Solving a Simple Transportation Problem Using LINGO

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Abstract

The transportation problem is a special type of linear programming problem where the objective is to minimise the cost of distributing a product from a number of sources or origins to a number of destinations. The origin of a transportation problem is the location from which shipments are despatched. The destination of a transportation problem is the location to which shipments are transported. Transportation problems can be classified into different groups based on their main objective and origin supply versus destination demand. Transportation problems whose main objective is to minimize the cost of shipping goods are called minimizing. An alternative objective is to maximize the profit of shipping goods, in which case the problems are called maximizing. In this paper we are going to solve a simple transportation problem using LINGO software.

Keywords: Transportation, shipments, origin, destination.

LINGO.

1. Introduction

The transportation problem is a special type of linear programming problem where the objective is to minimise the cost of distributing a product from a number of sources or origins to a number of destinations. Because of its special structure the usual simplex method is not suitable for solving transportation problems. These problems require a special method of solution. The origin of a transportation problem is the location from which shipments are despatched. The destination of a transportation problem is the location to which shipments are transported. The unit transportation cost is the cost of transporting one unit of the consignment from an origin to a destination. The transportation problem is a distribution-type problem, the main goal of which is to decide how to transfer goods from various sending locations (also known as origins) to various receiving locations (also known as destinations) with minimal costs or maximum profit. As long as the number of origins and destinations is low, this is a relatively easy decision. But as the numbers grow, this becomes a complicated linear programming problem. Transportation problems can be classified into different groups based on their main objective and origin supply versus destination demand. Transportation problems whose main objective is to minimize the cost of shipping goods are called minimizing. An alternative objective is to maximize the profit of shipping goods, in which case the problems are called maximizing. In a case where the supply of goods available for shipping at the origins is equal to the demand for goods at the destinations, the transportation problem is called balanced. In a case where the quantities are different, the problem is unbalanced. It is worth noting that sometimes problems that are solved using the transportation method have nothing to do with an actual movement of goods. What is crucial for applying the method is to recognize the network of connected elements. In this paper we are going to solve a simple transportation problem using LINGO software.

2. Literature Review


3. Mathematical Model

The transportation problem is defined as follows: There is a single item that has to be transported from m sources to n destinations. The supply in source i is \( a_i \) and the demand in destination j is \( b_j \). The unit cost of transportation from source i to destination j is \( C_{ij} \). The problem is to transport the item at minimum cost from the sources to the destinations. Let \( X_{ij} \) be the quantity of the item transported from source (supply) i to the destination (demand) j. The objective is to

\[
\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij}
\]

Subject to

\[
\sum_{j=1}^{n} X_{ij} = a_i \text{ where } i = 1, \ldots, m
\]

\[
\sum_{i=1}^{m} X_{ij} = b_j \text{ where } j = 1, \ldots, n
\]

\( X_{ij} \geq 0 \)

This is called balanced transportation problem where total supply is equal to total demand. Here all constraints are equations.

4. The Problem

Consider a transportation problem with three sources (rows) and three demand points (columns). The availability in the supply points are 30, 40 and 60 respectively and the requirements are 20, 60 and 50 respectively. Table 1 shows the data including the unit cost of transportation, which is shown in left-hand corner of the elements of the matrix.

| Table 1: Data including unit cost of transportation |
|-----------------|-----------------|-----------------|-----------------|
| 4               | 6               | 8               | 30              |
| 6               | 7               | 6               | 40              |
| 4               | 8               | 12              | 60              |
| 20              | 60              | 50              |

5. About LINGO

LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems concisely, solve them, and analyze the solution. Optimization helps you find the answer that yields the best result; attains the highest profit, output, or happiness; or achieves the lowest cost, waste, or discomfort. Often these problems involve making the most efficient use of your resources—including money, time, machinery, staff, inventory, and more. Optimization problems are often classified as linear or nonlinear, depending on whether the relationships in the problem are linear with respect to the variables. LINGO includes a set of built-in solvers to tackle a wide variety of problems. Unlike many modeling packages, all of the LINGO solvers are directly linked to the modeling environment. This seamless integration allows LINGO to pass the problem to the appropriate solver directly in memory rather than through more sluggish intermediate files. This direct link also minimizes compatibility problems between the modeling language component and the solver components. Local search solvers are generally designed to search only until they have identified a local optimum. If the model is non-convex, other local optima may exist that yield significantly better solutions. Rather than stopping after the first local optimum is found, the Global solver will search until the global optimum is confirmed. The Global solver converts the original non-convex, nonlinear problem into several convex, linear subproblems. Then, it uses the branch-and-bound technique to exhaustively search over these subproblems for the global solution. The Nonlinear and Global license options are required to utilize the global optimization capabilities.

6. LINGO Program

Model:

\[
\begin{align*}
\text{Min} &= 4\times X_{11} + 6\times X_{12} + 8\times X_{13} + 6\times X_{21} + 7\times X_{22} + 6\times X_{23} + 4\times X_{31} + 8\times X_{32} + 12\times X_{33}; \\
X_{11} + X_{12} + X_{13} &= 30; \\
X_{21} + X_{22} + X_{23} &= 40; \\
X_{31} + X_{32} + X_{33} &= 60; \\
X_{11} + X_{21} + X_{31} &= 20; \\
X_{12} + X_{22} + X_{32} &= 60; \\
X_{13} + X_{23} + X_{33} &= 50; \\
\end{align*}
\]

@GIN(X_{11});@GIN(X_{12});@GIN(X_{13});

@GIN(X_{21});@GIN(X_{22});@GIN(X_{23});

@GIN(X_{31});@GIN(X_{32});@GIN(X_{33});
7. Result and Discussion

Table 2: Result

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>40</td>
</tr>
</tbody>
</table>

The above is the quantity transported and the source and destination.

8. Conclusion

Thus we have solved the transportation problem using linear programming method using LINGO software.

References


