

# Shock Models with ODL Class of Life Distributions

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## Abstract

The overall decreasing life distributions are studied in relation to a shock model where shocks are arriving according to a homogeneous and a non-homogeneous Poisson process. Shock model based on probability generating function is also studied.

**Keywords:** Overall decreasing life distribution, Overall increasing life distribution, Homogeneous Poisson shock model, Non- Homogeneous Poisson shock model, Generating function shock model.

## 1. Introduction

Dating back to 1963, till now, various classes of life distributions have been introduced in reliability theory. The main classes of life distributions which have been introduced in the literature are based on failure rate as increasing (decreasing) failure rate *IFR* (*DFR*), increasing (decreasing) failure rate in average *IFRA* (*DFRA*), decreasing (increasing) mean residual life *DMRL* (*IMRL*). Based on the conditional survival function as new better (worse) than used *NBU* (*NWU*), new better (worse) than used in expectation *NBUE* (*NWUE*), harmonic new better (worse) than used in expectation *HNBUE* (*HNWUE*). Deshpand et al. (1986), are introduced another set of classes in terms of stochastic dominance. These classes are new better (worse) than used of second order *NBU(2)* (*NWU(2)*) or new better (worse) than used in average *NBUA* (*NWUA*), harmonic new better (worse) than used in expectation of third order *HNBUE(3)* (*HNWUE(3)*). Ahmed (1990), introduced the class of general harmonic new better (worse) than used in expectation *GHNBU* (*GHNWUE*). Al-Nachawati (1996), introduced the class of new better (worse) than used in average at specific interval *NBUASI* (*NWUASI*). El-Battal (2002), introduced the class of exponential better (worse) than used *EBU* (*EWU*). Mahmoud et. al (2005), introduced the class of exponential better (worse) than used in average *EBUA* (*EWUA*). Abdul-Moniem (2007), introduced the class of exponential better (worse) than used in average at specific interval *EBUASI* (*EWUASI*). For detailed discussions on properties and some possible applications we refer to Bryson and Siddiqui (1969), Rolski (1975), Barlow and Prochan (1981), Loh (1984), Deshpand et al. (1986), Abouammoh and Ahmed (1988), Ahmed (1990), Al-Nachawati (1996),

El-Battal (2002), Li (2004), Mahmoud et. al (2005) and Abdul-Moniem (2007).

Sepehrifar et al. (2012) are defined new class of life distributions called overall decreasing life (*ODL*) and its dual overall increasing life (*OIL*). And they investigated the probabilistic characteristics of this class of life distributions. A nonparametric testing procedure for exponentiality against an alternative *ODL* distribution is obtained.

**Definition 1.1:** A life distribution  $F$  on  $(0, \infty)$ , with  $F(0^-) = 0$  is called *ODL*, if

$$\int_x^\infty \bar{W}(t) dt \leq \mu \bar{W}(x); \quad x \geq 0, \tag{1}$$

where  $\bar{W}(z) = \mu^{-1} \int_z^\infty \bar{F}(u) du$  and  $\mu = \int_0^\infty \bar{F}(u) du$ .

Inequality (1) equivalent to

$$\int_x^\infty \int_t^\infty \bar{F}(u) dudt \leq \mu_H \int_x^\infty \bar{F}(u) du; \quad x > 0. \tag{2}$$

Let  $\bar{H}(t)$  be the survival function of a device which is subject to a sequence of independent shocks occurring randomly in time. Let  $N = \{N(t), t \geq 0\}$  be a general counting process during  $[0, t]$  and  $\{\bar{P}_m\}_{m=0}^\infty$  is the probability that the device survives the first  $k$  shocks,  $\bar{P}_k$  is assumed to be decreasing in  $k$  and  $\bar{P}_0 = 1$ . Then the survival probability that the device survives beyond time  $t$  can be expressed in the form

$$\bar{H}(t) = \sum_{k=0}^\infty P\{N(t) = k\} \bar{P}_k. \tag{3}$$

This paper is organized as follows: In Section 2, we present a homogeneous Poisson shock model. A non-homogeneous Poisson shock model is studied in Section 3. In Section 4, shock model based on probability generating function is also studied.

## 2. Homogeneous Poisson shock model

In this section we assume that the shock model is given by (3) such that shocks occurring randomly in time according

to Poisson process with constant intensity  $\lambda$ . Thus the shock model (3) becomes

$$\bar{H}(t) = \sum_{k=0}^{\infty} \exp(-\lambda t) \frac{(\lambda t)^k}{k!} \bar{P}_k \tag{4}$$

Esary et. al (1973) have shown that if  $\{\bar{P}_m\}_{m=0}^{\infty}$  has a discrete *IFR*, *IFRA*, *NBU*, *NBUE* or *DMRL* property, then this property will be reflected to  $\bar{H}(t)$  given by (4).

Similarly, results hold for the *HNBUE* class by Klefsjo (1981), *NBUFR* and *NBAFR* classes by Abouammoh et. al (1993), *NRBUE* class by Hendi (1995), *NBUASI* class by Al-Nachawati (1996), *EBU* class by El-Batal (2002), *EBUA* class by Mahmoud et. al (2005) and *EBUASI* class by Abdul-Moniem (2007). Now, we show that the same is true for the *ODL* class.

**Definition 2.1**

A discrete distribution and its survival probabilities  $\bar{P}_m = \sum_{j=m+1}^{\infty} P_j$ ,  $m=0,1,2,\dots$  with finite mean  $\mu = \sum_{j=0}^{\infty} j P_j$  are called discrete *ODL* if

$$\sum_{j=k}^{\infty} \sum_{i=j}^{\infty} \bar{P}_i \leq \mu \sum_{j=k}^{\infty} \bar{P}_j \tag{5}$$

**Lemma 2.2**

For  $\mu = \sum_{j=0}^{\infty} j P_j$  and  $\mu_H = \int_0^{\infty} \bar{H}(t) dt$ ,  $\mu_H = \frac{\mu}{\lambda}$ , where  $\bar{H}(t)$  is defined in (4)

**Proof**

Since  $\mu = \sum_{j=0}^{\infty} j P_j$  and  $\mu_H = \int_0^{\infty} \bar{H}(t) dt$ , then

$$\begin{aligned} \mu_H &= \int_0^{\infty} \sum_{k=0}^{\infty} \exp(-\lambda t) \frac{(\lambda t)^k}{k!} \bar{P}_k dt \\ &= \frac{1}{\lambda} \sum_{k=0}^{\infty} \bar{P}_k \int_0^{\infty} \exp(-\lambda t) \frac{(\lambda t)^k}{k!} d \lambda t \\ &= \frac{1}{\lambda} \sum_{k=0}^{\infty} j P_j \end{aligned}$$

So,  $\mu_H = \frac{\mu}{\lambda}$ .

Now, we prove that the discrete *ODL* properties of  $\bar{P}_k$ ,  $k=0,1,2,\dots$  is preserved for  $\bar{H}(t)$  under model (4).

**Theorem 2.3**

The survival function  $\bar{H}(t)$  in model (4) is *ODL* if  $\{\bar{P}_m\}_{m=0}^{\infty}$  is discrete *ODL*.

**Proof**

$\bar{H}(t) \in ODL$  if

$$\int_x^{\infty} \int_t^{\infty} \bar{H}(u) dudt \leq \mu_H \int_x^{\infty} \bar{H}(u) du \tag{6}$$

L.H.S of (6) is

$$\begin{aligned} \int_x^{\infty} \int_t^{\infty} \bar{H}(u) dudt &= \int_x^{\infty} \sum_{i=0}^{\infty} \exp(-\lambda u) \frac{(\lambda u)^i}{i!} \bar{P}_i dudt \\ &= \int_x^{\infty} \sum_{i=0}^{\infty} \bar{P}_i \left( \int_t^{\infty} \exp(-\lambda u) \frac{(\lambda u)^i}{i!} du \right) dt \\ &= \int_x^{\infty} \sum_{i=0}^{\infty} \frac{\bar{P}_i}{\lambda} \sum_{j=0}^i \exp(-\lambda t) \frac{(\lambda t)^j}{j!} dt \\ &= \frac{1}{\lambda} \int_x^{\infty} \sum_{j=0}^{\infty} \exp(-\lambda t) \frac{(\lambda t)^j}{j!} dt \sum_{i=j}^{\infty} \bar{P}_i \\ &= \frac{1}{\lambda^2} \sum_{j=0}^{\infty} \sum_{k=0}^j \exp(-\lambda x) \frac{(\lambda x)^k}{k!} \sum_{i=j}^{\infty} \bar{P}_i \\ &= \frac{1}{\lambda^2} \sum_{k=0}^{\infty} \sum_{j=k}^{\infty} \exp(-\lambda x) \frac{(\lambda x)^k}{k!} \sum_{i=j}^{\infty} \bar{P}_i \end{aligned}$$

Using condition (5), we get

$$\begin{aligned} L.H.S &\leq \frac{\mu}{\lambda^2} \sum_{k=0}^{\infty} \sum_{j=k}^{\infty} \exp(-\lambda x) \frac{(\lambda x)^k}{k!} \bar{P}_j \\ &= \frac{\mu}{\lambda^2} \sum_{j=0}^{\infty} \bar{P}_j \sum_{k=0}^j \exp(-\lambda x) \frac{(\lambda x)^k}{k!} \\ &= \frac{\mu}{\lambda} \sum_{j=0}^{\infty} \bar{P}_j \left( \frac{1}{\lambda} \sum_{k=0}^j \exp(-\lambda x) \frac{(\lambda x)^k}{k!} \right) \\ &= \frac{\mu}{\lambda} \sum_{j=0}^{\infty} \bar{P}_j \int_x^{\infty} \exp(-\lambda u) \frac{(\lambda u)^j}{j!} du \\ &= \mu_H \int_x^{\infty} \bar{H}(u) du \end{aligned}$$

This is complete of proof.  $\square$

**3. Non- Homogeneous Poisson shock model**

In this section we assume that the shock model given by (3) such that shocks occur according to a non-homogenous Poisson process with mean value function  $\Lambda(x)$  and event

rate  $\lambda(x) = \Lambda'(x)$  both defined on  $[0, \infty)$ .  $\lambda = \Lambda'(0)$  is taken as the right derivative of  $\Lambda(x)$  at  $x = 0$ . Thus the shock model (3) is given by

$$\bar{H}(x) = \sum_{k=0}^{\infty} \exp(-\Lambda(x)) \frac{\Lambda^k(x)}{k!} \bar{P}_k \tag{7}$$

**Theorem 3.1**

The survival function  $\bar{H}(x)$  in model (7) is *ODL* if  $\{\bar{P}_m\}_{m=0}^{\infty}$  is discrete *ODL*,  $\Lambda'(x) \geq 0$  for  $x \geq 0$  and  $\Lambda'(x) \geq \Lambda'(0)$

**Proof**

The survival function  $\bar{P}_k$  has the discrete *ODL* if

$$\sum_{j=k}^{\infty} \bar{P}_j \geq \frac{1}{\mu} \sum_{j=k}^{\infty} \sum_{i=j}^{\infty} \bar{P}_i$$

i.e.

$$\bar{P}_k + \bar{P}_{k+1} + \bar{P}_{k+2} + \dots \geq \frac{1}{\mu} [\bar{P}_k + 2\bar{P}_{k+1} + 3\bar{P}_{k+2} + \dots]$$

Multiplying both sides of the above inequality by the kernel  $\exp(-\Lambda(x)) \frac{\Lambda^k(x)}{k!}$  and taking summation over  $k = 0, 1, 2, \dots$ , we get

$$\bar{H}(x) + \sum_{j=0}^{\infty} \bar{H}_j(x) \geq \frac{1}{\mu} \left[ \bar{H}(x) + \sum_{j=0}^{\infty} (j+1) \bar{H}_j(x) \right], \quad (8)$$

where  $\bar{H}_j(x) = \sum_{k=0}^{\infty} \exp(-\Lambda(x)) \frac{\Lambda^k(x)}{k!} \bar{P}_{k+j}$ ,  $j = 1, 2, \dots$

From (2) the survival function  $\bar{H}(t)$  has continuous ODL if

$$\int_x^{\infty} \bar{H}(u) du \geq \frac{1}{\mu_H} \int_x^{\infty} \int_x^{\infty} \bar{H}(u) du dt, \quad (9)$$

where

$$\begin{aligned} \mu_H &= \int_0^{\infty} \bar{H}(u) du \\ &= \int_0^{\infty} \sum_{k=0}^{\infty} \exp(-\Lambda u) \frac{\Lambda^k(u)}{k!} \bar{P}_k du \\ &= \sum_{k=0}^{\infty} \bar{P}_k \int_0^{\infty} \exp(-\Lambda u) \frac{\Lambda^k(u) d\Lambda(u)}{k! \Lambda'(u)} \\ &= \frac{\mu}{\Lambda'(0)}. \end{aligned} \quad (10)$$

The result (10) is given by applying 2<sup>nd</sup> mean value theorem Gradshteyn and Ryzhik (2007). Let  $\frac{1}{\Lambda'(u)}$  be

bounded and monotonic when  $u \geq 0$  and let  $\Lambda'(0) \neq 0$  and  $\Lambda'(\infty) = \infty$ .

Using (10) in (9), we get

$$\begin{aligned} \sum_{k=0}^{\infty} \bar{P}_k \int_x^{\infty} \exp(-\Lambda u) \frac{\Lambda^k(u) d\Lambda(u)}{k! \Lambda'(u)} &\geq \\ \frac{\Lambda'(0)}{\mu} \int_x^{\infty} \sum_{k=0}^{\infty} \bar{P}_k \int_t^{\infty} \exp(-\Lambda u) \frac{\Lambda^k(u) d\Lambda(u)}{k! \Lambda'(u)} dt & \\ \sum_{k=0}^{\infty} \frac{\bar{P}_k}{\Lambda'(x)} \exp(-\Lambda(x)) \sum_{j=0}^k \frac{\Lambda^j(x)}{j!} &\geq \\ \frac{\Lambda'(0)}{\mu} \int_x^{\infty} \sum_{k=0}^{\infty} \frac{\bar{P}_k}{\Lambda'(t)} \exp(-\Lambda(t)) \sum_{j=0}^k \frac{\Lambda^j(t)}{j!} dt & \\ \frac{1}{\Lambda'(x)} \sum_{j=0}^{\infty} \exp(-\Lambda(x)) \frac{\Lambda^j(x)}{j!} \sum_{k=j}^{\infty} \bar{P}_k &\geq \\ \frac{\Lambda'(0)}{\mu} \int_x^{\infty} \frac{1}{[\Lambda'(t)]^2} \sum_{j=0}^{\infty} \frac{\Lambda^j(t)}{j!} \exp(-\Lambda(t)) d\Lambda(t) \sum_{k=j}^{\infty} \bar{P}_k & \end{aligned}$$

$$\begin{aligned} \frac{1}{\Lambda'(x)} \left[ \bar{H}(x) + \sum_{j=1}^{\infty} \bar{H}_j(x) \right] &\geq \\ \frac{\Lambda'(0)}{\mu [\Lambda'(x)]^2} \exp(-\Lambda(x)) \sum_{j=0}^{\infty} \sum_{i=0}^j \frac{\Lambda^i(x)}{i!} \sum_{k=j}^{\infty} \bar{P}_k & \\ \frac{1}{\Lambda'(x)} \left[ \bar{H}(x) + \sum_{j=1}^{\infty} \bar{H}_j(x) \right] &\geq \\ \frac{\Lambda'(0)}{\mu [\Lambda'(x)]^2} \exp(-\Lambda(x)) \sum_{i=0}^{\infty} \frac{\Lambda^i(x)}{i!} \sum_{j=i}^{\infty} \sum_{k=j}^{\infty} \bar{P}_k & \\ \bar{H}(x) + \sum_{j=1}^{\infty} \bar{H}_j(x) &\geq \frac{\Lambda'(0)}{\mu \Lambda'(x)} \left[ \bar{H}(x) + \sum_{j=1}^{\infty} (j+1) \bar{H}_j(x) \right] \end{aligned} \quad (11)$$

From (8) and (11) we get

$$\begin{aligned} \frac{1}{\mu} \left[ \bar{H}(x) + \sum_{j=0}^{\infty} (j+1) \bar{H}_j(x) \right] &\geq \\ \frac{\Lambda'(0)}{\mu \Lambda'(x)} \left[ \bar{H}(x) + \sum_{j=1}^{\infty} (j+1) \bar{H}_j(x) \right] & \end{aligned}$$

This implies that

$$\Lambda'(x) \geq \Lambda'(0). \quad \square$$

#### 4. Generating function shock model

Let  $P_0 = 1 - \bar{P}_0$  and  $P_i = \bar{P}_{i-1} - \bar{P}_i$ ,  $i = 1, 2, \dots$  be the probability mass function (pmf) of nonnegative random variable  $X$ . the probability generating function of  $X$  is

$$\begin{aligned} \psi(\theta) &= E(\theta^X) = \sum_{i=0}^{\infty} \theta^i P_i = P_0 + \sum_{i=1}^{\infty} \theta^i P_i \\ &= 1 - \bar{P}_0 + \sum_{i=1}^{\infty} \theta^i (\bar{P}_{i-1} - \bar{P}_i) \\ &= 1 - \bar{P}_0 + \theta \bar{P}_0 + \sum_{i=2}^{\infty} \theta^i \bar{P}_{i-1} - \sum_{i=1}^{\infty} \theta^i \bar{P}_i \\ \psi(\theta) &= 1 - (1 - \theta) \bar{P}_0 + \sum_{i=1}^{\infty} \theta^{i+1} \bar{P}_i - \sum_{i=1}^{\infty} \theta^i \bar{P}_i \\ &= 1 - (1 - \theta) \bar{P}_0 - \sum_{i=1}^{\infty} (1 - \theta) \theta^i \bar{P}_i \\ &= 1 - \sum_{i=0}^{\infty} (1 - \theta) \theta^i \bar{P}_i. \end{aligned} \quad (12)$$

We note that the random variable  $X$  have geometric distribution with parameter  $\theta$ , i.e

$$f(x) = P(X = i) = (1 - \theta) \theta^i; \quad i = 0, 1, 2, \dots \quad (13)$$

Relation (12), can be written in the form

$$\psi(\theta) = 1 - \sum_{i=0}^{\infty} P(X = i) \bar{P}_i. \quad (14)$$

Let  $X_i$ ;  $i = 1, 2, \dots, n$  be iid random variable with common pmf given by (13) then the variable  $Z = \sum_{i=1}^n X_i$  has the negative binomial distribution

$$P(z = n + i) = \binom{n+i-1}{i} \theta^i (1-\theta)^{n-i}; \quad i = 0, 1, 2, \dots \quad (15)$$

Next, define

$$B_n(\theta) = \begin{cases} \sum_{i=0}^{\infty} \binom{n+i-1}{i} \theta^i (1-\theta)^{n-i} \bar{P}_i & \text{for } n = 1, 2, \dots \\ 1 & \text{for } n = 0. \end{cases} \quad (16)$$

The form (16) has the following interesting physical meaning. Suppose that a device is subjected to two different types of shocks *I* and *II* say. At every time unit a shock of type *I* occurs with probability  $1-\theta$ . If  $X_i$  denote the number of type *I* shocks between  $(i-1)^{th}$  and  $i^{th}$ ,  $i \in N$  of type *II* shocks, then  $X_i$  has geometric distribution with pmf given by (13) and  $Z$  has a negative binomial distribution with pmf given by (15). Hence,  $B_n(\theta)$ ;  $n \in N$ , represents the probabilities that the device survives  $n$  shocks of type *I*, where  $\bar{P}_i$  represents the probability that the device survives the first  $i$  shocks of type *II*. See Hendi and Al-Nachawati (1996).

**Theorem 4.1**

Let  $B_n(\theta)$  be given by (16),  $\{\bar{P}_m\}_{m=0}^{\infty}$  is discrete ODL iff

$$B_n(\theta) \geq \sum_{i=1}^{\infty} \left( \frac{1+i-\mu}{\mu-1} \right) B_{n,i}(\theta),$$

where  $B_{n,i}(\theta) = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \theta^k (1-\theta)^{n-k} \bar{P}_{k+i}$ .

**Proof**

The survival function  $\bar{P}_k$  has the discrete ODL if

$$\sum_{j=k}^{\infty} \bar{P}_j \geq \frac{1}{\mu} \sum_{j=k}^{\infty} \sum_{i=j}^{\infty} \bar{P}_i$$

i.e.

$$\bar{P}_k + \bar{P}_{k+1} + \bar{P}_{k+2} + \dots \geq \frac{1}{\mu} [\bar{P}_k + 2\bar{P}_{k+1} + 3\bar{P}_{k+2} + \dots]$$

Multiplying both sides of the above inequality by the kernel  $\binom{n+k-1}{k} \theta^k (1-\theta)^{n-k}$  and taking summation

over  $k = 0, 1, 2, \dots$ , we get

$$B_n(\theta) + \sum_{i=1}^{\infty} B_{n,i}(\theta) \geq \frac{1}{\mu} \left[ B_n(\theta) + \sum_{i=1}^{\infty} (1+i) B_{n,i}(\theta) \right]$$

This implies that

$$\left( \frac{\mu-1}{\mu} \right) B_n(\theta) \geq \left( \frac{1+i-\mu}{\mu} \right) \sum_{i=1}^{\infty} B_{n,i}(\theta)$$

This is equivalent to

$$B_n(\theta) \geq \sum_{i=1}^{\infty} \left( \frac{1+i-\mu}{\mu-1} \right) B_{n,i}(\theta) \cdot \square$$

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