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An Optimal Control Approach to Operator Driven Reliability

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Abstract

The paper presents an optimal control approach to operator driven reliability (ODR) which has an impact on the overall operation of a facility. Experienced and/or properly trained operators are able to assess and manage the reliability aspects of assets and systems. On the other hand, inexperienced and improperly trained operators typically struggle to manage plant performance. Degradation of asset reliability, due to gaps in leadership support, technology utilization and employee competency, typically leads to significant, undesirable economic, safety and environmental consequences (e.g. in aviation sector).

Keywords: *Manufacturing equipment; Operator reliability; Training; Performance.*

1. Introduction

As part of their day-to-day activities, operators perform tasks that include process parameter inspections, minor adjustments and general observations of machine performance. Plant operators play a critical role within process manufacturing facilities. These team members oversee the equipment, assets and personnel necessary to run a successful chemical, petrochemical, refining facility or aviation manufacturing plants. Operators maintain and record readings and measurements of process control instrumentation and equipment to ensure an optimal level of performance and production, while also scheduling and coordinating maintenance efforts as necessary. Ultimately, an operator's goal is to improve the plant's product quality, efficiency and safety, while complying with applicable regulatory requirements [1]. In some facilities, the potential impact that the operations department has on the health of plant equipment is minimized, due to a lack of understanding of the value of operator's duties. For instance, in some cases, operators are simply appointed as "valve turners" or "meter readers." Facilities that do not enforce stringent training programs requiring operators to thoroughly understand aspects of the equipment to which they are assigned, the chemistry behind what they are doing and how external forces can impact the facility's processes, are far more likely to fail at optimizing and maximizing efficiency. As a result, overall performance of the plant will be negatively impacted. At these facilities, the operators are not encouraged or required to understand the complex processes that they are assigned in order to maintain and control the equipment or assets [2]. This practice results in process inefficiency, downtime and a higher risk for safety issues. Operations departments must ensure that their personnel are trained to quickly troubleshoot and correct problems before they get out of hand. For example, if an issue requires the operator to "call out" someone else to initiate corrective tasks, the chances of a quick resolution are slim if the operator is not trained or otherwise guided to be able to quickly identify these situations and take effective and timely action [1]. In addition to process training, it is also vital that facilities provide mechanical and fundamental instrument & electrical (I&E) training relevant to the operator's assigned area. This training enables operators to fully understand the inner workings of their equipment. With this knowledge, they will better recognize changes in sound, temperature, vibration, output and other variables, which can facilitate the early detection of degradation and pending failure and initiate proactive intervention. Operators can also better control or eliminate external, often random failure causes (e.g. oil condition, operating envelopes, etc.) which can significantly increase overall equipment availability and economic life. Recognizing the impact of proper operator driven reliability (ODR) on the overall operation of a facility cannot be understated. Experienced and/or properly trained operators are able to assess and manage the reliability aspects of assets and systems. On the other hand, inexperienced and improperly trained operators typically struggle to manage plant performance. Degradation of asset reliability, due to gaps in leadership support, technology utilization and employee competency, typically leads to significant, undesirable economic, safety and environmental consequences [9].

The operator's reliability (human's reliability) is a very important part of System's reliability. It considers the human faults in the whole process. The main goal of the study is to investigate an optimal control approach that can help to define an optimal fault rate reduction series during the training duration of the operator under given conditions which maximizes the objective function.

2. Theoretical Background

Suppose the operator's reliability function after a training for a time duration ***t*** is defined in the following expression [2]:

$$R_0(t,\tau) = e^{-\int_0^t \lambda(z,\varepsilon)dz . e^{-\int_0^\tau \upsilon(l,\xi)dl.}}$$

The term under the first exponent is associated with the operator's reliability:

$$R(t,\varepsilon;\tau,\xi) = \int_0^t \lambda(z,\varepsilon)dz . e^{-\int_0^\tau \upsilon(l,\xi)dl.}$$

in which the term $\int_0^t \lambda(z,\varepsilon)dz = r(t,\varepsilon)$ is the consumed reliability for a time t under conditions ε [2]. The term $\gamma(\tau,\xi) = \int_0^\tau \upsilon(l,\xi)dl$ is called restored reliability which is obtained in the training process for a time τ under conditions ξ. The operator's reliability can be then expressed in the following way:

$$R_0(t,\tau) = e^{-r.e^{-\gamma}} = R_0(r,\gamma)$$

This is leading to the conclusion that the operator's reliability $R_0(r,\gamma)$ is decreasing when the consumed reliability r is higher and is increasing when the restored reliability γ is higher. Let's consider in our case study the following: the restored reliability term $\gamma(\tau,\xi) = \int_0^\tau \upsilon(l,\xi)dl$ to be modelled as utility function which needs to be maximized. An optimal control theory will further be applied on that purpose.

Suppose an optimal control problem in discrete time with the following periods: *0,1,2…T*, and also assume the following [3-6], [10]:

$$\Theta = \{0,1,2,...,T-1\}$$

x_t - *n*- component vector-column of state variable; $t = 0,1,2,...,T$ (1)

u_t - *m*- component vector-column of control variable; $t = 0,1,2,...,T-1$

b_t - *s*- component vector-column of constants; $t = 0,1,2,...,T-1$

In Eq.(1) it is assumed that the state variable x_t is measured at the beginning of each period t and the control u_t is applied during this period t. This notation is shown on Fig.1:

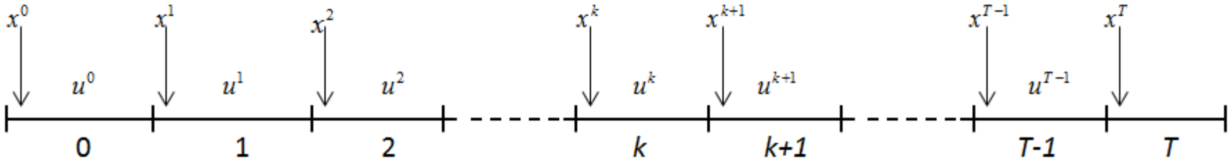


Fig.1. Overview of state variable x_t and control variable u_t

Let's also define continuously differentiable functions:

$$f : E^n \times E^m \times \Theta \to E^n,\ F : E^n \times E^m \times \Theta \to E^1,\ g : E^m \times \Theta \to E^s,\ S : E^m \times \Theta \cup \{T\} \to E^1$$

Hence, the optimal control problem in discrete form can be defined in the following way:

$$\max\left\{ J = \sum_{t=0}^{T} F(x_t, u_t, t) + S(x_T, T) \right\} \tag{2}$$

subject to constraints:

$$\Delta x_t = x_{t+1} - x_t = f(x_t, u_t, t),\ t = 0,1,...,T-1$$

$$x_0 \text{ - given} \tag{3}$$

$$g(u_t, t) \ge b_t,\ t = 0,1,...,T-1$$

In our case we consider the problem in a discrete time. For many economics tasks it is necessary to be considered in discrete time since sometimes it is impossible to have records in continuous time [7-10]. In case of annual/monthly/weekly/daily information, the time series of fault rate management can be expressed as:

$$\vec{u} = \{\ u_0, u_1, u_2, ..., u_T\ \} \tag{4}$$

The resource value is the sum of discounted cash flow obtained during the lifetime of the considered subject- from 0 to T. Suppose in our case T=10 days (i.e. the considered subject (training period) is 10 days). Therefore, the resource value is [11]:

$$V(x_0, \vec{u}) = \sum_{t=0}^{T} \beta^t C(u_t) \tag{5}$$

where: $C(u_t)$ - cash flow for ***t**-th* day;

$\beta = 1/(1+r)$ - discounting factor

r - interest rate of the resource.

The fundamental concept from the interest rate theory is the net present value Eq.(5) of the cash flow over time. It should be noted that the arbitrage absence supposes that the value of obligation (agreement, contract) should be the net present value of the cash flow.

Consider a cash flow, i.e. the series of periodic payments $C(u_t)$, discrete in time $t = 0,1,2,...,T$. Let the interest rate r is given in discrete complexity and it is applied on the payment periods. Then the net present value is defined by the following expression [11]:

$$V(x_0,\vec{u}) = \sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t} \tag{6}$$

Another important point related to the cash flow analysis is the return rate. Suppose the following values $C(u_t) > 0$, $t = 0,1\ldots T$, then the return rate $\bar{r}$ can be found by solving of the following non-linear equation:

$$\sum_{t=0}^{T} \frac{C(u_t)}{(1+\bar{r})^t} = 0 \tag{7}$$

And performing a substitution:

$$h = \frac{1}{(1+\bar{r})} \tag{8}$$

Then we get an equation Eq.(7) in the following modified form:

$$\sum_{t=0}^{T} C(u_t)h^t = 0 \tag{9}$$

and this equation Eq.(9) has unique positive root.

Therefore, it is very important to obtain a precise evaluation of the unique positive root of given algebraic equation. In order to evaluate the unique positive root of Eq.(9), then the equation Eq.(6) has to be expressed in the following way:

$$V(x_0,\vec{u}) - \sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t} = 0 \tag{10}$$

and it possess unique positive root, where:

$V(x_0,\vec{u})$ - the present value of the project;

$\sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t}$ - the sum of discounted cash flows over time.

From financial modelling point of view, the cash flow can be expressed as a “power function”, i.e. $C(u_t) = u_t^{\alpha}$, which has diminishing returns since $0 < \alpha < 1$. Substituting $C(u_t) = u_t^{\alpha}$ in Eq.(5), then we get the following objective function [11]:

$$V(x_0,\vec{u}) = \sum_{t=0}^{T} \beta^t u_t^{\alpha} \tag{11}$$

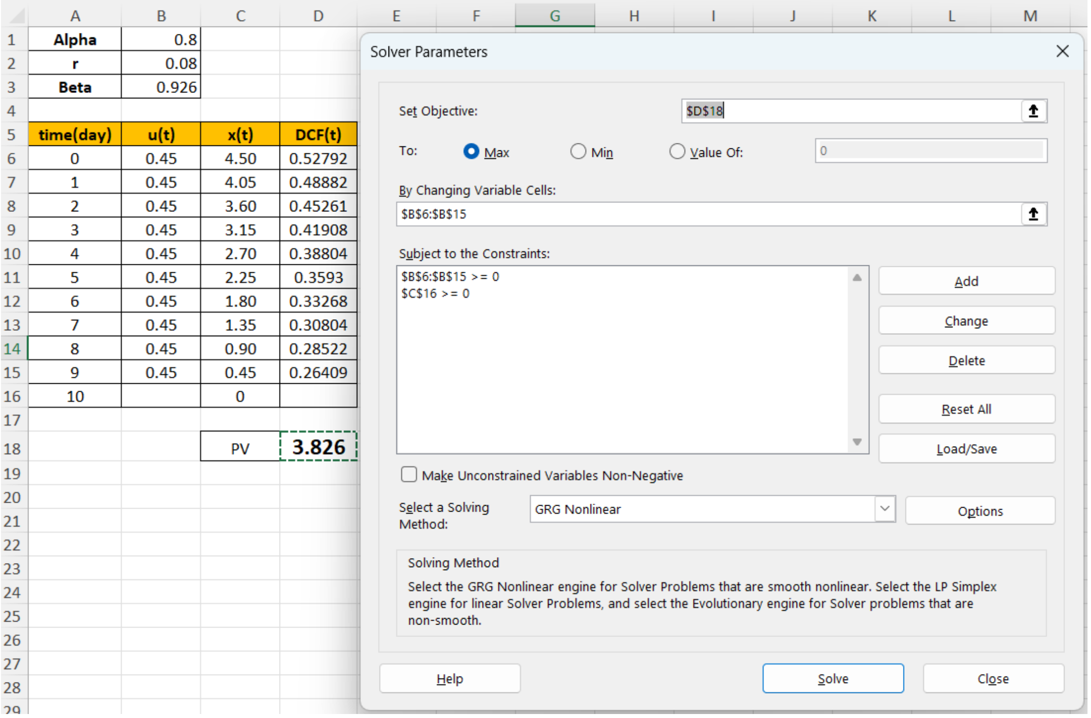
The trainer can choose the time series of fault rate management Eq.(4) which maximizes the present value functional: $V(x_0,\vec{u})$ with the following constraint [11]:

$$x_{t+1} - x_t = -u_t \tag{12}$$

The term u_t is a control variable in our study. It should be noted that equation Eq.(12) says that the decision-maker controls the reduction in the fault rate "reserve" from one day to the next. As well as, a numerical solution to a dynamic optimization problem requires two endpoint conditions. Suppose the initial(not trained) operator's fault rate is 4.5 [1/day] and the trainer/instructor decides to perform the training for time period of 10 days. Then the initial condition is $x_0 = 4.5$ and final one is $x_T = x_{10} \ge 0$ (the final condition is associated with the constraint $g(u_t,t) \ge b_t$ from Eq.(3)). It is optimal to reduce the whole fault rate after training period of time *T*. In our study it also assumed that the fault rate is non-negative value during the whole training period: $\vec{u} = \{u_T / u_T \ge 0 \text{ for } t = 0,1,...,T\}$.

3. Applications

An application of the proposed modeling for Eq.(5)-Eq.(12) is given in this chapter. Solving for the task is performed by using the Excel® software. Table 1 shows the picture of dynamic optimization problem of the fault rate reduction:



	A	B	C	D
1	**Alpha**	0.8		
2	**r**	0.08		
3	**Beta**	0.926		
4				
5	**time(day)**	**u(t)**	**x(t)**	**DCF(t)**
6	0	0.45	4.50	0.52792
7	1	0.45	4.05	0.48882
8	2	0.45	3.60	0.45261
9	3	0.45	3.15	0.41908
10	4	0.45	2.70	0.38804
11	5	0.45	2.25	0.3593
12	6	0.45	1.80	0.33268
13	7	0.45	1.35	0.30804
14	8	0.45	0.90	0.28522
15	9	0.45	0.45	0.26409
16	10		0	
17				
18			PV	**3.826**

Solver Parameters

Set Objective: `$D$18`

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells: `$B$6:$B$15`

Subject to the Constraints:
`$B$6:$B$15 >= 0`
`$C$16 >= 0`

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

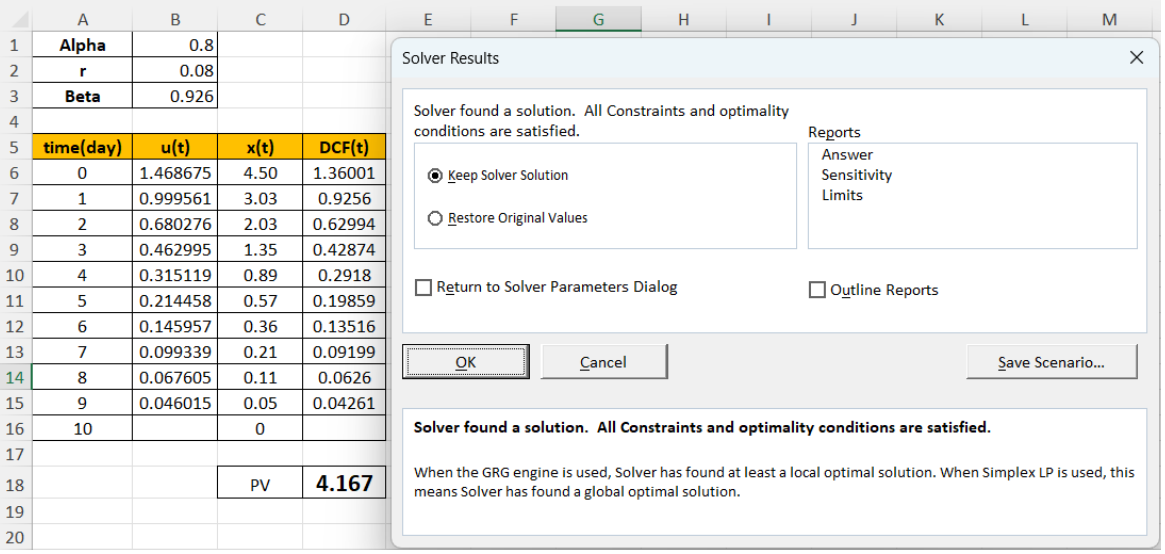
Table 1: Optimal Control Task in spreadsheet format

The parameter α which shows the curve of the daily function of cash flow, is assumed to be 0.8 and the interest rate *r* is 8 %. The formula in cell B3 calculates the discounting factor $\beta = 1/(1+r)$. The column C implements equation Eq.(12) which represents the time series of the fault rate. At the beginning of each day, the fault rate is equal to the rate of the beginning of previous day minus the fault rate in previous day, i.e. $x_{t+1} = x_t - u_t$.

Suppose the initial (before training) operator's fault rate is $x_0 = 4.5$ which is shown in cell C6. The function "Solver" in Excel® is using an iterative algorithm which requires an assignment of initial values of the optimal time series of fault rate. And suppose that the operator's fault rate is managed uniformly each day, for example, with 0.45 [1/day] (cells B6:B15, see Table 1). In the last column D- the DCF(t) means discounted cash flow earned during each day ***t***, i.e. $\beta^t u_t^\alpha$. The cell D18 contains the net present value $V(x_0,\vec{u})$.

The net present value $V(x_0,\vec{u})$ depends on the initial fault rate x_0 and on the fault control $\vec{u}$, i.e. $V(x_0,\vec{u})$ =3.8 if x_0 =4.5 and fault rate of 0.45 [1/day]. However, in that case the uniform fault rate series is not optimal when the future cash flows are discounted. Applying the function "Solver" in that case, we can find the optimal fault rate series $\vec{u}$ which maximizes $V(x_0,\vec{u})$. The constraints in our case are implemented in Solver dialog box: „B6:B15>=0" и „C16>=0".

The results from the optimization are shown in Table 2. Then the optimal fault rate reduction series is the following: 1.47 [1/day] in day *t=0*; 1 [1/day] in day *t=1*; 0.68 [1/day] in day *t=2*; etc. The objective function (cell D18) increases from 3.8 up to 4.2 if the optimal fault rate series is applied. It is interesting to point out that the optimal fault rate series is with a slope that is falling down due to the fact that the discounting factor stimulates to reduce the fault rate faster. And it can be seen that the optimal solution satisfies the constraints- to be non-negative the fault rate and the process stops when the fault rate is exhausted/minimized.



	A	B	C	D
1	**Alpha**	0.8		
2	**r**	0.08		
3	**Beta**	0.926		
4				
5	**time(day)**	**u(t)**	**x(t)**	**DCF(t)**
6	0	1.468675	4.50	1.36001
7	1	0.999561	3.03	0.9256
8	2	0.680276	2.03	0.62994
9	3	0.462995	1.35	0.42874
10	4	0.315119	0.89	0.2918
11	5	0.214458	0.57	0.19859
12	6	0.145957	0.36	0.13516
13	7	0.099339	0.21	0.09199
14	8	0.067605	0.11	0.0626
15	9	0.046015	0.05	0.04261
16	10		0	
17				
18			PV	**4.167**
19				
20				

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution

○ Restore Original Values

☐ Return to Solver Parameters Dialog

Reports
Answer
Sensitivity
Limits

☐ Outline Reports

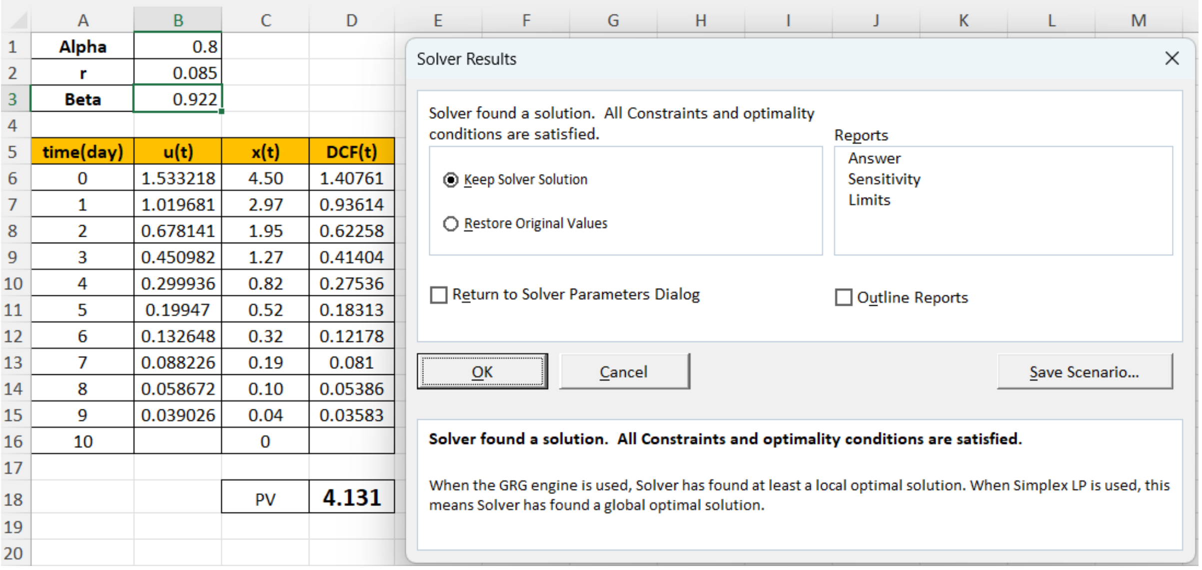
Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Table 2: Optimal Control Task (after running optimization)

It is also practically interesting to analyze the sensitivity, i.e. to show the impact of the interest rate over the optimal solution. Returning to Table 1 with changing the interest rate (cell B2), and again to start the optimization algorithm. Table 3 shows the new optimal solution with respect to the modified (increased) interest rate.

The increasing of the interest rate stimulates faster fault rate reduction: 1.53 [1/day] for day *t=0*; 1.02 [1/day] in day *t=1*; then smaller reduction according to the newer interest rate (8.5%). The new optimal fault rate series is with higher slope since the future cash flows are discounted with higher value. The net present value increases up to 4.13 with the new optimal series. The high discounting value explains why now the DCF value is 4.13, even with optimal values.



Alpha	0.8
r	0.085
Beta	0.922

time(day)	u(t)	x(t)	DCF(t)
0	1.533218	4.50	1.40761
1	1.019681	2.97	0.93614
2	0.678141	1.95	0.62258
3	0.450982	1.27	0.41404
4	0.299936	0.82	0.27536
5	0.19947	0.52	0.18313
6	0.132648	0.32	0.12178
7	0.088226	0.19	0.081
8	0.058672	0.10	0.05386
9	0.039026	0.04	0.03583
10		0	
		PV	**4.131**

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution
○ Restore Original Values

Reports
Answer
Sensitivity
Limits

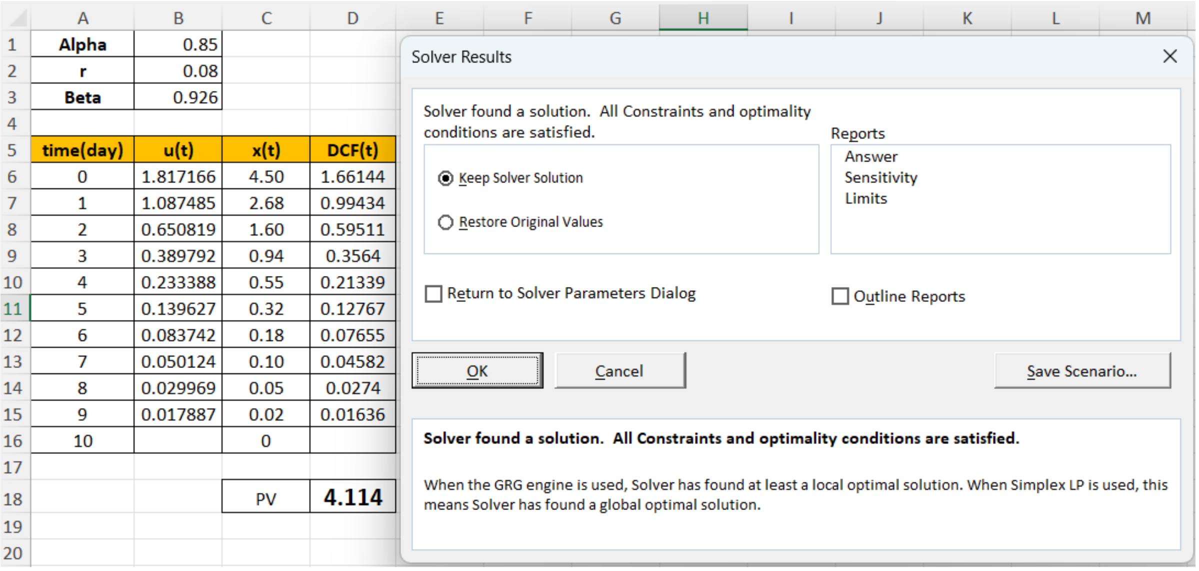
☐ Return to Solver Parameters Dialog
☐ Outline Reports

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Table 3: Optimal Control Task with change in interest rate

Two factors play an important role over the optimal fault rate series. The discounting factor induces that the fault rate has to be reduced faster, but this effect is counterbalanced by the diminishing returns of the daily cash flow.



Alpha	0.85
r	0.08
Beta	0.926

time(day)	u(t)	x(t)	DCF(t)
0	1.817166	4.50	1.66144
1	1.087485	2.68	0.99434
2	0.650819	1.60	0.59511
3	0.389792	0.94	0.3564
4	0.233388	0.55	0.21339
5	0.139627	0.32	0.12767
6	0.083742	0.18	0.07655
7	0.050124	0.10	0.04582
8	0.029969	0.05	0.0274
9	0.017887	0.02	0.01636
10		0	
		PV	**4.114**

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution
○ Restore Original Values

Reports
Answer
Sensitivity
Limits

☐ Return to Solver Parameters Dialog
☐ Outline Reports

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Table 4: Optimal Control Task with change in alpha parameter

Table 4 shows an optimal fault rate series for the case when the daily cash flow function is with lower value of diminishing returns. To represent this situation, the parameter α is changed from 0.8 to 0.85 and the optimization algorithm is started again. The new optimal fault rate series is steeper: 1.82 [1/day] for day t=0; 1.09 [1/day] for day t=1; 0.65 [1/day] for day t=2; etc. At about 85% from the whole fault rate will be reduced during the first 4 days of training. The objective function (cell D18) in this case increasing up to ~4.1 with the new optimal solution.

Consider the results summarized in Tables 1 and 2: uniform series of fault rate reduction vs optimal fault rate reduction series, the fault rate vs training day is plotted in Fig. 2:

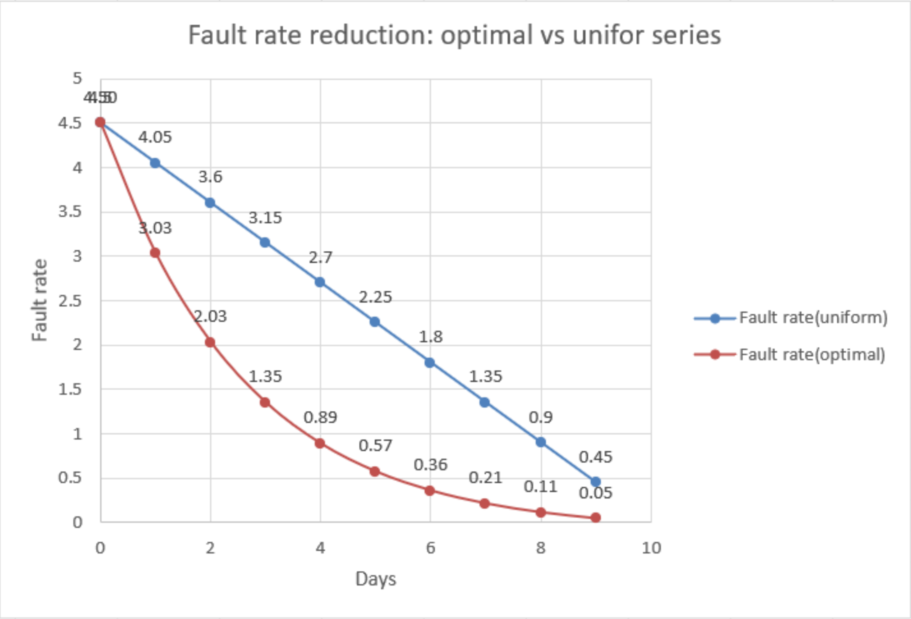


Fig. 2. Fault rate reduction vs training day

4. Conclusions

The following major outcomes have been obtained by the performed analysis:

- The increasing in the operator's restored reliability shows steeper slope for the case of optimal fault rate reduction series compared to the uniform fault rate reduction series (Fig.2). The considered application example shows that it is worth applying the optimal fault rate reduction series rather than the uniform fault rate reduction series.
- The influence of the interest rate over the optimal solution has been analyzed via performing a sensitivity study. Faster fault rate reduction can be accelerated by increasing the interest rate (see Table 3): approximately 85% from the (initial) fault rate is expected to be reduced during the first 4 days of the operator's training.

The proposed study suggests an overlapping between today's very important and modern subjects like financial modeling, applied optimization and human reliability.

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