Motion Control and the Skidding of Mecanum-Wheel Vehicles

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Abstract

A Mecanum-wheel vehicle can move in any direction and re-orient on the spot. The wheel consists of many passive rollers at its rim oriented at an angle of 45° to the wheel circumference. The omni-directional vehicle was invented in 1970s but has limited commercial applications. In recent years, hobbyists and students in robotics competitions have intensified interests in Mecanum-wheel vehicles, fueled in part by reasonably priced Mecanum wheels from on-line vendors. In this paper, the motion control of this vehicle is introduced from the resolved force control. The tractive force at each wheel will determine the resultant tractive force that propel the vehicle to any direction. The tractive forces at the wheels can also account for the vehicle’s tendency in slippage as the lateral component of the wheel tractive force will generate a tipping moment in the vehicle’s roll direction. In the most prevalent movements (forward and backward travels and turning on the spot), the wheel’s tipping moments will be balanced at the chassis, and there is no adverse effect. In a sideway motion, the tipping moment will bend the chassis, and the vehicle will experience slip and vibration. With the cause of the slipping identified, the chassis should be stiffened to minimize the slippage caused by the chassis bending. This insight should be helpful to hobbyist and students in robotics competition.

Keywords: Mecanum-wheel, omni-directional vehicle, motion control, sliding.

1. Introduction

An omni-directional vehicle can move in any direction and can re-orient to any directions at the same location. Ilon [1] first designed a Mecanum-wheel in 1975 for an omni-directional vehicle with orbiting rollers as shown in Figure 1. Each roller is keg-shaped with its axle obliquely angled about the periphery of the hub. The omni-directional motion of the vehicle is achieved as the vector summation of propelling (tractive) forces on the ground-engaging rollers can be in any direction by adjusting the wheel rotation direction and torque magnitude of the four wheels.

In 2005, Airtrax [2] started the commercial applications of Mecanum-wheel with omni-directional forklifts. In recent years, hobbyists and students in robotics competitions have intensified interests in Mecanum-wheel vehicles, fueled in part by reasonably priced Mecanum wheels from on-line vendors [3, 4, 5].

In this paper, the geometry of Mecanum wheels and rollers are first introduced. The motion control of this vehicle will be derived from the resolved force control. The tractive force at each wheel will determine the resultant tractive force that propel the vehicle to any direction. Using the principle of virtual work, the force control can be used to formulate the motion control.

Mecanum-wheel vehicles have a tendency of slippage, and the slippage was attributed by researchers to friction between the roller and the road. However, the friction alone cannot explain why the forward/backward and turning on the spot experience no problems, while slippage in sideways directions are noticeable.

In this paper, the slippage and skidding will be explained with the lateral component of the tractive force at the wheel. The lateral component of the tractive forces will generate a net bending moment to deform the chassis. The bended chassis will cause vibration, slippage, and skidding in all directions except for those prevalent movements. With the cause of slippage identified, corrective measures can be taken.

2. Mecanum wheels

In Figure 1, a Mecanum wheel is shown with seven passive rollers at its rim oriented at an angle of 45° to the wheel circumference in three different views: (a) the front view, (b) the di-metric view, and (c) the side view. As shown in Figure 1a, the peripheral of the rollers are tangent to a cylindrical surface.
Each roller is defined by an elliptical arc revolving about its axle. The elliptical arc is the intersection of a cylinder and a plane at a 45° angle as shown in Fig 2b, 2c and 2d.

In Figure 3a, the cylinder is cut by the plane as shown in Figure 2b. The resulting cut is rotated to view the cut plane as in Figure 3c. The curves on the top and bottom are part of an ellipse, as shown in Figure 3d, a line segment is also added as the roller’s axle. In Figure 3e, the elliptical profile is revolving about the roller axle to get the roller. In Figure 3g, the view in Figure 3f is rotated to better show the roller in a 3D model.

In Figure 4a, a translucent enveloping cylinder is superimposed on a Mecanum wheel. In Figure 4b, the Mecanum wheel is shown with a sectional view at a 45° angle to reveal the roller’s profile. The top of the sectional view is an elliptical arc that defines the contour of the roller and is obtained from Figure 3c.

In Figure 5a, the enveloping cylinder superimposed on a Mecanum wheel shown in Figure 4a becomes opaque and is reduced in diameter just enough to reveal the rollers. The same cylinder became translucent in Figure 5b. The intent of this figure is to show the rollers touching ground are enclosed in a cylindrical surface as in Figure 4a.

Figure 6 shows Mecanum-wheel vehicle. Note that there are two types of Mecanum wheels, left handed and right handed, used in the vehicle. The two diagonal wheels are of the same hand.
3. Force Control

The resolved force control algorithm was introduced by Wang and Brown in [6] for the resolved motion rate control as in [7]. The force control algorithm is derived from tractive forces at each wheel.

Figure 7a shows a Mecanum-wheel vehicle’s top view. Wheels 1 and 3 are left-handed, and Wheels 2 and 4 are right-handed. Figure 7b shows only the bottom roller which is in contact with the ground. Note that the bottom roller aligned in the direction opposite to the top roller, as viewed from top. Figure 7b shows tractive forces at each wheel. For example, F_1 is the tractive force at the bottom of the Wheel 1.

The wheel arrangement of the Mecanum-wheel vehicle shown in Figure 6 and 7 are in an X-configuration. If the chassis is square, the diagonal wheel forces will align and the rotation around the z-axis will not be possible. In an O-configuration, as shown in Figure 8, Wheels 1 and 3 are right-handed, and Wheels 2 and 4 are left-handed, and the tractive forces will not intersect, even the chassis is square. This will also give a larger turning radius for rotation on the spot.

Figure 8a shows that tractive force at each Mecanum-wheel resolved to x and y components. Figure 8b shows the resultant force and moment (force and moment at the vehicle’s center) to propel and rotate the vehicle.

The y component of the tractive force, shown in figure 8a, is to be balanced by the input (motor) torque, as shown in Figure 9. The figure shows the side view of the wheel’s FBD (Free Body Diagram). T_1 is the torque from the shaft to the wheel.

From Figure 8b, the resultant force and moment at the vehicle’s center can be related to the propelling force of each wheel as:

\[
\begin{align*}
F_{Rx}^x &= -F_1^x + F_2^x - F_3^x + F_4^x \\
F_{Rx}^y &= F_1^y + F_2^y + F_3^y + F_4^y \\
M_R^x &= \begin{bmatrix} a \\ -a \\ -a \\ a \end{bmatrix} - \begin{bmatrix} b \\ b \\ b \\ b \end{bmatrix} = \begin{bmatrix} F_1^y \\ F_2^y \\ F_3^y \\ F_4^y \end{bmatrix}^T \begin{bmatrix} a \\ -a \\ -a \\ a \end{bmatrix} \end{align*}
\]  

where \(a\) and \(b\) are half the vehicle’s width and length respectively as shown in Fig. 8b.

Since rollers are at a 45° angle with the wheel, the x and y components of tractive forces are related as:

\[
\begin{bmatrix} F_1^x \\ F_2^x \\ F_3^x \\ F_4^x \end{bmatrix} = \begin{bmatrix} -F_1^y \\ F_1^y \\ -F_3^y \\ F_3^y \end{bmatrix}
\]  

Substituting Equation (3) into (2), the resultant moment can be expressed as:
Combining Equations (1), (2), (3), and (4) into a vector-matrix form, and we get:

\[
M_R^T = \begin{bmatrix}
F_1^y \\ F_2^y \\ F_3^y \\ F_4^y \\
\end{bmatrix} = \begin{bmatrix}
\alpha + b \\ -(\alpha + b) \\ \alpha + b \\ -(\alpha + b) \\
\end{bmatrix}
\]

(4)

From Figure 9, we can relate wheel torque to wheel force at the center of the axle in the longitudinal direction by the wheel radius \( R \).

\[
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\
\end{bmatrix} = R \begin{bmatrix}
F_1^y \\ F_2^y \\ F_3^y \\ F_4^y \\
\end{bmatrix}
\]

(6)

Substituting Equation (6) into (5) yields the force control algorithm,

\[
\begin{bmatrix}
F_{R1} \\ F_{R2} \\ F_{R3} \\ F_{R4} \\
\end{bmatrix} = \frac{1}{R} \begin{bmatrix}
-1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\
\end{bmatrix} \begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\
\end{bmatrix}
\]

(7)

Equation (7) can be written in a vector-matrix format as

\[
F_R = J^T \Gamma
\]

J is called the Jacobian matrix, which relates the wheel torque to a vehicle's resultant force and moment.

\[
F_R = \begin{bmatrix}
F_{R1} \\ F_{R2} \\ F_{R3} \\ F_{R4} \\
\end{bmatrix}
\]

is the vehicle's resultant force and moment,

\[
T = \begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\
\end{bmatrix}
\]

is the torque applied to each wheel,

\[
J = \frac{1}{R} \begin{bmatrix}
-1 & 1 & \alpha + b \\ 1 & -1 & -(\alpha + b) \\ -1 & -1 & \alpha + b \\ 1 & 1 & \alpha + b \\
\end{bmatrix}
\]

(9)

4. Motion Control

The principle of virtual work is that the input power is equal to the output power. The input power of the vehicle is \( T^T \omega \), and the output power of the vehicle is \( F_R^T v \).

Therefore,

\[
F_R^T v = T^T \omega
\]

Taking the transpose of Equation (8) yields

\[
F_R^T = T^T J
\]

Substituting Equation (11) into (10), we get

\[
\omega = J v_R
\]

J relates the vehicular velocity \( v_R \) to the wheel velocity \( \omega \).

Equation (12) can be written in a vector-matrix format as

\[
\begin{bmatrix}
\omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\
\end{bmatrix} = \frac{1}{R} \begin{bmatrix}
-1 & 1 & \alpha + b \\ 1 & -1 & -(\alpha + b) \\ -1 & -1 & \alpha + b \\ 1 & 1 & \alpha + b \\
\end{bmatrix} \begin{bmatrix}
v_{R1}^y \\ v_{R2}^y \\ v_{R3}^y \\ v_{R4}^y \\
\end{bmatrix}
\]

(13)

Equation (13) is the same as the equation derived by Muir and Neuman [7]. A few examples will be given below about the application of this equation:

If the vehicle is moving forward, \( v_R = \begin{bmatrix} 0 \\ v_{R1}^y \end{bmatrix} \)

Substituting this \( v_R \) into Equation (13),

\[
\begin{bmatrix}
\omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\
\end{bmatrix} = \frac{1}{R} \begin{bmatrix}
v_{R1}^y \\ v_{R2}^y \\ v_{R3}^y \\ v_{R4}^y \\
\end{bmatrix}
\]

Therefore, all wheels are turning in the positive direction, clockwise viewed from right the wheel’s propelling force are shown in Figure 10b.

If the vehicle is moving to the right, \( v_R = \begin{bmatrix} v_{R1}^x \\ 0 \end{bmatrix} \)

Substituting this \( v_R \) into Equation (13),

\[
\begin{bmatrix}
\omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\
\end{bmatrix} = \frac{1}{R} \begin{bmatrix}
-v_{R1}^x \\ v_{R2}^x \\ -v_{R3}^x \\ -v_{R4}^x \\
\end{bmatrix}
\]

Therefore, Wheels 1 and 3 are turning in the negative direction, and Wheels 2 and 4 are turning in the positive direction. The wheel’s propelling force is shown in Figure 10f.
If the vehicle is rotating counter-clockwise,

\[
v_R = \begin{bmatrix} 0 \\ 0 \\ \omega_R \end{bmatrix}
\]

Substituting this \(v_R\) into Equation (13),

\[
\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{a + b}{R} \begin{bmatrix} \omega_R \\ -\omega_R \\ -\omega_R \\ \omega_R \end{bmatrix}
\]

Wheels 1 and 4 are turning in the positive direction, and Wheels 2 and 3 are turning in the negative direction. The wheel’s propelling force are shown in Figure 11a.

If the vehicle is moving in a diagonal direction,

\[
v_R^x = v_R^y \quad \text{and} \quad v_R = \begin{bmatrix} v_R^x \\ v_R^y \end{bmatrix}
\]

Substituting this \(v_R\) into Equation (13),

\[
\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} 0 \\ 2v_R^x \\ 2v_R^y \\ 0 \end{bmatrix}
\]

This means Wheels 1 and 3 will be locked and Wheels 2 and 4 will be rotating at the same speed. The wheel’s propelling force is shown in Figure 10c.

Figure 10e shows a Mecanum-wheel vehicle built with Lego blocks connected to Lego motors and 3D printed rollers. The remaining figures showed the vehicle’s motion in cardinal and inter-cardinal directions. As shown in the figure, the vehicle moves in the cardinal directions (Figure 10b, 10d, 10f and 10h) with two pairs of tractive forces and moves in the inter-cardinal directions (Figure 10a, 10c, 10g and 10i) with one pair of force. In these 8 directions, Wheels 1 and 3 are paired, and Wheels 2 and 4 are paired. In each case, the paired wheels are rotating in the same direction with the same torque.

By reversing the direction of rotation, the tractive force will change. For instance, forward motion in Figure 10b and reverse motion in Figure 10h are related by simply reversing the direction of wheel rotation. Likewise, diagonal motion in Figure 10a is reversed to motion in Figure 10i.

In Figure 11, all propelling forces \(F_i\) create moments about the vehicle’s center in the same direction, the vehicle has a pure rotation. The vehicle turns with a zero turning radius as the wheels’ tractive forces are in anti-parallel pairs. In each case, the paired wheels are rotating with the same magnitude to rotate the vehicle in the counter-clockwise direction (a) and clockwise direction (b).

5. Slippage, Bouncing and Skidding

In Reference [8], the authors pointed out that wheel slip is a common problem with the Mecanum-wheel vehicle, particularly when the vehicle moves sideways. The authors built a Mecanum-wheel vehicle with passive suspensions that pivot on a central spine and experienced slippage in the lateral movement and vibrations.

In Reference [6], Wang and Brown experienced difficulties in the lateral movement of a Mecanum-wheel vehicle built with the Lego Mindstorms, as shown in Figure 10e, even though it has no problems in forward/backward motion and turning on the spot.
In Reference [9], the authors observed that a Mecanum-wheel vehicle was susceptible to slippage, and with the same amount of wheel rotation, the actual lateral traveling distance is shorter than actual the forward traveling distance.

In Reference [10], Ahlers showed in his YouTube video that his Mecanum-wheel vehicle’s motion. With the slippage, the vehicle could not return to the starting location as it was intended to.

The tractive forces at the wheels can explain the vehicle’s slippage or skidding. Because the tractive force, as shown in Figure 7, is at a 45° angle to the wheel, the x component that is perpendicular to the vehicle’s chassis, as shown in Figure 12a. Figure 12b shows the free body diagram (FBD) of the wheel with a tipping moment.

![Figure 10](image)

Figure 10. The FBD of a wheel showing the tipping moment caused by the lateral force

Moving in the forward/backward directions or a zero-radius turning, the tipping moments created by four wheels cancel each other relative to the chassis, and the vehicle can proceed without a net tipping moment and without slippage.

In a lateral motion, moving to the left or right, the lateral tractive forces contribute to the driving direction. The tipping moment created by the lateral tractive force will bend the chassis and reduce the wheel roller contact if the chassis is not rigid.

For example, if the vehicle is moving to the left, as shown in Fig 10 (d), the lateral force on wheels will have a resultant force to the left, as shown in Figure 13. Figure 14 shows the front view of the vehicle with FBD of wheels on the left and the right. Figure 15 shows the FBD of the chassis, showing the tipping moment and force from wheels. Figure 16 shows the exaggerated bending deformation of the chassis. Figure 17 shows the exaggerated bending deformation of the chassis and wheels. With wheels lifted, the normal contact force of the wheel will be reduced, and the contact area of the roller will be reduced. Note that if the vehicle is moving to the right, the chassis will bend downward, and the two wheels will bend inward, as opposite to the case in Figure 17.

![Figure 11](image)

Figure 11. A vehicle moving to the left, top view

![Figure 12](image)

Figure 12. The FBD of wheels on either side

![Figure 13](image)

Figure 13. The FBD of the chassis when moving to the left showing forces and moment from wheels to the chassis.

The chassis bending is not permanent, as the wheels are not fixed on the ground. As the chassis bends it stores potential energy. The energy will be released when it overcomes the friction resistance at the wheels, and the chassis shape will be restored to be flat. During this unbending (releasing potential energy) process there are noticeable sliding and bouncing if the chassis is not rigid.

![Figure 14](image)

Figure 14. When moving to the left, the FBD of the chassis shows a net moment bending the chassis downward
In the inter-cardinal directions, when two wheels are powered in the same direction, and the other two are locked to slide. The tractive forces of the driving wheels will bend the chassis along a line connecting Wheels 1 and 3.

Figure 18 is Figure 10(a) reproduced to label coordinate axes, traction forces and wheel numbers. Figure 19 shows the vehicle in Figure 18 in a 3D view. The same slippage and bouncing will happen to inter-cardinal movement, as in the lateral movement.

In summary, the slipping and skidding is absent for forward/backward direction motion and turning on the spot, as their lateral tractive force will be balance for vehicle motion. In the lateral direction, if the chassis is not rigid, as those in Reference [6,8,9], the vehicle motion will experience vibration and the distance traveled will be less from the expected values. If a rigid chassis is used, as in industrial settings like AGV (Automated Guided Vehicles) [11, 12], chassis deformation and skidding will be minimized.

6. Conclusions

A Mecanum-wheel vehicle can move in any directions and can turn on the spot. The omni-directional vehicle can be used in a variety of applications ranging from a wheelchair to a forklift. In recent year, Mecanum wheel vehicles have been popular with hobbyists and students in robotics competitions where maneuverability in a tight space is prized and the reliability and durability are secondary. Even some skidding is tolerable.

In this paper, the motion control algorithm is derived based on the force control. This approach is easier to understand as it is more intuitive, helpful to hobbyists and high school students. The tractive force can also explain the skidding. Because the tractive force is at a 45° angle to the wheel, the component that is perpendicular to the wheel will create a tipping moment.

The tipping moment created by wheels will cancel each other in any of the forward/backward direction or turning on the spot. The lateral motion problem can be reduced somewhat with a rigid chassis to reduce bending.

References


[3] RobotShop Inc., Vex 4" Mecanum Wheel Set, http://www.robotshop.com/en/mecanum-wheel-set.html?gclid=Cj0KEQjw4cLKBRCCZmNTvyovvj-4BEiQAI_sgQiTLLsH0_jhuZUEi4POskv2KmQz7P_Tbj3bAIgLHyQ8aAmSe8P8HQAQ


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