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Monte Carlo Estimation of the Parameters of Two-Component Mixture Weibull Distribution

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Abstract
Mixture Weibull distribution is produced from combining k Weibull distributions by including a mixing parameter w_i where $i=1,2,\ldots,k$ is the number of component of Weibull distributions. This article involves a parameter estimation of mixture Weibull distribution using Monte-Carlo simulation. The study was based on 5000 replicates of generated data and the parameters were estimated using maximum likelihood method. The effects of sample size, mixing parameter and outliers on parameter estimates and bias were investigated. It was found that increasing the value of the mixing parameter caused a decrease in the mode and median, while increasing the sample size caused decrease in the bias, mean square error and standard deviation. On the other hand, it was also found that removing outliers caused a significant improvement in the accuracy of the parameter estimates and decreased the bias and mean square error.
Keywords: Mixture Weibull Distribution, Simulation, Bias, Outliers, Standard Deviation.

1. Introduction

Mixture Weibull distribution (MWD) is a linear combination of two or more Weibull distributions. It was first proposed in the late 1950’s by Kao, to allow more flexibility in data fitting with very little additional complexity over the single population model, [1]. MWD is considered a suitable model for life of components caused by more than one failure mode [2]. In many cases, MWD
provides better fit to certain data sets as compared to single component distribution particularly the data that do not fall on a straight line on a Weibull Probability Paper [3]. Jiang and Murthy [4,5,6] categorized the possible shapes of the failure rate function for a MWD in terms of parameters. Jiang et al. [7] dealt with a multi-component inverse mixture Weibull model, while Zhang et al. [8] used finite MWD to describe the diameter distributions of the rotated, sigmoid uneven-aged forest stands.
MWD plays a central role in Baysian statistics, not from physical mixing of several populations but from a lack of precise knowledge of the exact distribution of the sample data. An excellent reference for the applications of MWD in reliability, engineering, environmental and other fields can be found in Murthy et al. [4]. Jaramillo and Borja [9] used a bi-modal probability density function of a two-component MWD to represent wind speed analysis. Al Salih and Agarwal [10] presented the shapes of the density function of an extended Weibull distribution for various values of parameters. Carta and Ramirez [11] used the two-component MWD for estimation of wind speed distributions in the island of Gran Canaria. Lai and Xie [12] studied the univariate models derived from two or more Weibull distributions. Suhaila and Jemain [13] introduced a statistical study for fitting distributions of rainfall in Malaysia using mixed gamma, mixed Weibull and mixed exponential distributions. Razali et al. [14] submitted a simulation study of MWD where parameters were estimated using maximum likelihood method. Razali and Salih [15] explained how to produce MWD by including a mixing parameter with a number of parameters. Razali et al. [16] highlighted the use of several versions of MWD in the fields of statistics and engineering. Akdag et al. [17] discussed the use of two-component MWD in the analysis of wind speed data in the Eastern Mediterranean. Sultan et al. [18] investigated the properties of a mixture of two inverse Weibull distributions through Monte Carlo simulations using generated data with different sample size and different values of the set parameters. Furthermore, MWD have extensively been used in the analysis of time of death for individuals, injury analysis and traffic accidents.
Sometimes, in practical applications the available data can be detected as data coming from two or more sub-populations and can be treated as a mixture model. In such cases, it is important to assess the precision of parameter estimates by studying the properties of sampling distribution of estimators. This can be done through a simulation study on generated data of different sample sizes, where the effect of sample size and mixing parameters are analyzed and the accuracy of parameter estimates can be judged by evaluating the biasness, mean square error and outliers.
The first part of this study investigates the properties of MWD while the second part discusses the effects of sample size and mixing proportions on the parameter estimates as well as finding the unbiasedness property of the parameter estimates then evaluating the effect of outliers. These properties were explored through a simulation study on generated data based on three sets of parameters representing uni-modal and bi-modal MWD with different sample sizes, small, medium and large and different values of the mixing parameter. The estimation procedure was based on maximum likelihood estimation method and the precision was judged using 5000

replicates. This number of replicates is believed to be adequate to give the best estimates. Other researchers used 500, 1000, 3000, 5000 or 10000. However, it was found that outliers remained the same when the number of replicates was increased beyond 5000.
The density function of the MWD, $g(x)$, is given by;

$$g(x) = \sum_{i=1}^{n} w_i f_i(x) \tag{1}$$

where $f_i(x)$ is the density function of sub-population i of the two-parameter Weibull distribution, and w_i is the mixing parameter of the ith sub-population.
For two-component MWD the density and cumulative distribution functions are given by;

$$g(x) = w\left\{\frac{\beta_1}{\alpha_1}\left(\frac{x}{\alpha_1}\right)^{\beta_1-1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]\right\} + (1-w)\left\{\frac{\beta_2}{\alpha_2}\left(\frac{x}{\alpha_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]\right\} \tag{2}$$

$$G(x) = w\left\{1-\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]\right\} + (1-w)\left\{1-\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]\right\} \tag{3}$$

where α_i and β_i are the scale and shape parameters, respectively, and i=1,2.

2. Methodology

2.1 Properties of Mixture Weibull Distribution

a. Mean

The mean, $\bar{x}$, of the two-component MWD is given by;

$$\bar{x} = w\alpha_1\Gamma\left(1+\frac{1}{\beta_1}\right) + (1-w)\alpha_2\Gamma\left(1+\frac{1}{\beta_2}\right) \tag{4}$$

Proof:
The rth theoretical moment about the origin of the two-component MWD is given by;

$$\mu_r' = E(X^r) = w\int_0^\infty x^r f_1(x)dx + (1-w)\int_0^\infty x^r f_2(x)dx$$

r=1,2,3,…
where $f_1(x)$ and $f_2(x)$ can be defined as follows;

$$f_1(x) = \frac{\beta_1}{\alpha_1}\left(\frac{x}{\alpha_1}\right)^{\beta_1-1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]$$

$x > 0, \alpha_1 > 0, \beta_1 > 0$

$$f_2(x) = \frac{\beta_2}{\alpha_2}\left(\frac{x}{\alpha_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]$$

$x > 0, \alpha_2 > 0, \beta_2 > 0$ and w is the mixing parameter.

The first theoretical moment about the origin of the two-component MWD is given by;

$$\mu_1' = E(X) = w\int_0^\infty x f_1(x)dx + (1-w)\int_0^\infty x f_2(x)dx$$

or $$\mu_1' = w\mu_{11}' + (1-w)\mu_{12}'$$

where μ_{11}' and μ_{12}' are the first moment of the first and second Weibull distributions, respectively.

$$\mu_{11}' = \alpha_1\Gamma\left(1+\frac{1}{\beta_1}\right)$$

$$\mu_{12}' = \alpha_2\Gamma\left(1+\frac{1}{\beta_2}\right)$$

where Γ is a gamma function, $\Gamma(k) = \int_0^\infty y^{k-1}e^{-y}dy$ i.e.

Therefore, the first moment of the two-component MWD is

$$\bar{x} = \mu_1' = w\alpha_1\Gamma\left(1+\frac{1}{\beta_1}\right) + (1-w)\alpha_2\Gamma\left(1+\frac{1}{\beta_2}\right)$$

Q.E.D

b. Variance

The variance of the two-component MWD, σ_m^2 is given by;

$$\sigma_m^2 = w\left(\sigma_1^2 - (w-1)(\bar{x}_1 - \bar{x}_2)^2\right) - (w-1)\sigma_2^2 \tag{5}$$

Proof:
The variance of a function can be expressed as follows;

$$Var(x) = \sigma^2 = E\left\{[X - E(X)]^2\right\}$$
$$= E(X^2) - [E(X)]^2$$

For mixture distributions, Rinne [19] referred that the variance is given by;

$$\sigma_m^2 = \sum_{i=1}^{n} w_i E(X_i^2) - \left[\sum_{i=1}^{n} w_i E(X_i)\right]^2$$
$$= \sum_{i=1}^{n} w_i[Var(X_i) + E(X_i)]^2 - \left[\sum_{i=1}^{n} w_i E(X_i)\right]^2$$
$$= \sum_{i=1}^{n} w_i Var(X_i) + \sum_{i=1}^{n} w_i[E(X_i) - E(X)]^2$$

where $E(X_i) = \mu_{1i}$ and $E(X) = \mu_1$

For the two-component MWD, w_1=w and w_2=1-w, σ_1^2 and σ_2^2 are the variance of the first and second component Weibull distribution, respectively. Therefore, the variance of the MWD can be expressed as follows;

$$Var(X) = \sigma_m^2 = w\sigma_1^2 + (1-w)\sigma_2^2 + w(\mu_{11} - \mu_1)^2 + (1-w)(\mu_{12} - \mu_1)^2$$
$$= w\sigma_1^2 + (1-w)\sigma_2^2 + w[\mu_{11} - (w\mu_{11} + (1-w)\mu_{12})]^2$$
$$+ (1-w)[\mu_{12} - (w\mu_{11} + (1-w)\mu_{12})]^2$$

$$= w\sigma_1^2 + \sigma_2^2 - w\sigma_2^2 + w(\mu_{11} - w\mu_{11} - \mu_{12} + w\mu_{12})^2$$
$$+ (1-w)(\mu_{12} - w\mu_{11} - \mu_{12} + w\mu_{12})^2$$
$$= w\sigma_1^2 + \sigma_2^2 - w\sigma_2^2 + w[(\mu_{11} - \mu_{12}) - w(\mu_{11} - \mu_{12})]^2$$
$$+ (1-w)[- w(\mu_{11} - \mu_{12})]^2$$

$$= w\sigma_1^2 + w[(1-w)(\mu_{11} - \mu_{12})]^2 - [w(\mu_{11} - \mu_{12})]^2$$
$$+ w[w(\mu_{11} - \mu_{12})]^2 - (w-1)\sigma_2^2$$

Hence;

$$\sigma_m^2 = w\{\sigma_1^2 - (w-1)(\mu_{11} - \mu_{12})^2\} - (w-1)\sigma_2^2$$

Q.E.D

c. The Mean of the Cubes

The mean of the cubes of the two-component MWD is given by;

$$\overline{x^3} = w\alpha_1^3\Gamma\left(1 + \frac{3}{\beta_1}\right) + (1-w)\alpha_3^3\Gamma\left(1 + \frac{3}{\beta_2}\right) \quad (6)$$

Proof:

The third theoretical moment about the origin of the two-component MWD is given by;

$$\mu_3' = E(X^3) = w\int_0^\infty x^3 f_1(x)dx + (1-w)\int_0^\infty x^3 f_2(x)dx$$

where $f_1(x)$ and $f_2(x)$ can be defined as follows;

$$f_1(x) = \frac{\beta_1}{\alpha_1}\left(\frac{x}{\alpha_1}\right)^{\beta_1 - 1} \exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]$$

x > 0, $\alpha_1 > 0$, $\beta_1 > 0$

$$f_2(x) = \frac{\beta_2}{\alpha_2}\left(\frac{x}{\alpha_2}\right)^{\beta_2 - 1} \exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]$$

x > 0, $\alpha_2>0$, $\beta_2>0$ and w is the mixing parameter, $0<w<1$.

The third moment of MWD is expressed as follows;

$$\mu_3' = \sum_{i=1}^{n} w_i \mu_{3i}$$

For the two-component MWD, the third moment is given by;

$$\mu_3' = w\mu_{31} + (1-w)\mu_{32}$$

where μ_{31} and μ_{32} are the third moments of the first and second component Weibull distributions.

$$\mu_{31} = \alpha_1^3\Gamma\left(1 + \frac{3}{\beta_1}\right)$$

and

$$\mu_{32} = \alpha_2^3\Gamma\left(1 + \frac{3}{\beta_2}\right)$$

Hence,

$$\overline{x^3} = w\alpha_1^3\Gamma\left(1 + \frac{3}{\beta_1}\right) + (1-w)\alpha_3^3\Gamma\left(1 + \frac{3}{\beta_2}\right)$$

Q.E.D.

d. Mode

The mode of a distribution is the value of *x* corresponding to the maximum value of the probability density function. For a uni-modal distribution, *x* has only one value representing one mode, while for bi-modal distribution *x* has two values representing two modes.

The mode of the two-component MWD is represented by the value of *x* which satisfies the following equation;

$$0 = f_1(x).w.\left(\frac{\beta_1 - 1}{x} - .\frac{\beta_1}{\alpha_1}.\left(\frac{x}{\alpha_1}\right)^{\beta_1 - 1}\right) + f_2(x).(1-w).\left(\frac{\beta_2 - 1}{x} - .\frac{\beta_2}{\alpha_2}.\left(\frac{x}{\alpha_2}\right)^{\beta_2 - 1}\right) \quad (7)$$

where $f_1(x)$ and $f_2(x)$ can be defined as follows;

$$f_1(x) = \frac{\beta_1}{\alpha_1}\left(\frac{x}{\alpha_1}\right)^{\beta_1 - 1} \exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]$$

x > 0, $\alpha_1 > 0$, $\beta_1 > 0$

$$f_2(x) = \frac{\beta_2}{\alpha_2}\left(\frac{x}{\alpha_2}\right)^{\beta_2 - 1} \exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]$$

x > 0, $\alpha_2 > 0$, $\beta_2 > 0$

and w is the mixing parameter, $0 < w < 1$.

Proof:

To find the mode for the two-component MWD, the probability density function, *g(x)*, is differentiated with respect to *x*. Then, by equating the equation to zero we get the value of *x* which represents the mode(s);

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$$g(x) = w.f_1(x) + (1-w).f_2(x)$$

$$g(x) = w\left[\frac{\beta_1}{\alpha_1}\left(\frac{x}{\alpha_1}\right)^{\beta_1-1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]\right] + (1-w)\left[\frac{\beta_2}{\alpha_2}\left(\frac{x}{\alpha_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]\right]$$

$$\frac{\partial}{\partial x}g(x) = \frac{w\beta_1 x^{\beta_1-1}(\beta_1-1)\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]}{x\alpha_1^{\beta_1}} - \frac{w\left[\beta_1^2 x^{\beta_1-1}\left(\frac{x}{\alpha_1}\right)^{\beta_1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right]\right]}{x\alpha_1^{\beta_1}} + \frac{(1-w)\beta_2 x^{\beta_2-1}(\beta_2-1)\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]}{x\alpha_2^{\beta_2}} - \frac{(1-w)\left[\beta_2^2 x^{\beta_2-1}\left(\frac{x}{\alpha_2}\right)^{\beta_2}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right]\right]}{x\alpha_2^{\beta_2}}$$

$$0 = \frac{\beta_1}{\alpha_1^{\beta_1}}.x^{\beta_1-1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right].w\left(\frac{\beta_1-1}{x}\right) - \frac{\beta_1}{\alpha_1^{\beta_1}}.x^{\beta_1-1}\exp\left[-\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right].w.\frac{\beta_1}{x}.\left(\frac{x}{\alpha_1}\right)^{\beta_1} + \frac{\beta_2}{\alpha_2^{\beta_2}}.x^{\beta_2-1}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right].(1-w).\left(\frac{\beta_2-1}{x}\right) - \frac{\beta_2}{\alpha_2^{\beta_2}}.x^{\beta_2-1}\exp\left[-\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right].(1-w).\frac{\beta_2}{x}.\left(\frac{x}{\alpha_2}\right)^{\beta_2}$$

$$0 = \left(f_1(x).w.\left(\frac{\beta_1-1}{x}\right) - f_1(x).w.\frac{\beta_1}{x}.\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right) + \left(f_2(x).(1-w).\left(\frac{\beta_2-1}{x}\right) - f_2(x).(1-w).\frac{\beta_2}{x}.\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right)$$

$$0 = f_1(x).w\left(\frac{\beta_1-1}{x} - .\frac{\beta_1}{x}.\left(\frac{x}{\alpha_1}\right)^{\beta_1}\right) + f_2(x).(1-w).\left(\frac{\beta_2-1}{x} - .\frac{\beta_2}{x}.\left(\frac{x}{\alpha_2}\right)^{\beta_2}\right)$$

Hence,

$$0 = f_1(x).w.\left(\frac{\beta_1-1}{x} - .\frac{\beta_1}{\alpha_1}.\left(\frac{x}{\alpha_1}\right)^{\beta_1-1}\right) + f_2(x).(1-w).\left(\frac{\beta_2-1}{x} - .\frac{\beta_2}{\alpha_2}.\left(\frac{x}{\alpha_2}\right)^{\beta_2-1}\right)$$

Q.E.D

e. Median

The median is the middle value in a sample of size n. When the data points are put in rank order, there are $\frac{n-1}{2}$ values smaller and $\frac{n-1}{2}$ values larger. The median of two-component MWD is represented by the value of *x* which satisfies the following equation;

$$w.\left[1-\exp\left(-\frac{x}{\alpha_1}\right)^{\beta_1}\right] + (1-w)\left[1-\exp\left(-\frac{x}{\alpha_2}\right)^{\beta_2}\right] = 0.5 \quad (8)$$

Proof:

The median of two-component MWD cannot be found in an explicit form. The value of the median x was derived as the numerical solution which equates the cumulative distribution function (cdf) to 0.5.

2.2 Unbiasedness and Asymptotic Estimators

Let θ represent the set of parameters of MWD for data of size *n*. Any statistic whose mathematical expectation is equal to a parameter θ is called an unbiased estimator of the parameter θ. Otherwise, the statistic is said to be biased, Hogg and Craig [20].

Assuming that θ is a *k*-dimensional vector with components $\theta_1, \theta_2, \ldots, \theta_k$. The expression θ^ is called an estimator and is the numerical values obtained using the data for estimation.

An estimator $\hat{\theta}_i$ of θ_i is said to be unbiased if;

$$E(\hat{\theta}_i) = \theta_i \quad \forall \; i \quad (9)$$

Unbiasness means that repeating the estimation process a great number of times with the same sample size will lead to an average of the estimates obtained which is approximately equals to θ, Rinne [19].

An estimator for which $E(\hat{\theta}_i) \neq \theta_i$ is said to be biased. Bias defines and quantifies the accuracy or equivalently the systematic error of an estimator. An estimator is said to be more accurate as its bias is smaller. The bias $b(\hat{\theta}_i)$ of an estimator $\hat{\theta}_i$ is given by

$$b(\hat{\theta}_i) = E(\hat{\theta}_i) - \theta_i \qquad (10)$$

An estimator $\hat{\theta}_i$ of θ_i is said to be asymptotically unbiased if $E(\hat{\theta}_i) \rightarrow \theta_i$ as $n \rightarrow \infty$ for all possible values of θ_i. The term $\hat{\theta}_i$ is an asymptotically unbiased estimator of θ if $\hat{\theta}_i$ is asymptotically unbiased for i=1,2,…,k. In the case of $b(\hat{\theta}_i) \neq 0$ and $\lim_{n\to\infty} E(\hat{\theta}_i) = \theta$, then $\hat{\theta}_i$ is called asymptotically unbiased estimator.

2.3 Outliers

An outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs. A frequent cause of outliers is the use of mixture distributions. Outliers can lead to inaccurate estimates of the parameters and poor fitting of the model. This can be treated by removing outliers from the sample or replacing by corrected values. Outliers, being the most extreme observations, may include the sample maximum or sample minimum, or both, depending on whether they are extremely high or low. However, the sample maximum and minimum are not always outliers because they may not be unusually far from other observations.

Outliers can be determined using a number of criteria. One of the most common criterion is by the quartile range. Outliers can best be represented using a boxplot (or box-and-whisker plot) which is an excellent tool for conveying location and variation information in data sets, particularly for detecting and illustrating location and variation between different groups of data. A boxplot identifies the sample minimum, ($x_{\min}$), lower quartile (Q_1), median (Q_2), upper quartile (Q_3), and the sample maximum ($x_{\max}$). The lower quartile is the 25th percentile and the upper quartile is the 75th percentile. The spacings between the different parts of the box indicate the degree of spread (skewness) in the data and hence they identify outliers.

The interquartile range (IQR) in a boxplot is the difference between the third and first quartiles. i.e.

IQR = Q_3 - Q_1 . Therefore, upper outliers are the data points that fall above Q_3 + 1.5*IQR while lower outliers are the data points that fall below Q_1 - 1.5*IQR.

2.4 Simulation

Monte Carlo simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, non-linear or involves a number of parameters. This simulation is categorized as a sampling method because the inputs are randomly generated from a probability distribution to simulate the process of sampling from an actual population. The Monte Carlo simulation has been widely used to study the properties of the robust estimators and to test their performance. It allows to calculate parameter estimates and then compared to the true parameters of the distribution being studied.

This study involves generating random numbers of different sample sizes and using different values for the parameters to calculate parameter estimates based on 5000 Monte Carlo replicates. The average of the parameter estimates was compared with the true values to determine the bias, mean square error, standard deviation and total deviation. The iteration was terminated when the error was less than 10-5.

The best parameter estimates were determined for three sets of parameters using two-component MWD. Parameters were estimated by maximum likelihood estimation method using different sample sizes n=30, 50, 100 and 150 to represent small, medium and large sample sizes, and the mixing parameter, w, was varied (w=0.25, 0.50 and 0.75), for each set of parameters, θ = (w, β1, α1, β2, α2). The parameter values were chosen to represent uni-modal and bi-modal two-component MWD. The overview of the methodology for the simulation study is shown in Figure 1.

3. Results and Discussion

3.1 Mode and Median

Equations (7) and (8) were used to calculate the mode(s) and the median for each set of parameters, as shown in Table 1.

From Table 1, it is clear that sets 1 and 3 are bi-modal as they have two modes while set 2 is uni-modal as it has only one mode. It is also clear that the values of the mode and median decrease as the value of the mixing parameter increase.

3.2 Probability Density Function

Figures 2 show the shapes of the probability density function (pdf) of the two-component MWD for the three sets of parameters with different values of the mixing parameter. It is clear that sets 1 and 3 represent bi-modal and set 2 represents uni-modal.

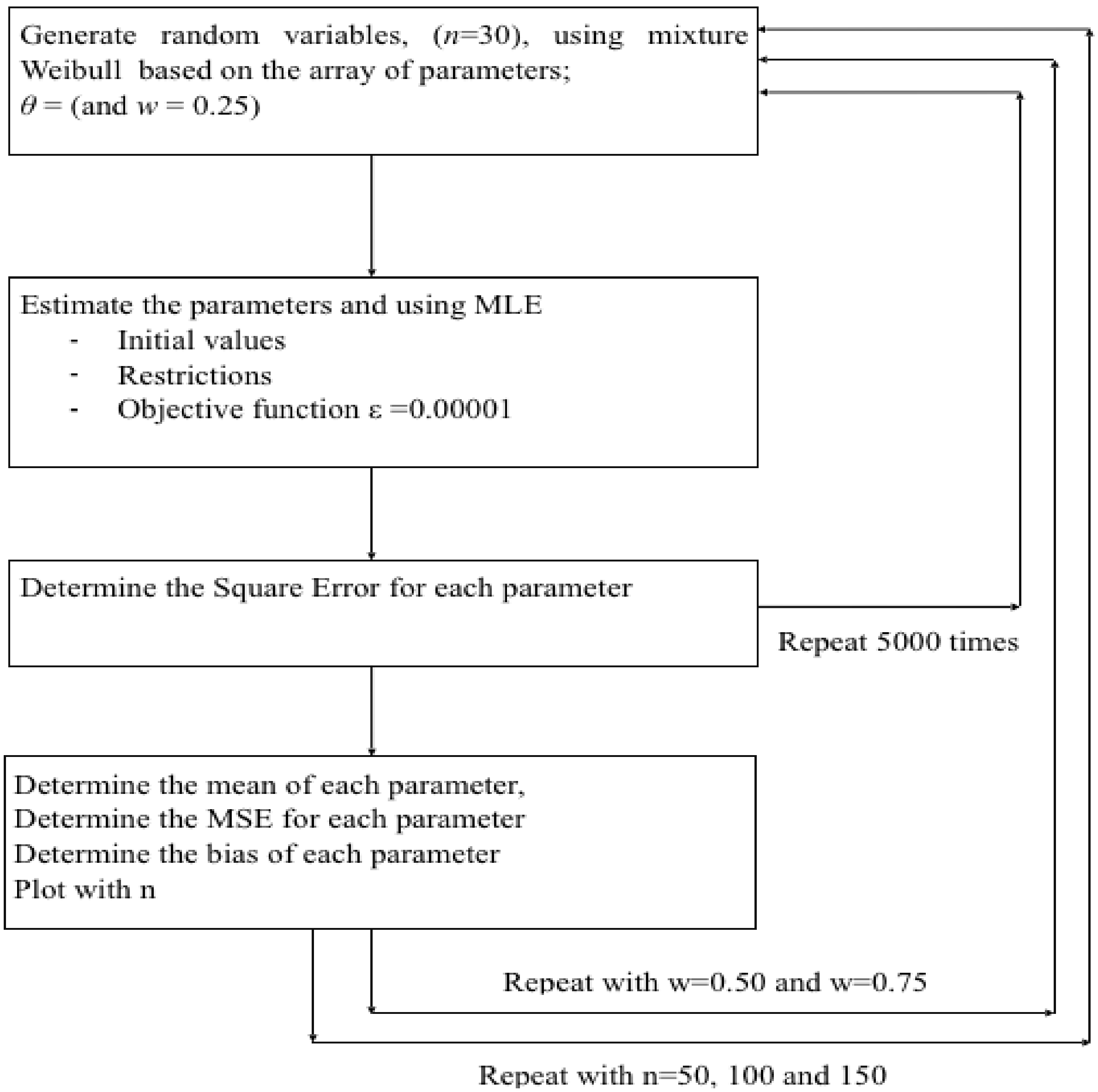

Figure 1: Flow chart of the methodology used for the simulation for each set of parameters

Table 1: Mode and Median of the MWD
for the three sets of parameters

Set	$\theta = (w, \beta_1, \alpha_1, \beta_2, \alpha_2)$	Mode(s)	Median
1	(0.25, 4.0, 0.8, 8.0,1.6)	0.7618, 1.5750	1.4293
	(0.50, 4.0, 0.8, 8.0,1.6)	0.7501, 1.5745	1.0720
	(0.75, 4.0, 0.8, 8.0,1.6)	0.7465, 1.5735	0.8182
2	(0.25, 2.0, 1.0, 2.0,2.0)	0.9516	1.3874
	(0.50, 2.0, 1.0, 2.0,2.0)	0.7985	1.1354
	(0.75, 2.0, 1.0, 2.0,2.0)	0.7389	0.9555
3	(0.25, 2.0, 25.0, 3.0,100.0)	19.3155, 87.3451	74.0198
	(0.50, 2.0, 25.0, 3.0,100.0)	18.1741, 87.3175	41.0965
	(0.75, 2.0, 25.0, 3.0,100.0)	17.8387, 87.2335	25.9971

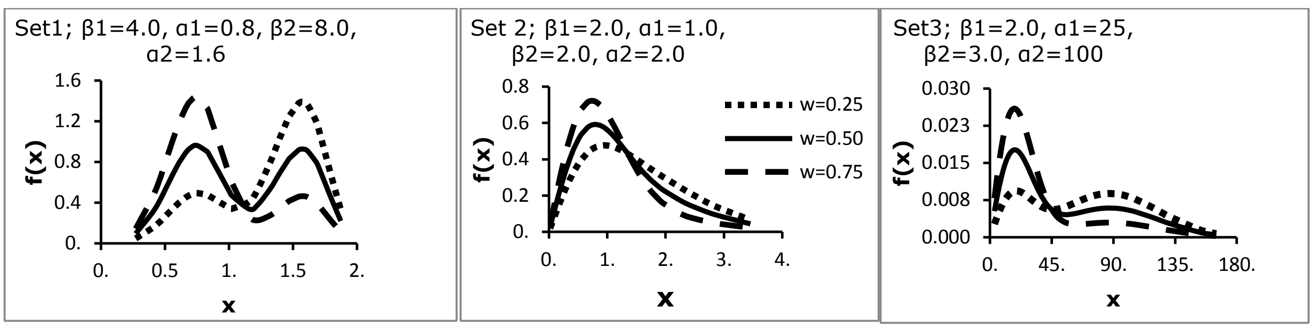

Figure 2: Probability density function for MWD
for Set1, Set2 and Set3

3.3 Parameter Estimates

For each set of parameters a random number of different sample size (n = 30, 50, 100 and 150) was generated with different values of the mixing parameter (w = 0.25, 0.50 and 0.75). The process was based on 5000 replications giving the average estimates of each parameter, as shown in Table 2.

Figures 3 (a)-(e) show, respectively, the 5000 estimates of the parameters of set 1 ($\beta1=4.0$, $\alpha1=0.8$,$w=0.25$, $\beta2=8.0$, $\alpha2=1.6$), when n=30 plotted with the true values of the estimates. These figures show the dispersion for the estimates of each parameter. This case was chosen as an example to illustrates the variation of the parameter estimates determined using the stated methodology.

Figures 4 (a) – (d) show, respectively the cumulative distribution function for the 5000 replicates for set1 ($\beta1=4.0$, $\alpha1=0.8$, $\beta2=8.0$, $\alpha2=1.6$), when w=0.25, and n=30, 50, 100 and 150, with the two solid lines representing the true values of the parameters and the average of the estimates. This case was chosen as an example to illustrate the cdf plots for each set of estimated values of the parameters with the true values and the average of the estimates. It is clear that the average line is much closer to the true values line when n=150 which indicates that for large sample sizes the estimates are closer to the true values giving lower bias.

Table 2: Parameter estimation

Set 1 (β_1=4.0, α_1=0.8, β_2=8.0, α_2=1.6)

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.3072	4.8686	0.7995	10.3062	1.4669
	50	0.2959	4.7511	0.8359	9.59861	1.5807
	100	0.2677	4.3938	0.8197	8.51994	1.6013
	150	0.2593	4.2460	0.8100	8.26195	1.6013
0.50	30	0.4856	4.9151	0.7702	10.0398	1.5394
	50	0.4952	4.6080	0.7947	9.20854	1.5828
	100	0.4995	4.3014	0.8006	8.59222	1.5939
	150	0.5012	4.1652	0.8012	8.33058	1.5994
0.75	30	0.6874	4.7387	0.7526	11.7585	1.4983
	50	0.7173	4.6042	0.7819	10.2266	1.5584
	100	0.7363	4.2862	0.7942	8.98184	1.5840
	150	0.7430	4.1660	0.7967	8.62996	1.5905

Set 2 (β_1=2.0, α_1=1.0, β_2=2.0, α_2=2.0)

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.4022	4.4913	0.8482	5.10716	2.2149
	50	0.3950	4.4433	0.9064	4.28686	2.2320
	100	0.3624	4.4093	0.9456	3.29116	2.2267
	150	0.3424	4.1691	0.9602	2.96056	2.1858
0.50	30	0.4252	4.6930	0.4764	4.81762	1.9934
	50	0.4261	4.6548	0.8107	4.22629	2.0519
	100	0.4162	4.6150	0.8687	3.25323	2.0572
	150	0.4207	4.2128	0.8939	2.90367	2.0581
0.75	30	0.4048	5.0264	0.6231	4.59088	1.5972

Set 3 (β_1=2.0, α_1=25.0, β_2=3.0, α_2=100.0)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.3137	3.2488	28.774	4.40239	101.395
	50	0.2938	2.7443	28.132	3.67942	101.843
	100	0.2704	2.3099	26.496	3.23152	100.891
	150	0.2626	2.1888	25.933	3.13161	100.607
0.50	30	0.4807	3.1137	24.531	4.20336	98.1394
	50	0.4823	2.7014	24.816	3.57018	98.4786
	100	0.4916	2.3214	25.041	3.24346	99.5603
	150	0.4944	2.1497	24.987	3.15122	99.6517
0.75	30	0.6161	3.6999	22.358	4.53386	87.3637
	50	0.6553	3.2011	23.536	3.95141	91.6980
	100	0.7067	2.5266	24.437	3.41227	96.4275
	150	0.7287	2.2711	24.743	3.26521	98.4291

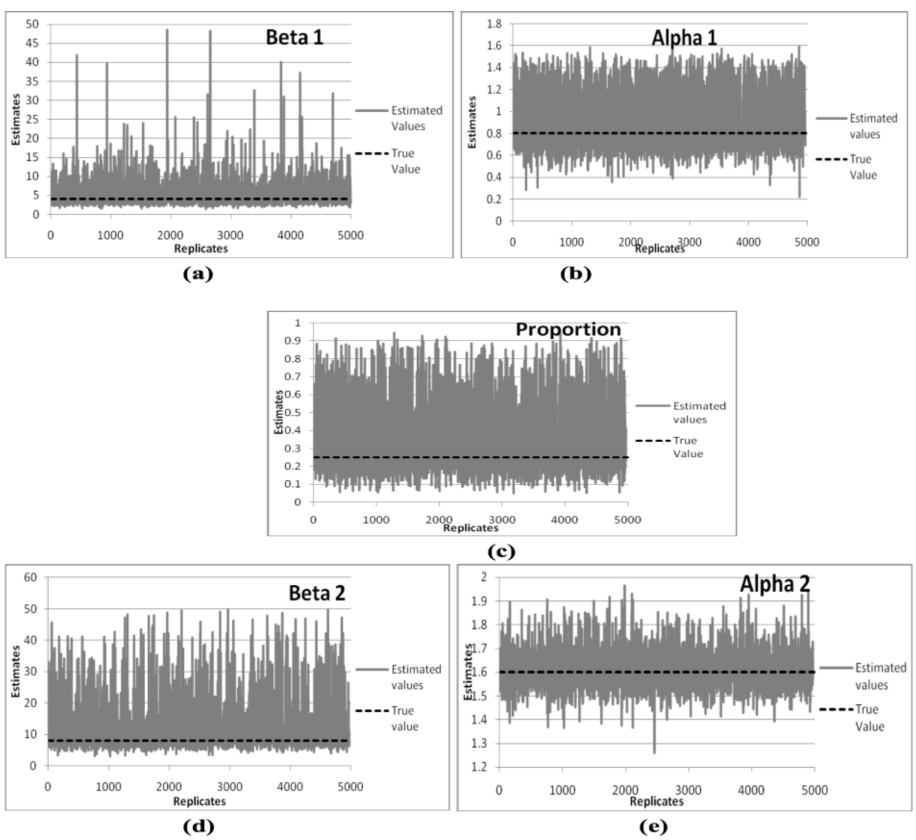

Figure 3: Variation of the 5000 replicates of the estimates of each parameter for set1 (β_1=4.0, α_1=0.8, β_2=8.0, α_2=1.6), when w=0.25, and n=30 with the true values

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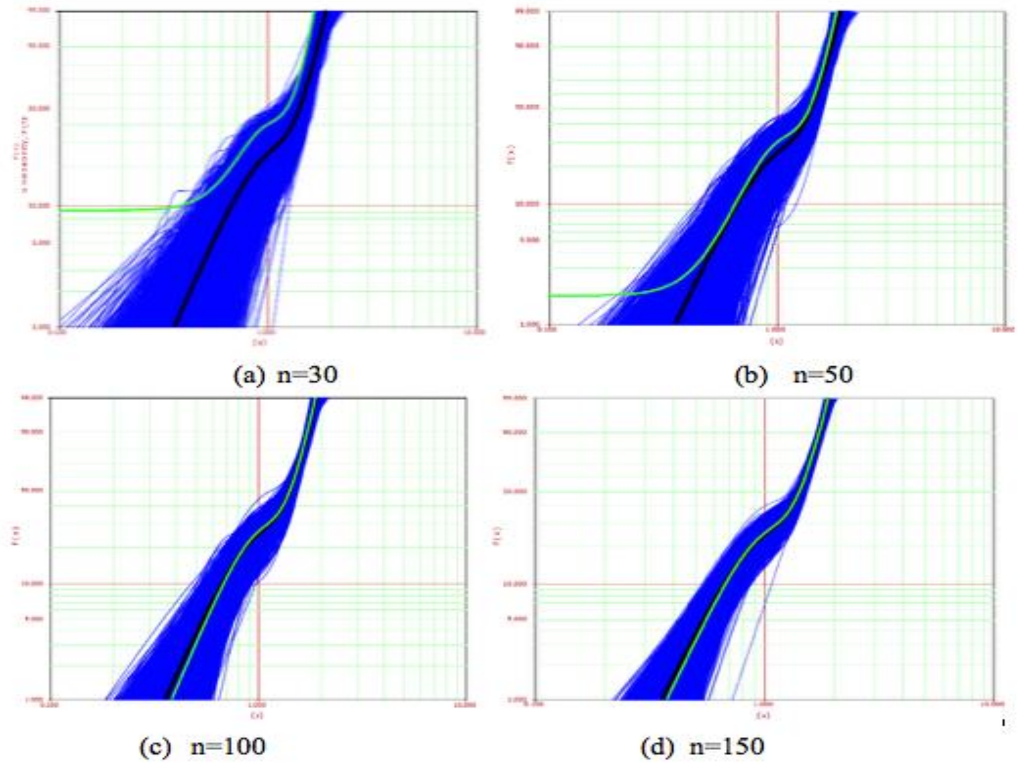

Figure 4: Cumulative distribution function of set1 (β_1=4.0, α_1=0.8, β_2=8.0, α_2=1.6),when w=0.25, with the true values (black) and average values (grey)

Figures 4 (a) – (d) show, respectively the cumulative distribution function for the 5000 replicates for set1 (β1=4.0, α1=0.8, β2=8.0, α2=1.6), when w=0.25, and n=30, 50, 100 and 150, with the two solid lines representing the true values of the parameters and the average of the estimates. This case was chosen as an example to illustrate the cdf plots for each set of estimated values of the parameters with the true values and the average of the estimates. It is clear that the average line is much closer to the true values line when n=150 which indicates that for large sample sizes the estimates are closer to the true values giving lower bias.

3.4 Bias of Estimates

The bias of the estimates of each parameter was determined using equation (10) for each set of parameters and for different sample sizes and different values of the mixing parameter. The results are shown in Table 3. It can be seen that the values of the bias in most cases are low indicating that the methodology proposed in this work for parameter estimation is reliable. It is also clear that the bias for each parameter decreases with increasing sample size.

3.5 Mean Square Error

The mean square error (MSE) of the parameter estimates can be calculated from the following equation;

$$MSE\left(\hat{\theta}\right) = \frac{\sum_{i=1}^{5000}\left(\hat{\theta} - \theta\right)^2}{5000} \tag{11}$$

Tables 4 show the MSE of the estimates of the three sets of parameters of MWD. It can be observed that increasing the sample size has a significant effect on decreasing the value of MSE. However, the mixing proportion effect on the value of MSE does not follow a specific pattern.

ISSN 2348 – 7968

Table 3: Bias of the estimates of parameters

Set1 (β_1=4.0, α_1=0.8, β_2=8.0, α_2=1.6)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0572	0.8688	-0.0005	2.30615	-0.1331
	50	0.0459	0.7511	0.0359	1.59861	-0.0193
	100	0.0168	0.3938	0.0197	0.51994	0.0013
	150	0.0093	0.2460	0.0100	0.26195	0.0013
0.50	30	-0.0145	0.9151	-0.0298	2.03977	-0.0606
	50	-0.0047	0.6080	0.0053	1.20854	-0.0172
	100	-0.0005	0.3014	0.0006	0.59222	-0.0061
	150	0.0012	0.1652	0.0012	0.33058	-0.0006
0.75	30	-0.0626	0.7387	-0.0474	3.75853	-0.1017
	50	-0.0327	0.6042	-0.0181	2.22661	-0.0416
	100	-0.0137	0.2862	-0.0058	0.98184	-0.0160
	150	-0.0070	0.1660	-0.0034	0.62996	-0.0095

Set2 (β_1=2.0, α_1=1.0, β_2=2.0, α_2=2.0)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.1513	2.4913	-0.1518	3.10716	0.2149
	50	0.1450	2.4433	-0.0936	2.28686	0.2320
	100	0.1124	2.4093	-0.0544	1.29116	0.2267
	150	0.0924	2.1691	-0.0398	0.96056	0.1858
0.50	30	-0.0747	2.6930	-0.2537	2.81762	-0.0066
	50	-0.0739	2.6548	-0.1893	2.22629	0.0519
	100	-0.0838	2.6150	-0.1313	1.25323	0.0572
	150	-0.0793	2.2177	-0.1062	0.90367	0.0581
0.75	30	-0.3452	3.0264	-0.3769	2.59088	-0.4028
	50	-0.3210	3.0999	-0.2874	2.03689	-0.2857
	100	-0.3032	3.2073	-0.2123	1.53116	-0.2115
	150	-0.2810	2.7247	-0.1644	1.02712	-0.1812

Set3 (β_1=2.0, α_1=25, β_2=3.0, α_2=100)

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0637	1.2488	3.7739	1.40239	1.3950
	50	0.0438	0.7443	3.1322	0.67942	1.8429
	100	0.0204	0.3099	1.4960	0.23152	0.8909
	150	0.0126	0.1888	0.9326	0.13161	0.6065
0.50	30	-0.0193	1.1137	-0.4693	1.20336	-1.8606
	50	-0.0178	0.7014	-0.1842	0.57018	-1.5214
	100	-0.0084	0.3214	0.0405	0.24346	-0.4397
	150	-0.0056	0.1497	-0.0134	0.15122	-0.3483
0.75	30	-0.1339	1.6999	-2.6424	1.53386	-12.636

Table 4: MSE of the estimates of parameters

Set1 (β_1=4.0,α_1=0.8,β_2=8.0, α_2=1.6)

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0368	11.444	0.0601	54.1185	0.0049
	50	0.0211	5.2304	0.0363	23.8805	0.0027
	100	0.0072	1.5271	0.0143	3.59462	0.0011
	150	0.0039	0.7540	0.0076	1.28845	0.0007
0.50	30	0.0232	10.338	0.0172	37.2884	0.0095
	50	0.0156	5.2527	0.0102	16.9387	0.0066
	100	0.0084	2.9853	0.0052	6.05791	0.0033
	150	0.0046	1.1754	0.0023	2.21982	0.0019
0.75	30	0.0295	13.033	0.0089	79.7122	0.0258
	50	0.0204	10.180	0.0054	35.7998	0.0185
	100	0.0107	5.2046	0.0022	9.59995	0.0093
	150	0.0060	3.1007	0.0013	4.51565	0.0051

Set2 (β_1=2.0,α_1=1.0,β_2=2.0,α_2=2.0)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.1193	46.127	0.1792	66.0759	0.6273
	50	0.1185	45.665	0.1781	41.0069	0.5755
	100	0.1010	42.858	0.1652	17.8127	0.4624
	150	0.0922	36.088	0.1490	12.6775	0.3729
0.50	30	0.0868	58.448	0.1513	62.5087	0.5669
	50	0.0934	51.328	0.1417	45.9203	0.5624
	100	0.0985	46.575	0.1126	19.5699	0.4882
	150	0.0954	36.775	0.0952	12.1621	0.4375
0.75	30	0.1706	69.069	0.1713	68.1482	0.5077
	50	0.1806	63.081	0.1454	45.2282	0.5860
	100	0.1889	65.605	0.1095	31.8689	0.6455
	150	0.1780	48.343	0.0865	19.3733	0.6006

Set3 (β_1=2.0,α_1=25,β_2=3.0,α_2=100)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0377	12.716	259.302	21.1325	194.141
	50	0.0239	5.3822	169.610	6.44668	113.714
	100	0.0106	0.9905	71.6680	0.64831	47.4342
	150	0.0066	0.5818	43.0361	0.37675	30.4818
0.50	30	0.0332	13.464	67.7612	15.2848	296.957
	50	0.0253	7.4316	46.5092	4.70435	222.568
	100	0.0138	3.1018	22.6429	0.91473	119.683
	150	0.0089	0.7449	13.5650	0.66417	71.8398
0.75	30	0.0671	29.983	45.7708	24.8795	882.005
	50	0.0540	21.037	28.9221	11.7814	684.137
	100	0.0272	8.2461	13.8271	2.20149	332.836
	150	0.0143	4.2859	7.17099	0.94703	197.087

3.6 Standard Deviation

The standard deviation of the parameter estimates of MWD were calculated for each set of parameters to determine the effect of sample size and mixing proportion. Results showed that in almost all cases, the values of the standard deviations of each parameter decreased when the sample size increased indicating that the estimates are more accurate at large sample sizes. However, no specific trend was found in the relation between the standard deviation and the mixing parameter. Table 5 illustrates the values of the standard deviation of each parameter for different sample sizes and different values of the mixing proportion.

3.7 Total Deviations

The total deviation (TD) of the estimates quantifies the absolute sum of the differences between the average of the parameter estimates and its corresponding true value, Al-Fawzan [21], as follows;

$$TD = \left|\frac{\hat{w}-w}{w}\right| + \left|\frac{\hat{\beta}_1-\beta_1}{\beta_1}\right| + \left|\frac{\hat{\alpha}_1-\alpha_1}{\alpha_1}\right| + \left|\frac{\hat{\beta}_2-\beta_2}{\beta_2}\right| + \left|\frac{\hat{\alpha}_2-\alpha_2}{\alpha_2}\right| \quad (12)$$

Table 6 lists the values of the total deviation for two-component MWD for the three sets of parameters with different sample sizes and different values of the mixing parameter. Results show that the total deviation decreases when the sample size increases. However, variation of the mixing parameter has an arbitrary effect on the total deviation.

3.8 Outliers

Upper and lower outliers of the parameter estimates for each set of parameters were determined as explained in section 2.3. Figure 5 represents the boxplots of the outliers of the parameter estimates for set 1 (β_1=4.0,α_1=0.8,β_2=8.0, α_2=1.6), when n = 30, 50, 100, 150 and w=0.25 with 5000 replicates. Table 7 shows the number of upper and lower outliers found in 5000 replications. It also shows the first quartile Q1, the third quartile Q3 and the IQR for set1, when n=30, 50, 100, 150 and w=0.25 with 5000 replicates. This set was chosen as an example to show the representation of outliers and to illustrate the idea and effect of outliers on the parameter estimates.

By removing the outliers, a new average for the estimates of each parameter was obtained for set 1 (β_1=4.0,α_1=0.8,β_2=8.0, α_2=1.6), n=30, 50, 100, 150 and w=0.25. These new estimates showed better qualities represented by lower values of bias, MSE, standard deviation and total deviation compared to the estimates obtained with outliers. Table 8 shows the new estimates of the parameters after ignoring the outliers together with the bias of the estimates, the mean square error, the standard deviation and the total deviation of the new estimates without outliers.

From these tables, we can see that the parameter estimates obtained after ignoring the outliers had lower bias, MSE, standard deviation, and total deviation as compared to the estimates obtained with outliers. The improvement in the accuracy of the estimates was more than double that determined with outliers.

4. Conclusions

This article illustrates the properties (mean, variance, mode, and median) of MWD and the effect of sample size and mixing parameter on the parameter estimates as well as the effect of outliers. The study was based on simulation of generated data for three sets of parameters representing uni-modal and bi-modal MWD with different sample sizes (n = 30, 50, 100 and 150) and different mixing parameters (w = 0.25, 0.50 and 0.75). The simulation was based on 5000 replicates for each case where the average, bias, mean square error, standard deviation and the total deviation of the parameter estimates were determined.

It was found that larger sample sizes gave more accurate estimates represented by lower bias, mean square error, standard deviation and total deviation. On the other hand the effect of the mixing parameter was most pronounced on the mode and median as increasing the value of the mixing parameter resulted in a decrease in the values of both the median and the mode. Increasing the value of the mixing parameter also affected the shape of the density function by increasing its peakness. However, the effect of the mixing parameter on the bias and on the mean square error did not show a specific trend. Outliers for each parameter were found and were illustrated by boxplots and their effect on the parameter estimates was discussed. It was found that by removing the outliers, the estimates had lower bias, mean square error, standard deviation and total deviation and the accuracy of the estimates was doubled.

As an overall conclusion, it was found that MWD provides a better fit for certain data sets as it can represent both uni-modal and bi-modal. Maximum likelihood estimation was found to give very accurate estimates for the parameters. To improve the accuracy of the estimates of the parameters, it is recommended to increase sample size and remove the outliers. This study can be extended to cove data sets of different ranges by using different values of the mixing parameter ranging from 0.10 to 0.90. Outlier analysis can be tested at different levels, i.e. 5%, 10% and 15%

ISSN 2348 – 7968

Table 5: Standard deviation of the estimates of parameters

Set1 (β_1=4.0,α_1=0.8,β_2=8.0, α_2=1.6)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.1895	3.3328	0.3362	7.05063	0.4694
	50	0.1405	2.2028	0.2131	4.69249	0.2153
	100	0.0830	1.1739	0.1189	1.83006	0.0460
	150	0.0617	0.8328	0.0864	1.10457	0.0269
0.50	30	0.1754	3.1138	0.1941	5.83508	0.3050
	50	0.1306	2.2250	0.1186	3.97154	0.1488
	100	0.0934	1.7068	0.0778	2.40613	0.0828
	150	0.0679	1.0716	0.0510	1.45291	0.0439
0.75	30	0.2153	3.5531	0.1880	8.05514	0.3613
	50	0.1612	3.1446	0.1132	5.58501	0.2185
	100	0.11113	2.2716	0.0674	2.97030	0.1330
	150	0.0838	1.7624	0.0505	2.05879	0.1007

Set2 (β_1=2.0,α_1=1.0,β_2=2.0,α_2=2.0)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.3009	5.9783	0.4876	7.08951	0.9574
	50	0.3059	6.0638	0.4767	5.76008	0.8735
	100	0.2950	5.9756	0.4375	3.95828	0.7309
	150	0.2879	5.5219	0.4138	3.39325	0.6633
0.50	30	0.3166	6.6983	0.4310	6.91150	0.9832
	50	0.3146	6.3397	0.4078	6.16606	0.8955
	100	0.3118	6.1747	0.3593	4.17267	0.7864
	150	0.3056	5.5654	0.3303	3.33479	0.7302
0.75	30	0.3334	7.0723	0.3942	7.18905	0.9365
	50	0.3358	6.9315	0.3761	6.10686	0.9048
	100	0.3451	7.1898	0.3404	5.28070	0.8938
	150	0.3370	6.2658	0.3063	4.21234	0.8341

Set3 (β_1=2.0,α_1=25,β_2=3.0,α_2=100)

w	*n*	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.1848	3.3129	15.8994	4.34911	19.9549
	50	0.1486	2.1979	12.6999	2.44836	11.9156
	100	0.1012	0.9462	8.34068	0.77233	6.97429
	150	0.0804	0.7391	6.49421	0.59959	5.48817
0.50	30	0.1870	3.4834	8.54498	3.71185	19.7145
	50	0.1604	2.6324	6.96073	2.09636	15.9232
	100	0.1175	1.7318	4.77140	0.92587	11.0223
	150	0.0929	0.8505	3.69998	0.80193	8.58596
0.75	30	0.2563	5.1241	7.58730	4.69219	31.9053
	50	0.2258	4.4042	5.81536	3.29325	26.9311
	100	0.1679	2.8191	4.10045	1.43828	19.2964
	150	0.1255	2.0524	3.03437	0.95092	15.0995

Table 6: Total deviation of the three sets of parameter estimates (β1,α1,β2,α2)

w	*n*	**(4.0,1.8,8.0,1.6)**	**(2.0,1.0,2.0,2.0)**	**(2.0,25,3.0,100)**
0.25	30	0.81793	3.66345	1.51167
	50	0.62804	3.15459	0.91749
	100	0.25589	2.46753	0.38225
	150	0.14480	2.06733	0.23182
0.50	30	0.58783	3.16167	1.03398
	50	0.32985	2.80359	0.59892
	100	0.15485	2.26167	0.26463
	150	0.08694	1.85443	0.14543
0.75	30	0.86074	3.84714	1.77186
	50	0.52160	3.42353	1.18427
	100	0.22974	3.09153	0.51676
	150	0.13973	2.05558	0.27841

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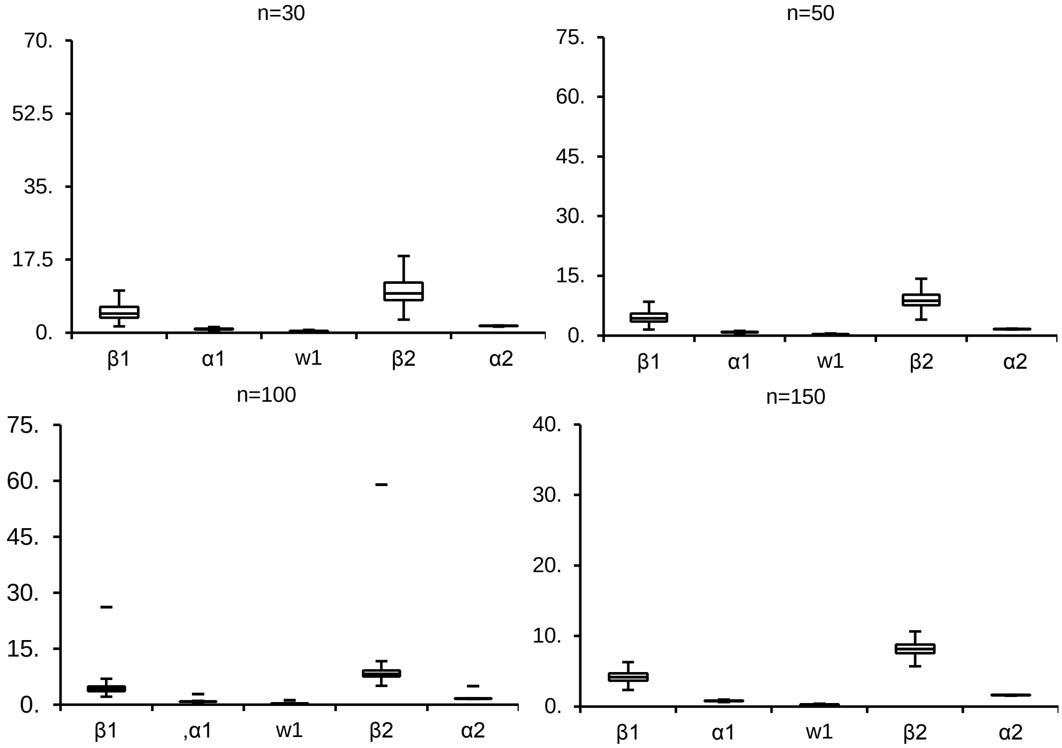

Figure 5: Box plot of the estimates of parameters for set 1 (β_1=4.0,α_1=0.8,β_2=8.0, α_2=1.6), w=0.25

Table 7: Upper and lower outliers for set 1, w=0.25

n=30

Parameter	β_1	α_1	*w1*	β_2	α_2
Min	1.49460	0.23951	0.05792	3.15863	1.26078
Q1	3.56546	0.72016	0.21746	7.79959	1.57206
Median	4.62136	0.81213	0.29100	9.40514	1.61039
Q3	6.17661	0.98019	0.41733	12.0254	1.65161
Max	48.5112	1.59550	0.94062	49.9525	1.96398
IQR	2.61115	0.26003	0.19987	4.22580	0.07955
Upper Outliers	264	220	197	419	114
Lower Outliers	0	4	0	0	38
Q3+1.5*IQR	10.0934	1.37023	0.71713	18.3641	1.77094
Q1-1.5*IQR	-0.35126	0.33011	-0.08234	1.46090	1.45273

n=50

Parameter	β_1	α_1	w1	β_2	α_2
Min	1.50301	0.38917	0.05359	3.96097	1.42453
Q1	3.50193	0.73581	0.21170	7.58339	1.57665
Median	4.34841	0.80267	0.26910	8.70163	1.60767
Q3	5.50702	0.90092	0.34643	10.2726	1.63872
Max	40.6477	1.55075	0.93544	49.5913	2.01017
IQR	2.00510	0.16511	0.13473	2.68917	0.06207
Upper Outliers	250	481	346	356	75
Lower Outliers	0	9	0	0	35
Q3+1.5*IQR	8.51467	1.14858	0.54852	14.3063	1.73183
Q1-1.5*IQR	0.49428	0.48815	0.00961	3.54964	1.48355

n=100

Parameter	β_1	α_1	w1	β_2	α_2
Min	2.12291	0.53183	0.08660	4.77805	1.45488
Q1	3.60103	0.75084	0.21543	7.52328	1.58030
Median	4.21431	0.79733	0.25515	8.24782	1.60234
Q3	4.92532	0.85728	0.30180	9.16553	1.62276
Max	21.1799	1.45302	0.91468	45.0556	1.88036
IQR	1.32429	0.10645	0.08638	1.64225	0.04246
Upper Outliers	179	318	196	156	35
Lower Outliers	0	16	0	4	31
Q3+1.5*IQR	6.91177	1.01696	0.43136	11.6289	1.68644
Q1-1.5*IQR	1.61458	0.59116	0.08586	5.05991	1.51662

n=150

Parameter	β_1	α_1	w1	β_2	α_2
Min	2.36026	0.60005	0.10228	4.82031	1.48724
Q1	3.65649	0.75769	0.21958	7.55586	1.58409
Median	4.14403	0.79713	0.25185	8.14045	1.60144
Q3	4.71358	0.84391	0.28935	8.79769	1.61822
Max	10.0654	1.50466	0.84355	24.0266	1.75952
IQR	1.05708	0.08623	0.06977	1.24183	0.03413
Upper Outliers	100	227	138	130	48
Lower Outliers	0	12	4	3	38
Q3+1.5*IQR	6.29920	0.97326	0.39401	10.6604	1.66942
Q1-1.5*IQR	2.07087	0.62834	0.11492	5.69311	1.53289

Table 8: Estimates, Bias, MSE, standard deviation and total deviation of the parameters for set 1 (β_1=4.0, α_1=0.8, β_2=8.0, α_2=1.6), w=0.25, without outliers.

Estimates

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.3167	4.8132	0.8513	9.59743	1.6103
	50	0.2730	4.4919	0.8028	8.78665	1.6068
	100	0.2568	4.2597	0.7975	8.32392	1.6018
	150	0.2534	4.1918	0.7972	8.16401	1.6011

Bias

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0667	0.8132	0.0513	1.59743	0.0103
	50	0.0230	0.4919	0.0028	0.78665	0.0068
	100	0.0068	0.2597	-0.0025	0.32392	0.0018
	150	0.0034	0.1918	-0.0028	0.16401	0.0011

MSE

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.0242	3.6237	0.0424	10.1130	0.0034
	50	0.0087	2.0095	0.0134	3.96382	0.0021
	100	0.0037	0.8915	0.0061	1.43036	0.0009
	150	0.0024	0.5878	0.0038	0.79429	0.0006

Standard Deviation

w	***n***	$\hat{w}$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\beta}_2$	$\hat{\alpha}_2$
0.25	30	0.1405	1.7214	0.1994	2.75010	0.0577
	50	0.0903	1.3296	0.1158	1.82914	0.0455
	100	0.0607	0.9079	0.0778	1.15139	0.0310
	150	0.0491	0.7424	0.0615	0.87610	0.0248

Total deviation

w	***N***	TD
0.25	30	0.74018
	50	0.32101
	100	0.13743
	150	0.08635

References

[1] J. H. K. Kao, A graphical estimation of mixed Weibull parameters in life testing of electron tubes, Technometrics, I (1959), 389-407
[2] S. Jiang and D. Kececioglu, Maximum likelihood estimates from censored data for mixed-Weibull distributions. IEEE T Reliab 41(2) (1992), 248-255.
[3] J. Zheng and C. Frey, Quantitative analysis of variability and uncertainty in emission estimation: An illustration of methods using mixture distributions. Proceeding AWMA, June (2001) 24-28.
[4] D. N. P. Murthy, M. Xie and R. Jiang, Weibull Models. New Jersey. John Wiley & Sons. 2004.
[5] R. Jiang and D. N. P. Murthy, Modeling failure data by mixture of two Weibull distributions; A graphical approach. IEEE T Reliab 44(1995) 477-488.
[6] R. Jiang and D. N. P. Murthy, Mixture of Weibull distributions. Parametric characterization of failure rate function. Appl Stoch Model D A 14(1998) 47-65.
[7] R. Jiang, M. J. Zuo and H. X. Li, Weibull and inverse Weibull mixture models allowing negative weights, Reliab Eng Syst Safe 66(1999):227-234.
[8] L. Zhang, J. H. Gove, Ch. Liu and W. B. Leak, A finite mixture of two Weibull distributions for modeling the diameter distributions of rotated sigmoid, uneven-aged stands. NRC Research Press, 2001.
[9] O. A. Jaramillo, M. A. Borja, Wind speed analysis in La Ventosa, Mexico. A bimodal probability distribution case. Renew Energ 29(2004) 1613-1630.
[10] J. Al-Salih and S. K. Agarwal, Finite mixture of gamma distributions: A conjugate prior. Comput Stat Data An. 51(9) (2007) 4369-4378.
[11] J. A. Carta and P. Ramirez, Analysis of two-component mixture Weibull statistics for estimation of wind speed distributions. Renew Energ 32(2007) 518-531.
[12] C. D. Lai and M. Xie, Stochastic Ageing and Dependence for Reliability. Birkhuaser Publisher, 2007.
[13] J. J. Suhaila and A. A. Jemain, Fitting daily rainfall amount in peninsular Malaysia using several types of exponential distributions. J Appl Sci Res. 3(10)(2007) 1027-1036
[14] A. M. Razali, A. A. Salih, A.A. Mahdi, A. Zahrim, A. K. Ibrahim and K. Sopian, On simulation study of mixture of two Weibull distributions. 7th WSEAS International Conference on System Science and Simulation in Engineering, Venice, Italy.21-23 November 2008.
[15] A. M. Razali and A. A. Salih, Combining two distributions using a mixing parameter. Eur J Sci Res 31(2)(2009) 296-305.
[16] A. M. Razali, A. A. Salih and A. A. Mahdi, Some simulation results of Weibull and mixed Weibull distributions. Fareast J Theo Stat 28(1)(2009) 107-116
[17] S. A. Akdag, H. S. Bagiorgas and G. Mihalakakou, Use of two-component Weibull mixtures in the analysis of wind speed in the Eastern Medeterranian. Appl Energ 87(2010) 2566-2573.
[18] K. S. Sultan, M. A. Ismail and A. S. Al-Moisheer, Mixture of two inverse Weibull distributions: Properties and estimation. Comput Stat Data An 51(2007) 5377-5387
[19] R. V. Hogg and A. T. Craig, Introduction to Mathematical Statistics. 5th Ed. USA. 1995.
[20] H. Rinne, The Weibull distribution: A Handbook. Chapman & Hall/CRC, 2008.
[21] M. A. Al-Fawzan, Methods for estimating the parameters of the Weibull distribution. King Abdul-Aziz City for Science and Technology (2000) 1-11

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