IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 6 Issue 10,October 2019
ISSN (Online) 2348 – 7968 | Impact Factor (2019) – 6.248
www.ijiset.com

New Maxwell Quantum Distribution Law and New Energy Relation for Particle in a Medium

[1]Mohammed AwadAlkareim Yousif Balal, [2]Mubarak Dirar Abdallah, [3]Mohmmed Mamoun Abdalazeem & [4]Sawsan Ahmed Elhouri Ahmed

[1]University of Gezira-College of Education-Department of Physics - Gezira - Sudan
[2]Sudan University of Science &Technology-College of Science-Department of Physics & International University of Africa- College Of Science- Department of Physics- Khartoum-Sudan
[3]University of Elimam Elmahdi -College of Education-Department of Physics – Khartoum - Sudan
[4]University of Bahri - College of Applied & Industrial Sciences, Department of Physics-Khartoum-Sudan

Abstract

Schrödinger equation for particle in a finite media with uniform potential was solved. The solution which is based on the fact that the particle exists gives complex and cosine wave function with energy relations different from that of the ordinary sine solution. Maxwell distribution law has been also found using the expression for the wave function in a frictional medium, quantum energy average and integration by parts, another approach has been tackled using the general expression for quantum average and the ordinary differentiation

Keywords: Schrodinger equation, finite medium, Maxwell quantum distribution, friction.

Introduction

Momentum is a valuable tool for predicting the future of physical system because it related to physical quantity that controlled a system, like force and energy. According to Newton's second law the resultant force is equal to the rate of change of momentum. The momentum is a conserved vector quantity. Thus it differs from the energy, which is a conserved scalar quantity, and to forces, which are only conserved locally.
In quantum mechanics the energy cannot be continuous for a particle.
The energy of quantum particle can have minimum value but cannot be zero. The importance of momentum in quantum mechanics results from the uncertainty principle and De Broglie hypothesis .Also the equation of motion as Schrodinger equation can be extracted from it [1, 2, 3]
The momentum also plays an important role in statistical physics .In a dynamical system theory, a phase space is a space in which all possible states of a system are represented, with each possible state corresponding

to unique point in the phase space. For mechanical systems, the phase space usually consists of all possible values 0 position and momentum variables of [4, 5].In statistical mechanics, any choice of a generalized coordinates for the position defines conjugate generalized momentum which together define coordinates on the phase space the momentum representation, wave functions are Fourier transforms of the equivalent real-space wave functions. The continuity equation governs the conservation of mass, charge and probability of any closed system.
This equation involves the spatial distribution of the flux density that is related to the temporal variation of the particle density (charge, mass). Ordinarily, this equation is derived from the equation of motion. The motion of any continuous charge/mass distribution can be thought of as a continuum (field or fluid). The continuity equation guarantees that there is no loss or gain of such quantities. This equation provides us with information about the system. The information is carried from on point to another by a particle (field wave) [6, 7].The continuity and momentum beside lasing equations are derived from Newton's laws. Different attempts were made to derive statistical laws and lasing equations [8, 9, 10].This work is conceived with new derivation of this these equations using quantum wave function and Maxwell distribution.

Quantum and Statistical Laws to Derive Fluid and Lasing Equations

The equation of motion of a particle moving in a field of potential V and a frictional medium with relaxation time T is given by:

$$mv\frac{dv}{dx} = mv\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dt} = -\frac{dV}{dx} - \frac{m}{\tau}v \quad (1)$$

Considering the practical as vibrating string

$$x = x_\circ e^{-i\omega t}$$

$v = x^\circ = -iwx \quad (2)$

Thus:

$m\int vdv = -\int dV + \frac{m}{i\tau w}\int vdv$

$\frac{1}{2}mv^2 = -V + \frac{m}{2\tau wi}v^2 (3)$

Since the kinetic energy oscillator is $k=\frac{1}{2}mv^2$ and the potential energy is

$V=\frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 = K \quad (4)$

On the other hand equation (3) yields

$K + V + \frac{K}{\omega\tau i} = \text{constant} \qquad (5)$

This constant of motion is the total energy E of the system, which takes the form

$E = K + V - \frac{iK}{\omega\tau} = E_\circ - \frac{iK}{\omega\tau} \qquad (6)$

Where the non-frictional energy takes the form

$$E_\circ = K + V = 2K \quad (7)$$

Thus

$E = E_\circ - i\frac{E_\circ}{wt} = K + V - \frac{iE_\circ}{\omega\tau}$ (8)

IF the relation time is assumed to be proportional to the periodic time T , such that T=ατ .

It f0ll0ws that $E = E_\circ - i\frac{\alpha}{2\pi}E_\circ = E_\circ - i\alpha_\circ E_\circ = K + V - i\alpha_\circ E_\circ$(9)

Therefore $E = \frac{P^2}{2m} + V - i\alpha_\circ E_\circ$ (10)

For particle in a box Schrodinger equation in a frictional medium is given by :

$i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi + V_\circ\varphi - i\alpha_\circ E_\circ\varphi$ (11)

Where $V_\circ$the potential barrier and T is a relaxation time .sing the method of separation of variables one can write:

$\varphi = \varphi(r,t) = f(t)u(r) = fu$ (12)

A direct substitution in equation (11) gives:

$$i\hbar u\frac{df}{dt} = \left\{-\frac{\hbar^2}{2m}\nabla^2 u + V_\circ u i\alpha_\circ E_\circ\right\}f$$

Rearranging gives

$\frac{i}{f}\hbar\frac{df}{dt} + i\alpha_\circ E_\circ + V_\circ = -\frac{\hbar^2}{2mu}\nabla^2 u = E_\circ$ (13)

$\frac{i}{f}\hbar\frac{df}{dt} = (E_\circ - V_\circ - i\alpha_\circ E_\circ)f$ (14)

$-\frac{\hbar^2}{2m}\nabla^2 u = E_\circ u = Ef$ (15)

The solution of equation (14) can be given by :

$f = A_1 e^{-\frac{iEt}{\hbar}} = A_1 e^{-\frac{i}{\hbar}(E_\circ - V_\circ - i\alpha_\circ)t} = A_1 e^{-\alpha_\circ E_\circ t} e^{-\frac{i}{\hbar}(E_\circ - V_\circ)t}$ (16)

Two possible solutions can be suggested for equation (15) . In one of them

$u = A_2 sin\alpha x$ (17)

To get : $\frac{\hbar^2}{2m}\alpha^2 u = E_\circ u \ \alpha = \sqrt{\frac{2mE_\circ}{\hbar^2}} = \frac{\sqrt{2m_\circ E_\circ}}{\hbar}$ (18)

For a one dimensional box of length L just outside the box

$|u|^2 = |\varphi|^2 = 0$ (19)

$u(L) = \varphi(x = L) = A_2 sin\alpha L = 0$ (20)

Thus : $\alpha L = n\pi$

Using (18) gives $E_\circ = \frac{\hbar^2\alpha^2}{2m} = \frac{\hbar^2\pi^2 n^2}{2mL^2}$ (21)

$E_\circ = \frac{\hbar^2 n^2}{8mL^2}$ (22)

The other solution of equation (15) can be $u = A_3 e^{i\alpha x}$ (23)

This with the aid equation (15), gives:

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 6 Issue 10,October 2019
ISSN (Online) 2348 – 7968 | Impact Factor (2019) – 6.248
www.ijiset.com

$$\frac{\hbar^2}{2m}\alpha^2 u = E_\circ u$$

This solution with be consist with the first solution and gives the same energy relation when are assumes that the probability is equal are at the bound any just inside the box at $(x = L)$

i.e.

$u(\mathrm{L}) = A_3 e^{i\alpha L} = A_3 cos\alpha L + A_3 sin\alpha L = 1$ (24)

This requires:

$$A_3 cos\alpha L = 1$$

$A_3 sin\alpha L = 0$ (25)

This requires:

$\alpha L = 2n\pi\alpha = \frac{2n\pi}{L} A_3 = 1$ (26)

Thus according to equation (18) equation (16) gives

$$2\mathrm{m}E_\circ = \hbar^2\alpha^2 = \frac{4\hbar^2\pi^2\hbar^2}{L^2}$$

$E_\circ = \frac{\hbar^2 n^2}{2mL^2}$(27)

This equation is not completely conforming to equation (22), The full complete agreement requires rewriting (11) by suggesting

$\alpha L = 2n\pi$ (28)

This also satisfies equation (27) to get again:

$E_\circ = \frac{\hbar^2 n^2}{2mL^2}$ (29)

Another boundary condition can be obtained by suggesting that at the boundary just inside the box (medium) the probability of finding the particle inside the box is finite and is equal to$P_\circ$. This requires

$|u(L)|^2 = P_\circ$

$u(\mathrm{L}) = \sqrt{P_\circ}$ (30)

Thus: $u(L) = A_3 e^{i\alpha L} = A_3 cos\alpha L + iA_3 sin\alpha L + VP_\circ$ (31)

$A_3 cos\alpha L = \sqrt{P_\circ}, cos\alpha L = \frac{\sqrt{P_\circ}}{A_3}$, $sin\alpha L = 0$, $\alpha L = n\pi$

This requires:

$\frac{\sqrt{P_\circ}}{A_3} = cos\alpha L = \pm 1$ (32)

Where for

$$n = 1,3,5,7 cos\alpha L = -1$$

And for

$$n = 0,2,4,6 cos\alpha L = 1$$

This requires (33)

$A_3 = \pm\sqrt{P_\circ}$ (34)

This means that$u$ is real and is given by

$u = A_3 cos\alpha x$ (35)

But this is not consistent with the solution

$u{=}A_2 sin\alpha x$ (36)

Since they give different probability distributions. To have solutions typical to each other consider

$u = (x = L) = iP_\circ$ (37)

This satisfies

$|u(L)|^2 = P_\circ$ (38)

And gives also

$A_3 cos\alpha L + iA_3 sin\alpha L = i\sqrt{P_\circ}$ (39)

This means that

$cos\alpha L = 0 \; A_3 sin\alpha L = \sqrt{P_\circ}$ (40)

This requires

$sin\alpha L = 1 \; A_3 = \sqrt{P_\circ}$ (41)

$\alpha L = (n + 1/2)\pi$

$\alpha = \frac{(n+\frac{1}{2})\pi}{L}$(42)

This gives the energy in the form

$E_\circ = \frac{(n+\frac{1}{2})^2 \hbar^2}{8mL^2}$ (43)

Another solution based on the existence of particles inside the box can be suggested by assuming

$u(x) = A_4 cos\alpha x \quad , \nabla u = \alpha A_4 sin\alpha x$

$\nabla^2 u = -\alpha^2 A_4 cos\alpha x = -\alpha^2 u$ (44)

Inserting this expression in equation (15) gives

$\frac{\hbar^2}{2m}\alpha^2 = E_\circ u\alpha = \frac{\sqrt{2mE_\circ}}{\hbar}$ (45)

Since just inside the box at (x=L) the box at $(x = L)$ the box the particle exist, it flows that

$|u(x = L)|^2 = |A_4 cos\alpha L|^2 = P_\circ$ (46)

Which means that the probability of existence of the particle is $P_\circ$. One of the possible solutions is to suggest

$u(L) = A_4 cos\alpha L = \sqrt{P_\circ} \quad , \; A_4 = \sqrt{P_\circ}$(47)

$cos\alpha L = 1$ (48)

$\alpha L = 2n\pi$ (49)

According to equation (4)

$$\frac{L\sqrt{2mE_\circ}}{\hbar} = 2n\pi \quad , \qquad E_\circ = \frac{n^2h^2}{2mL^2} \qquad (50)$$

This solution gives the same energy from as that proposed by the exponential solution with real wave function shown in equation (23) , (27) {see equation (27)} In view of equations (12) , (16) and (23) the wave function in excited state takes the form

$$\varphi = A_1 e^{i\theta} e^{-\alpha E\tau} \qquad (51)$$

Where the collision for timeτ cause it to go in energy to be in an excited state with energy E and θ

$$\theta = \frac{(V_\circ - E_\circ)\tau + \alpha x_\circ}{\hbar} \qquad (52)$$

Here one assumes that all particles are in the ground state with $(n = 0\,, E = 0)$

Thus when they are excited their energy is. Can write the wave function as:

$$\varphi = Ae^{-\alpha E} \qquad (53)$$

Where $\alpha = \alpha_\circ \tau$ (54)

$$A_\circ = A_1 e^{i\theta} \qquad (55)$$

$$\overline{E} = \frac{\int_0^\infty \overline{\varphi} E \varphi dE}{\int_0^\infty \overline{\varphi}\varphi dE} = \frac{|A|^2 \int_0^\infty Ee^{-2\alpha E} dE}{|A|^2 \int_0^\infty e^{-2\alpha E} dE} \qquad (56)$$

Using the identity:$\int udv = uv - \int vdu$ (57)

With:

$$u = Edv = e^{-2\alpha E} du$$

$$v = -\frac{1}{2\alpha} e^{-2\alpha E} (58)$$

$$\int_0^\infty Ee^{-2\alpha E} dE = -\frac{E}{2\alpha} e^{-2\alpha E} \Big|_0^\infty + \frac{1}{2\alpha}\int_0^\infty e^{-2\alpha E} dE$$

$$= -\frac{\infty e^{-\infty}}{2\alpha} + \frac{0e^{-0}}{2\alpha} - \frac{1}{4\alpha^2} e^{-2\alpha E} \Big|_0^\infty$$

$$= -0 + 0 - \frac{1}{4\alpha^2}\{e^{-\infty} - e^{-0}\}$$

$= \frac{1}{4\alpha^2}$ *(59)*

Also:

$$\int_0^\infty e^{-2\alpha E} dE = -\frac{1}{2\alpha} e^{-2\alpha E} \Big|_0^\infty$$

$$= -\frac{1}{2\alpha}\{e^{-\infty} - e^{-0}\}$$

$= \frac{1}{2\alpha}$ *(60)*

Thus:

$\bar{E} = \frac{2\alpha}{4\alpha^2} = \frac{1}{2\alpha}$ (61)

In view of equation (53) the wave function takes the form

$\varphi = Ae^{-\frac{E}{2\bar{E}}}$ (62)

Thus the number of particle are given by

$$n = \varphi\bar{\varphi} = A^2 e^{-\frac{E}{\bar{E}}}$$

$= n_\circ e^{-\beta E} \beta = \frac{-1}{\bar{E}}$ (63)

The statistical laws can also be found by using equation (56) to get

$\bar{E} = \frac{\int_0^\infty \bar{\varphi} E \varphi dE}{\int_\circ^\infty \bar{\varphi}\varphi dE} = \frac{\int_0^\infty nEdE}{\int_0^\infty ndE} = \frac{\int_0^\infty EndE}{I}$ (64)

Where:

$I = \int_0^\infty ndE$ (65)

But one can write equation (64) in the form :

$\bar{E} = \frac{dLnI}{d\beta_\circ} = \frac{dLnI}{dI}\frac{dI}{d\beta_\circ} = \frac{1}{I}\frac{dI}{d\beta_\circ}$ (66)

This means that {compeer (64) and (66) }

$\frac{dI}{d\beta_\circ} = \int EndE$ (67)

But from (66) $\int dLnI = \int \bar{E} d\beta_\circ$

$\ln I = \int \bar{E} d\beta_\circ + C_\circ$, $I = e^{\int \bar{E} d\beta_\circ} e^{C_\circ} = e^{C_\circ} e^{\int \bar{E} d\beta_\circ}$

$I = C_1 e^{\int \bar{E} d\beta_\circ}$ (68)

For constant $\bar{E}$

$I = C_1 e^{\beta_\circ \bar{E}}$ (69)

When all values are near the average value

$I = C_1 e^{\beta_\circ E}$ (70)

But from (64) :

$I = \int \frac{dI}{dE} dE = \int ndE$ (71)

Thus from (70) and (71) :

$n = \frac{dI}{dE} = \frac{C_1}{\beta_\circ} e^{\beta_\circ E} = Ae^{\beta_\circ E}$ (72)

But from (66) and (70)

$\bar{E} = \frac{1}{I} \frac{dI}{d\beta_\circ} = \frac{1}{I} (\beta_\circ I) = \beta_\circ$ (73)

Hence from (72) :

$n = Ae^{-\frac{E}{\bar{E}}}$ (74)

Another approach is based on defining [see (67)}

$$\frac{dI}{d\beta_\circ} = S = \int EndE = \frac{1}{2} \int ndE^2 = \int \frac{dS}{dE^2} dE^2$$

This means that:

$\frac{dS}{dE^2} = \frac{1}{2} n$ (75)

From (72) and (75)

$\frac{dI}{dE} = 2\frac{dS}{dE^2}$ (76)

Thus

$\frac{1}{2}\frac{dE^2}{dE} = \frac{dS}{dI}$ (77)

From (75):

$$E = \frac{dS}{dI} = \frac{dS}{\left(\frac{dI}{d\beta_\circ}\right)d\beta_\circ} = \frac{dS}{Sd\beta_\circ}$$

$$\int \frac{dS}{S} = \int Ed\beta_\circ + C_3$$

$$\ln S = C_3 + \int Ed\beta_\circ$$

$S = e^{c_3}e^{\int Ed\beta_\circ} = C_4 e^{\int Ed\beta_\circ}$ (78)

Assuming E to be independent of $\beta_\circ$, one gets:

$S = C_4 e^{E\beta_\circ}$ (79)

Thus:

$n = 2\frac{dS}{dE^2} = 2\frac{dS}{dE}\frac{dE}{dE^2} = \beta_\circ C_4 e^{\beta_\circ E}\left(\frac{1}{E}\right) = \frac{C_4\beta_\circ e^{\beta_\circ E}}{E}$ (80)

$n = g(E)e^{-\beta_\circ E}$ (81)

Conclusion

Using the equation of motion of a particle in a frictional medium in equation (1) a useful expression for energy of string in a frictional medium was found in equation (8). This expression is used to find Schrodinger equation (11) in a frictional media. Using separation of variables for particle in a box subjected to constant potential the expressions of the wave functions were found in equation (16) and (17). Assuming that just outside the bulk matter (the box) no particles exist (see equation (20)). The energy is shown to be quantized, in this approach the spatial wave function. One can use an exponential spatial wave function to find the same energy expression by assuming that just inside the bulk matter the practical exists as shown by equations (24) a new

energy expression (27) is found by assuming u to be real. The same expression in (50) can be obtained it $u$is a cosine function as shown by equation (44).However it u is imaginary as equation (37) shows another energy quantization expression is found in equation (43). The statistical distribution Maxwell equation can be found by using the wave function (53) for frictional media. Using the quantum average (56)The number of particles was found in equation (63) by using integration by parts. Another useful expression for Maxwell distribution was found use ordinary differentiation and the quantum expression for average physical quantity in (64) to get the number of particles in equations (63) and (81).
The probability distribution for particle in a box or a medium with constant potential was found for complex wave function as well as cosine wave function using the fact that the particle exists inside the medium. This gives new different probability distribution and different energy relations. The Maxwell distribution was found also by using wave function for frictional medium and quantum average as well as integration by parts. The same distribution was found by using the quantum energy average and ordinary differentiation laws.

References

1. Griffiths, David J. . . Introduction to Quantum Mechanics (2nd ed.). 2004.

2. Hagen Kleinert. Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, 3rd ed. Singapore: World Scientific. Draft of 4th edition, 2004.

3. George Mackey. The mathematical foundations of quantum mechanics. 2004.

4. D. Greenberger, K. Hentschel, F. Weinert, eds. Compendium of quantum physics, Concepts, experiments, history and philosophy, Springer-Verlag, Berlin, Heidelberg. 2009.

5. Eckert, Michael. The Dawn of Fluid Dynamics: A Discipline Between Science and Technology. Wiley. 2006.

6. Jump up to:[a b] Anderson, J. D. Fundamentals of Aerodynamics (4th ed.). London: McGraw–Hill. 2007.

7. Landau, Lev Davidovich; Lifshitz, Evgenii Mikhailovich. Fluid Mechanics. London: Pergamon.1987.