2.6)$$

Then (by lower semi-continuity of φ)

$$\Phi(r) \le \lim_{n\to\infty} \inf \varphi(d(x_n, x_{n-1})).$$

We claim that $r = 0$. In fact taking upper limits as $n \to \infty$ on either side of the following inequality:

$$\psi(d(x_{n+1}, x_n)) \le \psi(d(x_n, x_{n-1})) - \varphi(d(x_n, x_{n-1})).$$

and using (2.6),We have

$$\psi(r) \le \psi(r) - \varphi(r),$$

i.e. $\varphi(r) \le 0$.

Then $\varphi(r) = 0$ by the property of function φ, and furthermore by property of function φ

$\varphi(r) = 0$ implies $r = 0$.

$$\text{So } \lim_{n\to\infty} d(x_{n+1}, x_n) = r = 0. \quad (2.7)$$

Next we claim that $\{x_n\}$ is Cauchy. Let

$$c_n = \sup\left\{d(x_j, x_k) : j, k \ge n\right\}.$$

Then $\{C_n\}$ is decreasing.

If $\lim_{n\to\infty} C_n = 0$, Then we are done.

Assume that $\lim_{n\to\infty} C_n = C > 0$.

Choose $\epsilon < \frac{C}{8}$ small enough and select N such that for all $n \ge N$,

$$d(x_{n+1}, x_n) < \varepsilon \text{ and } C_n < C + \varepsilon.$$

By the definition of C_{N+1}, there exists $m, n \ge N + 1$ such that

$$d(x_m, x_n) > C_n - \varepsilon \ge C - \varepsilon.$$

Replacing x_m by x_{m+1} if necessary, we have

$$d(x_n, x_{m+1}) > C - \varepsilon. \quad (2.8)$$

i.e. $d(x_n, x_{m+1}) - d(x_{m+1}, x_m) > C - \varepsilon - d(x_{m+1}, x_m)$

i.e.

$$d(x_n, x_m) \ge d(x_n, x_{m+1}) - d(x_{m+1}, x_m)$$
$$> C - \varepsilon - d(x_{m+1}, x_m)$$

i.e. $d(x_m, x_n) > C - \varepsilon - d(x_{m+1}, x_m)$

i.e. $d(x_m, x_n) > C - \varepsilon - \varepsilon$

i.e. $d(x_m, x_n) > C - 2\varepsilon$ (2.9)

We may assume that m is even, n is odd.

Then $d(x_{m-1}, x_{n-1}) > C - 4\varepsilon$ and since $d(x_{m-1}, x_m) \le d(x_{m-1}, x_{n-1})$ and $d(x_{n-1}, x_n) \le d(x_{m-1}, x_{n-1})$.

So $\frac{1}{2} \min\left\{d(x_{m-1}, x_m), d(x_{n-1}, x_n)\right\} \le d(x_{m-1}, x_{n-1})$

i.e.

$\frac{1}{2} \min \left\{d(x_{m-1}, Sx_{m-1}), d(x_{n-1}, Tx_{n-1})\right\} \le d(x_{m-1}, x_{n-1})$.

So from given assumption

$$\psi(d(Sx_{m-1}, Tx_{n-1})) \le \psi(M(x_{m-1}, x_{n-1})) - \varphi(M(x_{m-1}, x_{n-1}))$$

i.e. $\psi(d(x_m, x_n)) = \psi(d(Sx_{m-1}, Tx_{n-1}))$

$$\le \psi\left(\max\left\{\begin{array}{l} d(x_{m-1}, x_{n-1}), d(x_{m-1}, Sx_{m-1}), \\ d(x_{n-1}, Tx_{n-1}), \\ \dfrac{d(x_{m-1}, Tx_{n-1}) + d(x_{n-1}, Sx_{m-1})}{2} \end{array}\right\}\right)$$

$$-\varphi\left(\max\left\{\begin{array}{l} d(x_{m-1}, x_{n-1}), d(x_{m-1}, Sx_{m-1}), \\ d(x_{n-1}, Tx_{n-1}), \\ \dfrac{d(x_{m-1}, Tx_{n-1}) + d(x_{n-1}, Sx_{m-1})}{2} \end{array}\right\}\right)$$

.

$$\le \psi\left(\max\left\{\begin{array}{l} d(x_{m-1}, x_{n-1}), d(x_{m-1}, x_m), \\ d(x_{n-1}, x_n), \dfrac{d(x_{m-1}, x_n) + d(x_{n-1}, x_m)}{2} \end{array}\right\}\right)$$

$$-\varphi\left(\max\left\{\begin{array}{l} d(x_{m-1}, x_{n-1}), d(x_{m-1}, x_m), \\ d(x_{n-1}, x_n), \dfrac{d(x_{m-1}, x_n) + d(x_{n-1}, x_m)}{2} \end{array}\right\}\right)$$

.

i.e.

$$\psi(d(x_m, x_n)) \le \psi(d(x_{m-1}, x_{n-1})) - \varphi(d(x_{m-1}, x_{n-1}))$$

We have proved that $\psi\left(C_{N+1}\right) < \psi\left(C_N\right) - \varphi\left(\dfrac{C}{2}\right)$ (if ε is small enough).

This is impossible. Thus we must have $C = 0$.

That is, the sequence $\{x_n\}$ is Cauchy sequence. Since X is complete, so the sequence $\{x_n\}$ is convergent. That is, there exists $z \in X$ such that $x_n \to z$ as $n \to \infty$.

Moreover $x_{2n} \to z$ and $x_{2n+1} \to z$ $n \to \infty$.

Now we prove that z is fixed point of S and T.

Since $x_n \to z$, there exists $n_0 \in N$ such that

$$d(z, x_n) \le \frac{1}{3} d(z, y) \quad \text{for all } y \ne z \text{ with } n \ge n_0.$$

Then we have

$$\frac{1}{2} d(x_{2n-1}, Sx_{2n-1}) \le d(x_{2n-1}, Sx_{2n-1}) \le d(x_{2n-1}, x_{2n})$$
$$\le d(x_{2n-1}, z) + d(z, x_{2n})$$
$$\le \frac{2}{3} d(y, z) = d(y, z) - \frac{1}{3} d(y, z) \le d(y, z) - d(x_{2n-1}, z)$$
$$\le d(x_{2n-1}, y) \le \frac{1}{2} d(x_{2n-1}, Sx_{2n-1}) \le d(x_{2n-1}, y). \tag{2.10}$$

Now suppose if $d(y, Ty) \le d(x_{2n-1}, Sx_{2n-1})$.

Then $\frac{1}{2}\min\{ d(y, Ty), d(x_{2n-1}, Sx_{2n-1}) \} \le d(x_{2n-1}, y)$.

And if $d(x_{2n-1}, Sx_{2n-1}) \le d(y, Ty)$,

then $\frac{1}{2}\min\{ d(y, Ty), d(x_{2n-1}, Sx_{2n-1}) \} \le d(x_{2n-1}, y)$.

This implies (2.1), that is, we have

$$\psi(d(Sx_{2n-1}, Ty)) \le \psi(M(x_{2n-1}, y)) - \varphi(M(x_{2n-1}, y)).$$

$$\psi(d(Sx_{2n-1}, Ty)) \le$$
$$\left(\max\left\{\begin{array}{l} d(x_{2n-1}, y), d(x_{2n-1}, Sx_{2m-1}), \\ d(y, Ty), \dfrac{d(y, Sx_{2n-1}) + d(x_{2n-1}, Ty)}{2} \end{array}\right\}\right)$$
$$-\varphi\left(\max\left\{\begin{array}{l} d(x_{2n-1}, y), d(x_{2n-1}, Sx_{2m-1}), \\ d(y, Ty), \dfrac{d(y, Sx_{2n-1}) + d(x_{2n-1}, Ty)}{2} \end{array}\right\}\right)$$

Letting $n \to \infty$, we have

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$\psi\,(d(z,\ Ty)) \le \psi\,(\max\{d(z,\ y),\ d(y,\ Ty)\}) - \varphi\,(\max\{d(z, y), d(y, Ty)\})$

That is, $\psi\,(d(z, Ty)) \le \psi\,(\max\{d(z, y), d(y, Ty)\})$.

That is, $d(z, Ty) \le \max\{d(z, y), d(y, Ty)\}$. (2.11)

And by lemma 2.1, $d(y, Ty) \le d(y, z)$, (2.12)

Thus from (2.11) and (2.12), we conclude

$d(z, Ty) \le d(z, y)$ for all $y \in X-\{z\}$. (2.13)

Now $d(y, Ty) \le d(y, z) + d(z, Ty)$

$\le d(y, z) + d(y, z)$

i.e. $\frac{1}{2} d(y, Ty) \le d(y, z)$.

Now either $d(y, Ty) \le d(z, Sz)$ or $d(z, Sz) \le d(y, Ty)$.

If $d(y, Ty) \le d(z, Sz)$, then

$$\frac{1}{2}\min\{d(y,Ty), d(z,Sz)\} \le d(y,z).$$

And if $d(z, Sz) \le d(y, Ty)$, then

$$\frac{1}{2}\min\{d(y,Ty), d(z,Sz)\} \le d(y,z).$$

So by (2.1),

$$\psi(d(Sz,Ty)) \le \psi\left(\max\left\{\begin{array}{l} d(z,y), d(y,Ty), \\ d(z,Sz), \dfrac{d(y,Sz)+d(z,Ty)}{2}\end{array}\right\}\right)$$

$$-\varphi\left(\max\left\{d(z,y), d(y,Ty), d(z,Sz), \frac{d(y,Sz)+d(z,Ty)}{2}\right\}\right).$$

$$-\varphi\left(\max\left\{d(z,y), d(y,Ty), d(z,Sz), \frac{d(y,Sz)+d(z,Ty)}{2}\right\}\right).$$

Take $y = x_{2n}$.

$$\psi(d(Sz,Tx_{2n})) \le \psi\left(\max\left\{\begin{array}{l} d(z,x_{2n}), d(x_{2n},Tx_{2n}), \\ d(z,Sz), \dfrac{d(x_{2n},Sz)+d(z,Tx_{2n})}{2}\end{array}\right\}\right)$$

$$-\varphi\left(\max\left\{\begin{array}{l} d(z,x_{2n}), d(x_{2n},Tx_{2n}), \\ d(z,Sz), \dfrac{d(x_{2n},Sz)+d(z,Tx_{2n})}{2}\end{array}\right\}\right).$$

Letting $n \to \infty$, we have

$$\psi(d(Sz,z)) \le \psi\left(\max\left\{\begin{array}{l} d(z,z), d(z,z), \\ d(z,Sz), \dfrac{d(z,Sz)+d(z,z)}{2}\end{array}\right\}\right)$$

$$-\varphi\left(\max\left\{d(z,z), d(z,z), d(z,Sz), \frac{d(z,Sz)+d(z,z)}{2}\right\}\right).$$

$$\psi(d(Sz,z)) \le \psi(d(Sz,z)) - \varphi d(Sz,z),$$

which gives $z = Sz$. Analogously $z = Tz$.

The following examples show the generality of our results.

Example 2.1 Let $X = \{(0, 0), (0, 4), (4, 0), (0, 5), (5, 0), (4, 5), (5, 4)\}$ be endowed with the metric d defined by

$$d[(x_1, x_2), (y_1, y_2)] = |x_1 - y_1| + |x_2 - y_2|.$$

Let S and T be such that

$$S(x_1, x_2) = \begin{cases}(x_1, 0) & \text{if } x_1 \le x_2 \\ (0, 0) & \text{if } x_1 > x_2\end{cases} \quad \text{and}$$

$$T(x_1, x_2) = \begin{cases}(x_2, 0) & \text{if } x_1 \le x_2 \\ (0, x_2) & \text{if } x_1 > x_2\end{cases}.$$

Then S and T do not satisfy the condition (1.1) of Theorem 1.1 at $x = (4, 5)$, $y = (5, 4)$. However, this is readily verified that all the hypotheses of Theorem 2.1 are satisfied for the maps S and T with $\psi(t) = t$ and $\emptyset(t) = \frac{1}{7}t$.

Corollary 2.1. Let (X, d) be a complete metric space and let S and T be maps on X. Assume that for each $x, y \in X$,

$$\frac{1}{2}\min\{d(x, Sx), d(y, Ty)\} \le d(x, y)$$

implies

$$d(Sx, Ty) \le M(x, y) - \varphi(M(x, y)),$$

where φ is defined as above. Then S and T have a common fixed point.

Proof: It comes from Theorem 2.1 by taking ψ an identity map.

Corollary 2.2. Let (X, d) be a complete metric space and let T be a map on X. Assume that for each $x, y \in X$,

$$\frac{1}{2} d(x, Tx)\} \le d(x, y) \text{ implies}$$

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)),$$

where ψ and φ are defined as above. Then T has a unique fixed point.

Proof: It comes from Theorem 2.1 by taking $S = T$.

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