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String model to Derive Continuity and Momentum Fluid Equations ' from Quantum and Maxwell Distribution Laws and Lasing Process

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Abstract

Using Maxwell distribution Quantum law, and the Newtonian energy relation continuity and momentum fluid equation was done by differentiation the number density with respect to time and to coordinate. The momentum equation derivation requires the coefficient of the energy in the exponential power is equal to the thermal kinetic energy. This conforms with the statically value proposed by Maxwell distribution but with a positive sing. This number density function can successfully describes lasing. This is since it predicts population inversion and intensity of amplified light.

Key words: Maxwell statistical distribution, energy, continuity equation, momentum equation, fluid energy.

Introduction

Quantum mechanics is fundamental theory in physics which describes nature at the smallest scales of energy levels of atoms and subatomic particles equation that describes the changes over time of a physical system which is affected by the surrounding. This equation is considered as a back bone of quantum mechanics, which succeeded in describing the behavior of single particle but, but it fails to describe quantum system of many – body, because of the complex interaction between particles. As a consequence, the wave function of the system is complicated in nature having a large amount of information. On the other hand statistical mechanics is a branch of physics that uses method of probability theory and statistics to describe atoms and elementary particles. It uses mathematical tools for dealing with large amounts of particle in the physical system in solving physical problems. It can descried a wide variety of fields that consists of randomly be having particles. Its applications include electronics, laser and material science it is main purpose is to clarify the properties of matter in aggregate, in terms of the microscopic properties of individual constituents of the system. These properties of statistical physics make it in close link with fluids a fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces like liquids and gases. This branch of physics has two parts static and dynamic fluid. In fluid dynamic, the equation of motion is performed by principles of conservation in physics like mass, energy and momentum conservation, Bernoulli’s equation is one of important equations describing fluid in motion, which can be obtained by principles of energy and momentum conservation. There are equivalence between the laws of fluid, quantum and classical mechanics because all of them depend on principles of conservations, and deals with very small particles. This encourages doing this work which is devoted to derive fluid continuity and momentum equation from quantum and statistical laws.

String Model for Fluid Equations and Lasing

Consider the particles as small vibrating strings with kinetic and potential energies given by

$K=\frac{1}{2}mv_e^2$ (1)

$V=\frac{1}{2}kx_e^2 = \frac{1}{2}m\omega^2 x_e^2$ (2)

Where the effective values are related to the maximum valves according to the relations

$v_e = \frac{1}{\sqrt{2}}v_m x_e = \frac{1}{\sqrt{2}}x_m$ (3)

Since $x = x_m \sin\omega t$ (4)

$v = x^{\circ} = \omega x_m \cos\omega t = v_m cos\omega t$ (5)

$v_m = \omega x_m$ (6)

Thus:

$v_e^2 = \frac{v_m^2}{2} = \frac{\omega^2 x_m^2}{2} = \omega^2 x_m^2$ (7)

$x_e^2 = \frac{x_m^2}{2}$ (8)

From equations (1) and (2):

$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_e^2 = V$ (9)k=

Thus the total energy E is given to be

$E = K + V = 2K = mv_e^2 = mv^2$ (10)

One can also treat strings as subjected only to kinetic force such that its energy is related to the maximum velocity,

$E = \frac{1}{2} m v_m^2$ (11)

Where m is the mass and v_m is assumed to represent the maximum velocity, such that the average velocity is given by:

$v = \frac{v_m}{\sqrt{2}}$ (12)

Hence from (11) and (12) one can write the energy to be given by:

$E = mv^2 = m(v_x^2 + v_y^2 + v_z^2)$ (13)

Since the momentum P is given by:

$P = mv$ (14)

Thus one can write $E = mv.v = P.v$ (15)

In the x – direction

$E = mv_x^2 = P_x v_x = P.v$ (16)

Now multiply both sides by the Quantum expression for the particle density for resistive bulk matter which is given by:

$n = Ae^{\alpha px - \beta ET}$ (17)

To get

$En = Pnv$ (18)

Thus $\frac{\partial n}{\partial t} = -\beta En$

$En = \frac{-1}{\beta} \frac{\partial n}{\partial t}$ (19)

$\frac{\partial nv}{\partial x} = v \frac{\partial n}{\partial x} + n \frac{\partial v}{\partial x}$ (20)

$\frac{\partial n}{\partial x} = \alpha P n$ (21)

Here one assumes in (17) that $P = mv$ is independent of coordinates thus

$\frac{\partial P}{\partial x} = m \frac{\partial v}{\partial x} = 0 \, \frac{\partial v}{\partial x} = 0$ (22)

Thus equation (22) and (10) beside (11) gives

$\frac{\partial nv}{\partial x} = \alpha P n v$ (23)

Since the matter density ρ is given by

$\rho = mn$ (24)

And assuming uniform fluid particles with constant mass m equation (19), (23) and (18) gives

$$-\frac{m}{\beta} \frac{\partial n}{\partial t} = \frac{m}{\alpha} \frac{\partial nv}{\partial x}$$

$\frac{1}{\alpha} \frac{\partial \rho v}{\partial x} + \frac{1}{\beta} \frac{\partial \rho}{\partial t} = 0$ (25)

For :

$\alpha = \beta$ (26)

One gets the continuity equation

$\frac{\partial \rho v}{\partial x} + \frac{\partial \rho}{\partial t} = 0$ (27)

The momentum equation can be derived also from number density expression of Maxwell distribution

$$n = Ae^{\beta E} \qquad (28)$$

Where the mass density is given by

$$\rho = mn = mAe^{\beta E} = \rho_\circ e^{\beta E} (29)$$

Where the energy is given by

$$E = \frac{1}{2}\,\rho v_x^2 + V \qquad (30)$$

And

$$\frac{dE}{dx} = \frac{d(\frac{1}{2}\rho v_x^2)}{dx} + \frac{dV}{dx} \qquad (31)$$

But from (29)

$$\frac{d\rho}{dx} = \rho_\circ\, e^{\beta E}\left\{\left(\beta\frac{\partial(\frac{1}{2}\rho v_x^2)}{\partial x} + \beta\frac{\partial V}{\partial x}\right)\right\} \qquad (32)$$

$$\frac{\partial\rho}{\partial x} = \beta\rho\{\frac{1}{2}v_x^2\frac{\partial\rho}{\partial x} + \frac{1}{2}\rho\frac{\partial v_x^2}{\partial x} + \frac{\partial V}{\partial x}\} = \rho\beta\left\{\frac{1}{2}v_x^2\frac{\partial\rho}{\partial x} + \rho v_x\frac{\partial v_x}{\partial x} + \frac{\partial V}{\partial x}\right\}(33)$$

Since the force F is real ted to the potential according to the relation:

$$F = -\frac{\partial V}{\partial x} \qquad (34)$$

Rearranging (23) gives:

$$\{\frac{1}{\beta} - \frac{1}{2}\rho v_x^2\}\frac{\partial\rho}{\partial x} = \rho v_x^2\frac{\partial v_x}{\partial x} + \rho\frac{\partial V}{\partial x} \qquad (35)$$

But

$$\frac{\partial\rho v_x}{\partial t} = \frac{\partial\rho v_x}{\partial x}\frac{\partial x}{\partial t} = v_x\frac{\partial\rho v_x}{\partial x} \qquad (36)$$

By using the laws of statistical physics:

$$\frac{1}{\beta} = kT = \frac{1}{2}\rho v_x^2 \qquad (37)$$

Thus equation (35) gives:

$$\rho v_x\frac{\partial v_x}{\partial x} = -\frac{\partial V}{\partial x} = F \qquad (38)$$

If one assumes that:

$$\frac{dv_x}{dx} = \frac{\partial v_x}{\partial x} \qquad (39)$$

Then equation (38) gives:

$$\rho v_x\frac{dv_x}{dx} = \rho\frac{\partial x}{\partial t}\frac{dv_x}{dx} = \rho\frac{dv_x}{dt} = F \qquad (40)$$

Which is the ordinary momentum fluid equation .Another more direct approach can be obtained from equation (28) to get :

$$\rho = mn = \rho_\circ e^{\beta E} \qquad (41)$$

Where that total differentiation ω . r. t. x gives:

$$\frac{d\rho}{dx} = \beta\rho_\circ e^{\beta E}\frac{dE}{dx} = \beta\rho\frac{dE}{dx} \qquad (42)$$

Using the expression (20)

$$\frac{dE}{dx} = \frac{d\rho\frac{v_x^2}{2}}{dx} + \frac{dV}{dx} \qquad (43)$$

$$\frac{d(\rho\frac{v_x^2}{2})}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho v_x\frac{dv_x}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho\frac{dx}{dt}\frac{dv_x}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho\frac{dv_x}{dt} \quad (44)$$

Assuming that the potential depends on x only:

$$\frac{dV}{dx} = \frac{\partial V}{\partial x} = -F \qquad (45)$$

Inserting (44) and (45) in (43)

$$\frac{dE}{dx} = \frac{1}{2}v_x^2\frac{d\rho}{dx} + \rho\frac{dv_x}{dt} - F \qquad (46)$$

With the aid of equation (46) equation (32) give:

$$\frac{1}{\beta}\frac{d\rho}{dx} = \frac{1}{2}\rho v_x^2\frac{d\rho}{dx} + (\rho\frac{dv_x}{dt} - F)\rho$$

$$(\frac{1}{\beta} - \frac{1}{2}\rho v_x^2)\frac{d\rho}{dx} = (\rho\frac{dv_x}{dt} - F)\rho \qquad (47)$$

Thus

$$\frac{1}{\beta} = \frac{1}{2}\rho v_x^2 \quad , \quad \rho\frac{dv_x}{dt} = F \qquad (48)$$

A third approach can also be tackled by differentiating (41) $\omega.r.t$ (t)
Toget:

$$\frac{d\rho}{dt} = \beta\rho\frac{dE}{dt} \qquad (49)$$

In view of equation (30)

$$\frac{dE}{dt} = \frac{1}{2}\frac{d\rho v_x^2}{dt} + \frac{dV}{dt} = v_x\rho\frac{dv_x}{dt} + \frac{dx}{dt}\frac{dV}{dx} + \frac{1}{2}v_x^2\frac{d\rho}{dt}$$

$$= \rho v_x\frac{dv_x}{dt} + \frac{1}{2}v_x^2\frac{d\rho}{dt} + v_x\frac{dV}{dx} \qquad (50)$$

Inserting (50) in (49) gives

$$\frac{1}{\beta}\frac{d\rho}{dt} = \rho\left\{\rho v_x\frac{dv_x}{dt} + \frac{1}{2}v_x^2\frac{d\rho}{dt} + v_x\frac{dV}{dx}\right\} + \left\{\frac{1}{\beta} - \frac{1}{2}\rho v_x^2\right\}\frac{d\rho}{dt} = \rho v_x\{\rho\frac{dv_x}{dt} + \frac{dV}{dx}\} \quad (51)$$

One of the possible solutions is to set

$$\frac{1}{\beta} = \frac{1}{2}\rho v_x^2 = kT \qquad (52)$$

Thus equation (51) gives

$$\rho\frac{dv_x}{dt} = -\frac{dV}{dx} = -\frac{dV}{dx} = F \qquad (53)$$

This again represents the ordinary fluid momentum equation, according to equation (28) the light intensity is given by:

$$I = cnhf = cAe^{\beta E}(hf) = I_\circ e^{\beta E} \qquad (54)$$

Thus at ground state (E=0)

$$n = A \qquad (55)$$

But at excited state $(E = \infty)$, $(n \to \infty)$ (56)

Thus population inversion takes place. If one assumes that at ground state the energy is only due to potential part. Thus state the energy is only due to potential part.

Thus $E_\circ = V$ (57)

If it collides and gains kinetic energy:

$$E = E_\circ + \frac{1}{2}mv^2 = E_\circ + \beta = E_\circ + hf \qquad (58)$$

Thus from (28):

$$n = Ae^{\frac{E_\circ}{hf}}e^1 = Ae^1e^{\frac{E_\circ\tau}{h}} \qquad (59)$$

Where:$\Delta\tau\Delta E = h$, $\tau(hf) = h$

$$\tau = \frac{1}{f} \qquad (60)$$

This means that n at excited state increases with life and relaxation time in E which agrees with laser theory.

Conclusion

When deriving continuity equation one deals only with the relation between the changes of fluid density with its motion. Thus the kinetic term is important and one can ignore the role of potential energy. This is done by assuming that particles as in the form of strings. In this case that the potential energy effective value is equal to the effective value of the kinetic energy as shown by equation (9), equation (10) gives the total energy, in terms of the kinetic energy. Another typical energy from can be also found by assuming the string energy to be purely kinetic and related to the maximum velocity as shown by equation (11). The energy relation (16) resembles (11).Treating strings as travelling quantum waves moving in a medium , the number density is given in equation (17) as function of energy and momentum. Differentiating this expression $\omega.r.t$ space and time the ordinary continuity equation (27) for the fluid has been found. On the other hand the Maxwell distribution law was derived by some of this paper authors from quantum wave function. This expression (28) is used to derive the fluid momentum equation. This is done by using Newtonian energy density equation (31) , then multiplying both sides by m to find matter density (see equation (29)).A direct differentiation of the matter density partially$\omega.r.t$ (see equation (33))gives fluid momentum equation (40) This requires the parameter β to be equal to the kinetic thermal energy as shown by equation (37) .This conforms with what proposed in statistical physics . The same results can be obtained by differentiating the matter densityρ $\omega.r.t$ to x and t totally as shown by equations (48) and (53) .In all cases, equations (37), (48) and (52) shows that the parameter$\beta$is equal to kinetic thermal energy. These results agree with that proposal by statistical mechanics.

However the distribution law in this model has exponential coefficient with a positive sign, which is a like that of Maxwell with a negative sing. But fortunately this relation can describe light amplification and lasing process. This is very clear from equation (55) and (56), where the number

of particles in the excited state is larger than the number in the ground state. This means that population inversion takes place.
The intensity relation in equation (54) and the number expression (59) show also that the light intensity increases when the life time of Metastable state increases. This again agrees with the laser theories.
Using quantum laws and statistical physical laws continuity equation and momentum were derived. A new statistical law capable of describing lasing process was also obtained.

References

1.Chester, Marvin. Primer of Quantum Mechanics. John Wiley. (1987).
2.Griffiths, David J. Introduction to Quantum Mechanics (2nd ed.). A standard undergraduate text. 2004.
3.Hagen Kleinert. Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets, 3rd ed. Singapore: World Scientific. Draft of 4th edition. 2004.
4.Curtright, T. L.; Zachos, C. K. "Quantum Mechanics in Phase Space". Asia Pacific Physics Newsletter. 2012.
5.Laurendeau, Normand M. Statistical Thermodynamics: Fundamentals and Applications. New York: Cambridge University Press. 2005.
6.Jump up to:[a b c] Pedlosky, Joseph. Geophysical fluid dynamics. Springer. pp. 10–13. ISBN 978-0-387-96387-7. 1987.
7.George Mackey. The mathematical foundations of quantum mechanics. 2004.
8.D. Greenberger, K. Hentschel, F. Weinert, eds. Compendium of quantum physics, Concepts, experiments, history and philosophy, Springer-Verlag, Berlin, Heidelberg. 2009.
9. Eckert, Michael. The Dawn of Fluid Dynamics: A Discipline Between Science and Technology. Wiley. 2006.