ISSN (Online) 2348 – 7968 | Impact Factor – 6.72
www.ijiset.com

Filtering And Ranking Decision Factors Via Nano Topology and Mis-Analysis

***S. Dayana Mary & **M. Hannah Angelin**

*Assistant Professor, Department of Mathematics, Coimbatore Institute of Technology affiliated to Anna University, Coimbatore-641014, Tamil Nadu, South India. email:sdayanamary@cit.edu.in

**Assistant Professor, Department of Civil Engineering, Coimbatore Institute of Technology affiliated to Anna University, Coimbatore-641014, Tamil Nadu, South India. email:hannah@cit.edu.in

ABSTRACT

The goal of this paper is to analyze the application of nano-topology in selected models designed to address real-life problems. The central focus of the study is the utilization of nano-topology in the decision-making process for purchasing residential properties, where both external and internal attributes of a household are taken into consideration. By integrating nano-topological conditions into decision frameworks, this research demonstrates how innovative mathematical and computational approaches can optimize housing choices, reduce diminishing factors, and support sustainable property development. The findings indicate that nano-topology can provide a structured methodology for evaluating complex variables in real estate decisions, thereby contributing to innovation in the construction sector and advancing sustainable housing practices. Also, this work highlights the potential of nano topology as a bridge between abstract mathematical theory and practical data-driven applications in decision analysis.

KEY WORDS: Societal Challenges, Decision-making Algorithm, External factors and Internal factors, Nano topology, Mutual Information Score.

INTRODUCTION

Attribute reduction and feature selection have become crucial steps in pattern recognition and machine learning tasks. Classical rough set theory is a mathematical tool used to handle data sets that contain imprecision and uncertainty. The study of attribute reduction and feature selection in information systems can be approached using classical rough set theory. Equivalence relations serve as the foundation of rough set theory. These relations allow for the partitioning of objects in a universe into exclusive equivalence classes, which form the basic information granules for approximating any subset of the universe. The fundamental concept behind rough sets is to eliminate redundant information in the data in order to make accurate decisions or classifications. The research on rough set theory has gained significant attention in both theoretical and applied areas. In recent decades, topology has found applications in various fields such as medicine, engineering, economics, chemistry, computer science, and cosmology. Topology has proven its usefulness in simplifying human interventions and improving daily life. Palwak [5] provided a definition and discussion of rough sets. Wei - Zhi Wu [6] introduced the concept of attribute reduction. Yuhua Qianab et al., [7] conducted research on set-valued ordered information systems.

In recent years, topology has been increasingly employed by mathematicians to model and solve real-world problems. Levine [1] introduced the concept of semi-open sets and semi-continuity in topological spaces. Additionally, the author introduced generalized closed sets in [2], which served as a foundation for the study of closed sets in topology. Stone [3] and Tong [4] have provided definitions and conducted investigations on regular open sets and strong regular open sets, respectively. The concept of nano topological space was originally proposed by Lelli's Thivagar [8], who used lower and upper approximations as well as boundary regions. Thivagar applied set-valued ordered information systems for attribute reduction in nutrition modelling. Following Thivagar and Richard's work on nano near open sets [9], several mathematicians have devoted their attention to generalizing these sets.

Throughout this paper, let $(\mathcal{P}, \tau_{\mathcal{R}}(\mathcal{S})$ be a Nano Topological space with respect to $\mathcal{S}$, where $\mathcal{S} \subseteq \mathcal{P}$, $\mathcal{R}$ is an equivalence relation on $\mathcal{P}, \mathcal{P}/\mathcal{R}$ denotes the family of equivalence classes of $\mathcal{P}$ by $\mathcal{R}$ on which unless otherwise stated, no separation axioms are assumed. Similarly, let $(\mathcal{Q}, \tau_{\mathcal{R}}(\mathcal{T})$ be a Nano Topological space with respect to $\mathcal{T}$, where $\mathcal{T} \subseteq \mathcal{Q}, \mathcal{R}$ is an equivalence relation on $\mathcal{Q}, \mathcal{Q}/\mathcal{R}$ denotes the family of equivalence classes of $\mathcal{Q}$ by $\mathcal{R}$. Additionally, $(\mathcal{W}, \tau_{\mathcal{R}}(\mathcal{X})$ is a Nano Topological space with respect to $\mathcal{X}$, where $\mathcal{X} \subseteq \mathcal{W}$, $\mathcal{R}$ is an equivalence relation on $\mathcal{W}$, $\mathcal{W}/\mathcal{R}$ denotes the family of equivalence classes of $\mathcal{W}$ by $\mathcal{R}$. For a subset $\mathcal{M}$ of a space $(\mathcal{P}, \tau_{\mathcal{R}}(\mathcal{S})$, $\mathcal{N}cl(\mathcal{M})$ and $\mathcal{N}int(\mathcal{M})$ denote the nano closure $\mathcal{M}$ and nano interior of $\mathcal{M}$, respectively. In the following, we provide the definitions of some of these concepts, which are utilized in our present study.

II. PRELIMINARIES

In this section, we provide a brief overview of fundamental definitions and results in nano topological spaces. These concepts are essential in establishing the main results.

Definition 2.1 [5] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. The pair $(\mathcal{U}, \mathcal{R})$ is said to be the approximation space. Let $\mathcal{X} \subseteq \mathcal{U}$. Then,

(i) The lower approximation of $\mathcal{X}$with respect to $\mathcal{R}$ is the set of all objects which can be for certain classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and it is denoted by $\mathcal{L}_{\mathcal{R}}(\mathcal{X})$.

$\mathcal{L}_{\mathcal{R}}(\mathcal{X}) = \cup_{x \in U} \{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \subseteq X\}$, where $\mathcal{R}(\mathcal{X})$ denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be possibly classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and it is denoted by $\mathcal{U}_{\mathcal{R}}(\mathcal{X})$. $\mathcal{U}_{\mathcal{R}}(\mathcal{X}) = \cup_{x \in U} \{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \cap \mathcal{X} \neq \emptyset, x \in U\}$.

(iii) The boundary region of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be classified neither as $\mathcal{X}$ nor as not $\mathcal{X}$ with respect to $\mathcal{X}$and it is denoted by $\mathcal{B}_{\mathcal{R}}(\mathcal{X})$.

$\mathcal{B}_{\mathcal{R}}(\mathcal{X}) = \mathcal{U}_{\mathcal{R}}(\mathcal{X}) - \mathcal{L}_{\mathcal{R}}(\mathcal{X})$.

Property 2.2 [5] If (U, R) is an approximation space and$X, Y \subseteq U$, then

(i) $L_{\mathcal{R}}(X) \subseteq X \subseteq U_{\mathcal{R}}(X)$.
(ii) $L_{\mathcal{R}}(\emptyset) = U_{\mathcal{R}}(\emptyset) = \emptyset$.
(iii) $L_{\mathcal{R}}(U) = U_{\mathcal{R}}(U) = U$.
(iv) $U_{\mathcal{R}}(X \cup Y) = U_{\mathcal{R}}(X) \cup U_{\mathcal{R}}(Y)$.
(v) $U_{\mathcal{R}}(X \cap Y) \subseteq U_{\mathcal{R}}(X) \cap U_{\mathcal{R}}(Y)$.
(vi) $L_{\mathcal{R}}(X \cap Y) = L_{\mathcal{R}}(X) \cap L_{\mathcal{R}}(Y)$.
(vii) $L_{\mathcal{R}}(X \cup Y) \supseteq L_{\mathcal{R}}(X) \cup L_{\mathcal{R}}(Y)$.
(viii) $L_{\mathcal{R}}(X) \subseteq L_{\mathcal{R}}(Y)\ and\ U_{\mathcal{R}}(X) \subseteq U_{\mathcal{R}}(Y) whenever X \subseteq Y$.
(ix) $U_{\mathcal{R}}(X^c) = [L_{\mathcal{R}}(X)]^c and L_{\mathcal{R}}(X^c) = [U_{\mathcal{R}}(X)]^c$.
(x) $U_{\mathcal{R}}[U_{\mathcal{R}}(X)] = L_R[U_{\mathcal{R}}(X)] = U_{\mathcal{R}}(X)$.
(xi) $L_{\mathcal{R}}[L_{\mathcal{R}}(X)] = U_{\mathcal{R}}[L_{\mathcal{R}}(X)] = L_{\mathcal{R}}(X)$.

Definition 2.3 [8]Let $\mathcal{U}$ be the universe; $\mathcal{R}$ be an equivalence relation on U and $\tau_{\mathcal{R}}(X) = \{U, \emptyset, L_{\mathcal{R}}(X), U_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$, where $X \subseteq U$. Then $\tau_{\mathcal{R}}(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_{\mathcal{R}}(X)$.
(ii) The union of the elements of any sub collection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.

Then $\tau_{\mathcal{R}}(X)$ is a topology on U called the Nano Topology on U with respect to X. Then $(U, \tau_{\mathcal{R}}(X))$ is called the Nano topological space. Elements of the Nano Topology are known as Nano open sets in U. Elements of $[\tau_{\mathcal{R}}(X)]^c$ are called Nano closed sets with $[\tau_{\mathcal{R}}(X)]^c$ being called dual Nano Topology of $\tau_{\mathcal{R}}(X)$.

Definition 2.4 [8] If $\tau_{\mathcal{R}}$ is the Nano topology on U with respect to X, then the set $\beta_{\mathcal{R}}$= {U,$\emptyset$, $L_{\mathcal{R}}(X), B_{\mathcal{R}}(X)$} is the basis for $\tau_{\mathcal{R}}(X)$.

III APPLICATION OF NANO-TOPOLOGY IN THE DECISION-MAKING PROCESS FOR RESIDENTIAL HOUSE PURCHASES

In recent years, there has been growing interest in the application of nano topology in various fields. One area that has seen significant development is the use of nano-topology in decision making, particularly in the context of purchasing residential properties. This paper aims to explore the potential benefits and challenges associated with the integration of nano-topology into the decision making process for buying houses. The decision to purchase a residential property is a complex one, involving a range of factors such as location, price, size, and amenities. Traditionally, individuals rely on their own judgment and the advice of real estate agents to make informed decisions. However, with the emergence of nano-topology, there is now an opportunity to enhance the decision making process by leveraging the power of nano topology. **3.1 Decision-Making Algorithm**Step 1: Begin with a non-empty finite universe $\mathcal{U}$, and a finite set of attributes $\mathcal{A}$. Partition these attributes into two classes: Conditional Attributes (Ce-External or Ci-Internal) and Decision-Making Attribute ($\mathcal{D}$). Also, define an equivalence relation $\mathcal{R}$ on the universe. Step 2: Find the lower approximation, upper approximation, and boundary region with respect to $\mathcal{R}$, a subset $\mathcal{S}$ of the universe $\mathcal{U}$.Step 3: Generate the Nano topology. Step 4: Eliminate an attribute from the set of attributes and determine the lower approximation, upper approximation, and boundary region. Step 5: Generate the Nano topology. Step 6: Repeat steps 4 and 5 for each attribute in the conditional

attribute class. Step 7: Determine the set CORE in all cases, where the CORE attributes are those for which the equivalence relation holds.Step 8: The attributes in CORE are the main decision attributes.
Refer to Figure 3.1 for a visual representation.

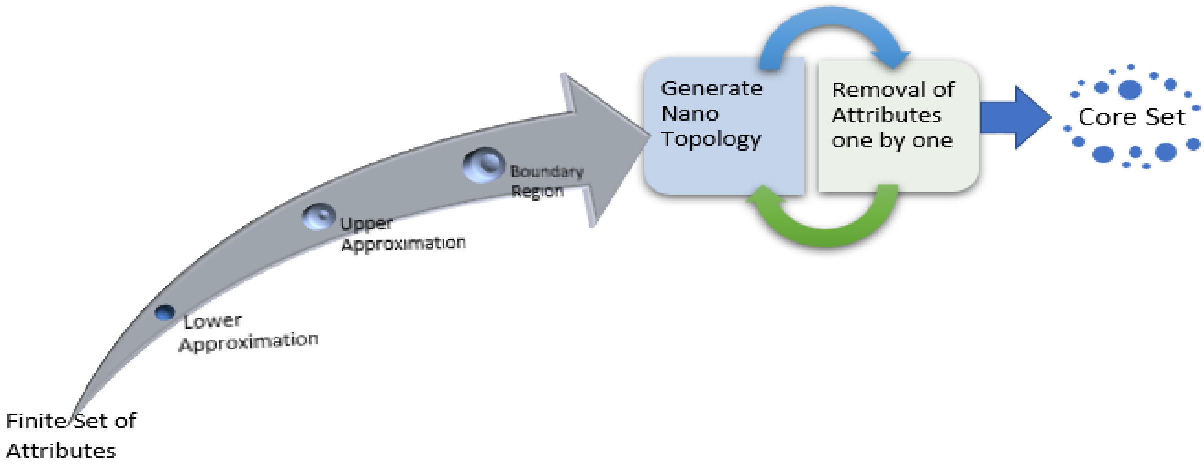


Fig. 3.1 Decision making algorithm

3.2 DECISION-MAKING PROBLEM

Decision-making techniques are made use in all areas of day-to-day life. Every stage needs decision-making. Buying a residential house is still considered as an important thing and a man's dream. Be it for the first time or the next. Arriving at a decision to purchase an immovable property lies on various attributes. A person has to plan, analyze, assess, choose and decide upon from a list of attributes before a final decision is made to own any property. There is a need to investigate not only as individual attributes but also as a whole. The algorithm is founded on eliminating redundant attributes in order to generate a successfully reduced set and formulate the core set of attributes.
In this study, eight residential houses $\{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4, \mathcal{H}_5, \mathcal{H}_6, \mathcal{H}_7, \mathcal{H}_8\}$ at different locations were selected. Real time data was collected to find the main attributes based on which the decision on purchasing the residential property is to be done. Conditional attributes are further categorized as external and internal. In this paper, Nano topology concept is used to identify the conditional attributes towards decision making. This is applied to decide upon the owning of residential house property.

The people of South India, before deciding upon to buy a residential property, focusses on various parameters considering the building and its environment. They have a belief that nature also influences the happiness and well-being of the inmates of the house. The deciding parameters or the conditional attributes that help in the decision to own a residential building as a whole is studied under two heads: external factors and internal factors.

External factors that are considered which decide-upon in owning the property are taken as: (i) the distance of the property from important places like that of hospitals, schools, places of worship, entertainment places- whether it is located less than 5 kilometers or more (ii) whether the structure is either a load-bearing structure or a non-load bearing structure or a temporary structure (iii) whether the age of the building is less than 5 years or more (iv) whether the building is an individual type or an apartment floor (iv) whether the basic units of the building is made of either bricks and river sand or bricks and manufactured sand (M-sand).

Internal factors that are considered which decide-upon in owning the property are taken as (i) whether water and electricity connections are already available (ii) whether the building stands on the land approved by the Directorate of Town and Country Planning (DTCP) (iii)whether there are possibilities of alteration or expansion in the future (iv) whether the Vasthu Shastra considerations are followed and (v) whether the orientation and planning of the building makes maximum use of the natural resources (sunlight, wind) that is whether the construction is sustainable. Vasthu Shastra is a science developed before many centuries for efficient design of buildings. It provides techniques to harness the energies from nature for the betterment of the users. The people of South India, believe that the concepts of Vasthu Shastra combine the five basic elements of nature, they are earth, water, air, fire and sky that enhances happiness and harmony making the living atmosphere pleasant, serene and prosperous. These form foundations of one's dream house. These are shown in Figure 3.2.

Fig. 3.2 Conditional Attributes

3.3 SURVEY 1

Let the distance from important places like hospitals, schools, places of worship, entertainment places ($\mathcal{A}$), Type of Construction ($\mathcal{B}$), Age of the building ($\mathcal{C}$), Building type ($\mathcal{D}$) and Basic units of construction ($\mathcal{E}$) be the Conditional Attributes-external (C_E) involved in decision-making in the purchase of a residential house.

That is, $C_e = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$.

Consider, $\mathcal{A} = \{A_1, A_2\}$ where A_1 = less than 5kms, A_2 = more than 5 kms,

$\mathcal{B} = \{B_1, B_2, B_3\}$ where B_1 = Load bearing structure, B_2 = Non load bearing structure, B_3=Temporary Structure

$\mathcal{C} = \{C_1, C_2\}$ where C_1 = less than 5years, C_2= more than 5 years

$\mathcal{D} = \{D_1, D_2\}$ where D_1 = Individual property, D_2= Apartment floor

$\mathcal{E} = \{E_1, E_2\}$ where E_1= Bricks and River Sand, E_2 = Bricks and M-sand

An information's decisions data set is presented in Table 3.1. Five conditional attributes are chosen as external factors for eight houses.

Table 3.1 **External Conditional Attributes**

Residential Houses	External Conditional Attributes (C_E)					Decision Making Attribute (D)
	Distance from important places ($\mathcal{A}$)	Type of Construction ($\mathcal{B}$)	Age of the building ($\mathcal{C}$)	Building Type ($\mathcal{D}$)	Basic Units of construction ($\mathcal{E}$)	Yes or No
H_1	A_1	B_1	C_1	D_1	E_1	Yes
H_2	A_1	B_1	C_2	D_1	E_1	No
H_3	A_2	B_2	C_1	D_2	E_1	Yes
H_4	A_2	B_1	C_1	D_2	E_2	No
H_5	A_2	B_3	C_1	D_2	E_1	No
H_6	A_2	B_1	C_1	D_2	E_2	Yes
H_7	A_2	B_2	C_2	D_2	E_2	No
H_8	A_2	B_2	C_1	D_2	E_1	Yes

3.4 SURVEY 2

Let the conditions of Water and electric supply connections availability ($\mathcal{F}$), Whether the building is constructed on DTCP approved land ($\mathcal{G}$), Possibility of future expansion ($\mathcal{I}$), Vasthu considerations ($\mathcal{J}$) and Sustainable construction ($\mathcal{K}$) be the Conditional Attributes-internal (C_I) involved in the decision-making process regarding the purchase of a residential house.

That is, $C_I = \{\mathcal{F}, \mathcal{G}, \mathcal{I}, \mathcal{J}, \mathcal{K}\}$.

$\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2\}$ where $\mathcal{F}_1$ = Yes, $\mathcal{F}_2$= No

$\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2\}$ where $\mathcal{G}_1$ = Yes, $\mathcal{G}_2$ = No

$\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2\}$ where $\mathcal{I}_1$ = Yes, $\mathcal{I}_2$ = No

$\mathcal{J} = \{\mathcal{J}_1, \mathcal{J}_2\}$ where $\mathcal{J}_1$ = Exactly followed, $\mathcal{J}_2$ = Partially followed

$\mathcal{K} = \{\mathcal{K}_1, K_2\}$ where $\mathcal{K}_1$= Yes, $\mathcal{K}_2$ = No

An information's decisions data set is presented in Table 3.2. Five conditional attributes are chosen as internal factors for eight houses.

Table 3.2 Internal Conditional Attributes

Residential Houses	Internal Conditional Attributes (C_I) change all H by I, all I by J, all J by K					Decision Making Attributes (D)
	Water and Electric supply connections availability ($\mathcal{F}$)	Building on DTCP approved land ($\mathcal{G}$)	Possibility of future expansion ($\mathcal{I}$)	Vasthu considerations ($\mathcal{J}$)	Sustainable Construction ($\mathcal{K}$)	Yes or No
H_1	$\mathcal{F}_1$	$\mathcal{G}_1$	$\mathcal{I}_2$	$\mathcal{J}_1$	$\mathcal{K}_2$	Yes
H_2	$\mathcal{F}_1$	$\mathcal{G}_2$	$\mathcal{I}_2$	$\mathcal{J}_1$	$\mathcal{K}_2$	No
H_3	$\mathcal{F}_1$	$\mathcal{G}_1$	$\mathcal{I}_2$	$\mathcal{J}_2$	$\mathcal{K}_1$	Yes
H_4	$\mathcal{F}_1$	$\mathcal{G}_2$	$\mathcal{I}_1$	$\mathcal{J}_2$	$\mathcal{K}_1$	No
H_5	$\mathcal{F}_1$	$\mathcal{G}_1$	$\mathcal{I}_2$	$\mathcal{J}_1$	$\mathcal{K}_2$	No
H_6	$\mathcal{F}_1$	$\mathcal{G}_2$	$\mathcal{I}_2$	$\mathcal{J}_1$	$\mathcal{K}_1$	Yes
H_7	$\mathcal{F}_1$	$\mathcal{G}_2$	$\mathcal{I}_2$	$\mathcal{J}_1$	$\mathcal{K}_1$	No
H_8	$\mathcal{F}_2$	$\mathcal{G}_1$	$\mathcal{I}_1$	$\mathcal{J}_2$	$\mathcal{K}_1$	Yes

Let $\mathcal{P} = \{H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8\}$

Case 1: Acceptance Criteria

Let $\mathcal{S} = \{H_1, H_3, H_6, H_8\}$ be the set of residential houses decided to buy by the individual

Let $\mathcal{R}$ represent the equivalence relation on $\mathcal{P}$ in relation to the set of all decision parameters.

The set of equivalence classes associated with $\mathcal{R}$ is provided as follows:

$\mathcal{P}/\mathcal{R} = \{\{H_2\}, \{H_3\}, \{H_4\}, \{H_8\}, \{H_1, H_5\}, \{H_6, H_7\}\}$.

Therefore, the nano topological space on $\mathcal{P}$ in relation to $\mathcal{S}$ is given by

$$\tau_{\mathcal{R}}(S) = \{\emptyset, \mathcal{P}, \{H_3, H_8\}, \{H_1, H_5, H_6, H_7\}, \{H_1, H_3, H_5, H_6, H_7, H_8\}\}$$

Step 1 If we remove the attribute $\mathcal{F}$, then

$\mathcal{P}/\mathcal{R} - (\mathcal{F}) = \{\{H_2\}, \{H_3\}, \{H_4\}, \{H_8\}, \{H_1, H_5\}, \{H_6, H_7\}\}$ and

$$\tau_{\mathcal{R}-(\mathcal{F})}(\mathcal{S}) = \{\emptyset, \mathcal{P}, \{H_3, H_8\}, \{H_1, H_5, H_6, H_7\}, \{H_1, H_3, H_5, H_6, H_7, H_8\}\}$$

Hence $\tau_{R-(\mathcal{F})}(\mathcal{S}\} = \tau_{\mathcal{R}}(\mathcal{S})$.

Step 2 If we remove the attribute $\mathcal{G}$, we get

$P/R - (\mathcal{G}) = \{\{H_3\}, \{H_4\}, \{H_8\}, \{H6, H7\}, \{H1, H2, H5\}\}$.

$$\tau_{\mathcal{R}-(\mathcal{G})}(\mathcal{S}) = \{\emptyset, \mathcal{P}, \{H_3, H_8\}, \{H_1, H_2, H_5, H6, H7\}, \{H1, H2, H3, H5, H6, H7, H8\}\} \neq \tau_{\mathcal{R}}(\mathcal{S})$$

Step 3 If we remove the attribute H, we have

$$\mathcal{P}/\mathcal{R} - (H) = \{\{H2\}, \{H3\}, \{H4\}, \{H8\}, \{H6, H7\}, \{H1, H5\}\}.$$

$$\tau_{\mathcal{R}-(H)}(\mathcal{S}) = \{\emptyset, P, \{H3, H8\}, \{H1, H5, H6, H7\}, \{H1, H3, H5, H6, H7, H8\}\} = \tau_{\mathcal{R}}(\mathcal{S})$$

Step 4 If we remove the attribute I, we have $\mathcal{P}/\mathcal{R} - (I) = \mathcal{P}/\mathcal{R}$ and hence $\tau_{R-(I)}(\mathcal{S}) = \tau_{\mathcal{R}}(\mathcal{S})$.

Step 5 If we remove the attribute J, we have $\mathcal{P}/\mathcal{R} - (J) = \mathcal{P}/\mathcal{R}$ and hence $\tau_{R-(J)}(\mathcal{S}) \neq \tau_{\mathcal{R}}(\mathcal{S})$.

From the above discussion, we have

CORE ($\mathcal{R}$) = {Water and Electric supply connections availability (F), Vasthu considerations (I)}

Case 2: Rejection Criteria

Let $\mathcal{S} = \{H2, H4, H5, H7\}$ be the set of residential houses rejected by the individual.

$$\tau_R(\mathcal{S}) = \{\emptyset, \mathcal{P}, \{H2, H4\}, \{H1, H5, H6, H7\}, \{H1, H2, H4, H5, H6, H7\}\}$$

Step 1 If we remove the attribute F, we get

$\mathcal{P}/\mathcal{R} - (F) = \{\{H2\}, \{H3\}, \{H4\}, \{H8\}, \{H1, H5\}, \{H6, H7\}\}$ and

$\tau_{\mathcal{R}-(F)}(\mathcal{S}) = \{\emptyset, \mathcal{P}, \{H2, H4\}, \{H1, H5, H6, H7\}, \{H1, H2, H4, H5, H6, H7\}\}$,

Hence $\tau_{\mathcal{R}-(F)}(\mathcal{S}\} = \tau_{\mathcal{R}}(\mathcal{S})$. .

Step 2 If attribute G is removed, we get

$$\mathcal{P}/\mathcal{R} - (G) = \{\{H3\}, \{H4\}, \{H8\}, \{H6, H7\}, \{H1, H2, H5\}\}$$

$\tau_{R-(G)}(S) = \{\emptyset, P, \{H4\}, \{H1, H2, H5, H6, H7\}, \{H1, H2, H4, H5, H6, H7\}\} \neq \tau_{\mathcal{R}}(\mathcal{S})$.

Step 3 If attribute H is removed, we have

$P/R - (H) = \{\{H2\}, \{H3\}, \{H4\}, \{H8\}, \{H1, H5\}, \{H6, H7\}\}$,

$\tau_{R-(H)}(S) = \{\emptyset, P, \{H2, H4\}, \{H1, H5, H6, H7\}, \{H1, H2, H4, H5, H6, H7\}\}$.

Hence $\tau_{R-(H)}(S) = \tau_{\mathcal{R}}(\mathcal{S})$..

Step 4 If we remove the attribute I, we have $\mathcal{P}/\mathcal{R} - (I) = \mathcal{P}/\mathcal{R}$and hence $\tau_{\mathcal{R}-(I)}(\mathcal{S}) = \tau_{\mathcal{R}}(\mathcal{S})$.

Step 5 If we remove the attribute J, we have $\mathcal{P}/\mathcal{R} - (J) = \mathcal{P}/\mathcal{R}$ and hence $\tau_{\mathcal{R}-(J)}(\mathcal{S}) \neq \tau_{\mathcal{R}}(\mathcal{S})$..

From the above discussion, we have CORE = {Water and Electric supply connections availability ($\mathcal{F}$), Vasthu considerations ($\mathcal{I}$)}

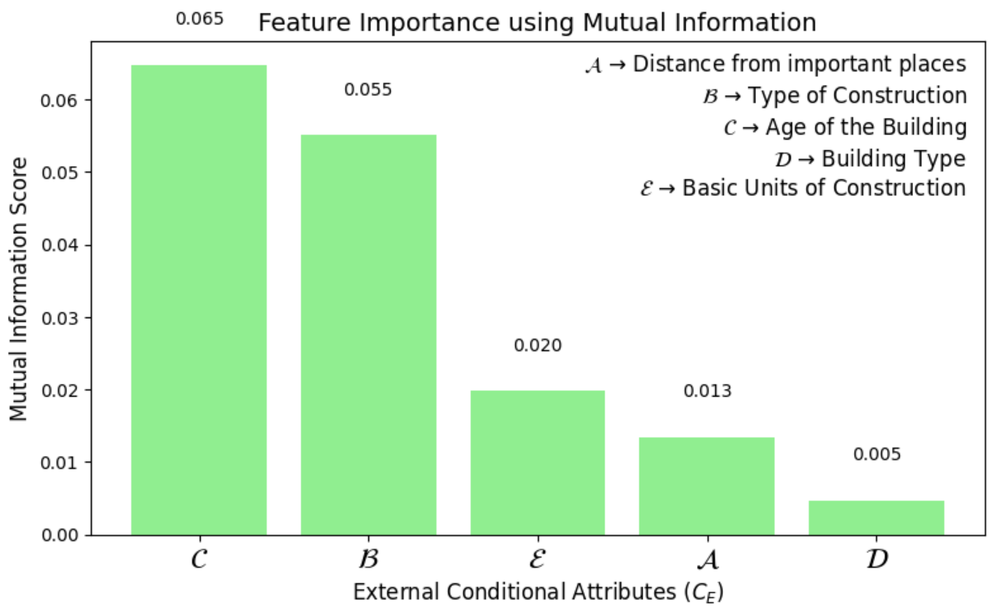
Observation: Discussions from the above two cases we observed that "Water and Electric supply connections availability" and "Vasthu considerations" are the main decision-making factors (internal) for an individual planning to purchase a residential property.

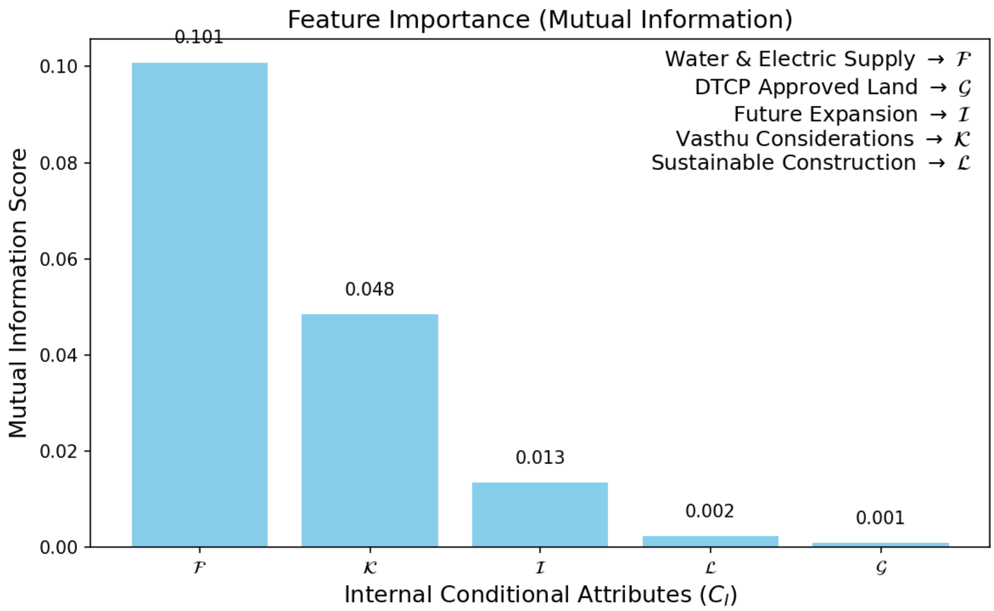
4. Empirical Validation using Mutual Information System

Nano topological concepts are applied to analyze factors influencing an individual's choice of purchasing a house. Primary data collected from a small sample of respondents are used to construct nano topological spaces and nano-open sets, which enables the identification of significant attributes through a rigorous theoretical framework. To establish the practical relevance and reliability of the nano topological results, an empirical justification is carried out using Mutual Information System (MIS) data involving a larger population that validate these findings against real-world large-scale data. The consistency between the nano topological results and the MIS-based empirical analysis demonstrates that nano topology can effectively identify key decision-making attributes even from limited data. MIS provides structured and comprehensive information collected from a larger population, allowing the observed nano topological patterns to be examined from a broader

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 13 Issue 1, January 2026
ISSN (Online) 2348 – 7968 | Impact Factor – 6.72
www.ijiset.com

perspective. The comparison between the theoretically derived key attributes and those obtained from MIS analysis helps in demonstrating the effectiveness of nano topology as a decision-support tool beyond purely mathematical settings.





CONCLUSION

In this study, nano topology has been successfully employed to identify the most influential factors affecting an individual's house-purchasing decision. Using a small primary dataset, nano topological structures were constructed to determine key attributes in a systematic and theoretical manner. The results obtained through this approach were further validated using MIS data collected from a significantly larger group, thereby establishing a strong connection between abstract mathematical analysis and practical data interpretation. The agreement between the nano topological findings and the MIS-based empirical results confirms that nano topology is capable of extracting meaningful insights even from limited information. This methodology can be extended to various other decision-making problems involving attribute selection, such as healthcare planning, consumer behavior analysis, and resource allocation. Hence, nano topology emerges as a powerful mathematical tool for bridging theoretical models and real-world applications.

REFERENCES

[1] N. Levine, Semi-open sets and semi-continuity in Topological Spaces, Amer. Math. Monthly, 70, 36-41, (1963).
[2] N. Levine, Generalized closed sets in topology, Rend. Cire. Math. Palermo, 19, 89-96, (1963).
[3] M. Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41, 374–481, (1937).
[4] J. Tong, Weak almost continuous mapping and weak nearly compact spaces, Boll. Un. Mat. Ital. 6, 385–391, (1982).
[5] Z. Pawlak, Rough Sets. International Journal of Information and Computer Science., 11(5): 341-356, (1982).
[6] Wei-Zhi Wu, Attribute reduction based on evidence theory in incomplete decision systems, Information Sciences 178, 1355 – 1371, (2008).
[7] Yuhua Qianab, Chuangyin Dangb, Jiye Lianga and Dawei Tance, Set-valued ordered information systems, Information Sciences, 179, 2809 – 2832, (2009).
[8] M. Lellis Thivagar and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1, 31-37, (2013).

Figure Captions:

- Fig. 3.1 shows the Decision-Making Algorithm used in the application of Nano Topology.
- Fig.3.2 shows the conditional attributes considered.

Table Captions:

- Table 3.1. shows external conditional attributes for eight houses.
- Table 3.2. shows internal conditional attributes for eight houses.