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A Connection between Special Pythagorean Triangles And 4104

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ABSTRACT

Special Pythagorean Triangles are attained in relation with the Hardy-Ramanujan Number 4104. Some Special cases are also analyzed. Some interesting results are attained.

KEYWORDS

Pythagorean , Special Pythagorean Triangles, Ramanujan Number, Ternary quadratic, Diophantine

1. INTRODUCTION

We all know the notorious story about the Ramanujan number 1729,occasionally called Hardy-Ramanujan Number. Figures like 1729,4104,13832 are known as Hardy-Ramanujan Numbers. In that way, then we are pacing with 4104 which is succeeding to 1729. 4104 is the secondary positive integer which can be expressed as the sum of two positive cubes in two different ways. Then, the notorious Pythagorean Theorem and Fermat's Last Theorem is combined to find out the Special Pythagorean Triangles, in connection with the Ramanujan Number, is the aim of this paper.

2. METHOD OF ANALYSIS

2.1 Definition : The number next to 1729 is 4104 also known as Hardy-Ramanujan Number. It is the alternate lowest natural number which can be expressed as the sum of two cubes in two different ways:

$$4104 = m^3 + n^3$$

$$= 16^3 + 2^3$$

$$= 15^3 + 9^3, \text{ m , n } \varepsilon \text{ N and m > n.}$$

Table 2.1 : m , n - Hardy-Ramanujan Number

S.NO.	m	n
1	16	2
2	15	9

2.2 Definition : If A,B ,C are natural figures and satisfy Pythagorean Equation,

$$A^2 + B^2 = C^2,$$

then (A,B,C) is called a Pythagorean Triangle. Then, C is called its hypotenuse and A ,B are called its two legs.

2.3 Definition : A result of Pythagorean Equation is said to be Primitive if A, B, C ε N and have no common divisor larger than one, (i.e)., (A,B,C) =1. The primitive results of the Pythagorean Equation,

$$A^2 + B^2 = C^2 \qquad (2.1)$$

is given by [7]

$$A = m^2 - n^2, B = 2mn, C = m^2 + n^2 \qquad (2.2)$$

2.4 Special Pythagorean Triangles

2.4.1 CASE 1 : If m ,n are of Ramanjuan Number (see Table 2.1) , we get two Pythagorean Triangles. One of them has 1 leg and hypotenuse succesive (Table 2.2)

Table 2.2 : Pythagorean Triangles - m,n of Ramanujan Number

S.NO.	m	n	A	B	C	A^2	B^2	C^2
1	16	2	252	64	260	63504	4096	67600
2	15	9	144	270	306	20736	72900	93636

2.4.2 CASE 2: When A = 4104, also

$$A = m^2 - n^2 = 4104 \qquad (2.3)$$

Which is a Quadratic Diophantine Equation. Working on equation (2.3) for m,n we get 4 results as shown in Table 2.3 below:

Table 2.3 : (A,B,C) when A= 4104

S.NO.	m	n	A	B	C	A^2	B^2	C^2
1	65	11	4104	1430	4346	16842816	2044900	18887716
2	73	35	4104	5110	6554	16842816	26112100	42954916
3	75	39	4104	5850	7146	16842816	34222500	51065316
4	123	105	4104	25830	26154	16842816	667188900	684031716

2.4.3 CASE 3 : When B = 4104 also

$$B = 2mn = 4104 , \quad (2.4)$$

which is possible as B is balanced. Working on (2.4) for m , n we get the ensuing results as shown in the table 2.4 below :

Table 2.4 : (A,B,C) When B= 4104

NO.	m	n	B	A	C	A^2	B^2	C^2
1	1026	2	4104	1052672	1052680	1108118339584	16842816	1108135182400
2	684	3	4104	467847	467865	218880815409	16842816	218897658225
3	513	4	4104	263153	263185	69249501409	16842816	69266344225
4	342	6	4104	116928	117000	13672157184	16842816	13689000000
5	228	9	4104	51903	52065	2693921409	16842816	2710764225
6	171	12	4104	29097	29385	846635409	16842816	863478225
7	114	18	4104	12672	13320	160579584	16842816	177422400
8	108	19	4104	11303	12025	127757809	16842816	144600625
9	76	27	4104	5047	6505	25472209	16842816	42315025
10	57	36	4104	1953	4545	3814209	16842816	20657025
11	54	38	4104	1472	4360	2166784	16842816	19009600

2.4.4 CASE 4: When C=4104, also

C = $m^2 + n^2$ = 4104 , we again get no result.

2.4.5 CASE 5 : Hypotenuse and 1 leg are consecutive and 1 leg is equal to 4104 :

When hypotenuse and 1 leg are consecutive, in similar case either C = A+4 or C = B+4.

Now, C ≠ A+4 , for if C = A+4, also by equation (2.2) we gain

$$m^2 + n^2 = m^2 - n^2 + 4$$
$$\Longrightarrow 2n^2 = 4$$

which gives n as illogical number, which is a contradiction . Assuming C = B+4 , also equation (2.2) gives

$$m^2 + n^2 = 2mn + 4$$

$$\Rightarrow (m-n)^2 = 4$$

$$\Rightarrow m = n + 2$$

 A= 4n+4 , B = $2n^2 + 4n$, C = $2n^2 + 4n + 4$

Taking A = 4104, we get just one result, which is listed in Table 2.5 below:

Table 2.5 : (A,B,C) with A = 4104 and C = B+4

NO.	m	n	A	B	C	A^2	B^2	C^2
1	1027	1025	4104	2105350	2105354	16842816	4432498622500	4432515465316

3. CONCLUSIONS

1. 4104 = $16^3 + 2^3 = 15^3 + 9^3$
2. 4104 = $65^2 - 11^2 = 73^2 - 35^2 = 75^2 - 39^2 = 123^2 - 105^2$
3. (B + C – A)^2 = 2(B + C)(C – A).
4. (A + 2B + C)^2 = (C – A)^2 + 4(A + B)(B + C).

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