IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 2, February 2015.
www.ijiset.com
ISSN 2348 – 7968

Weak Continuity Via Bioperation-Semiopen Sets

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ABSTRACT. *In this paper, we introduce and study the weak form of* $[\gamma,\gamma']$-$[\beta,\beta']$*-semicontinuous functions called weakly* $[\gamma,\gamma']$-$[\beta,\beta']$-*semicontinuous functions between bioperation-topological spaces.*

1. INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, seperation axioms etc. by utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces. Maki and Noiri [3] introduced the notion of $\tau_{[\gamma,\gamma']}$, which is the collection of all $[\gamma,\gamma']$-open sets in a topological space (X,τ). In this paper, we introduce and study the weak form of $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous functions called weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous functions between bioperation-topological spaces.

2 PRELIMINARIES

The closure and the interior of a subset A of (X,τ) are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.1 [1] Let (X,τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $\mathcal{P}(X)$ of X such that $V \subset V^{\gamma}$ for each $V \in \tau$, where V^{γ} denotes the value of τ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.

Definition 2.2 [3] A topological space (X,τ) equipped with two operations namely γ and γ' defined on τ is called a bioperation-topological space and it is denoted by (X,τ,γ,γ').

Definition 2.3 A subset A of a topological space (X,τ) is said to be $[\gamma,\gamma']$-open set [3] if for each $x \in A$ there exist open neighbourhoods U and V of x such that $U^{\gamma} \cap V^{\gamma'} \subset A$. The complement of a $[\gamma,\gamma']$-open set is called a $[\gamma,\gamma']$-closed set. Also $\tau_{[\gamma,\gamma']}$ denotes set of all $[\gamma,\gamma']$-open sets in (X,τ).

2000 Mathematics Subject Classification 54D05, 54C08, 54B05.
Keywords and phrases. $[\gamma,\gamma']$- *semiopen set, weakly* $[\gamma,\gamma']$-*semicontinuous functions.*
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Definition 2.4. [3] For a subset A of (X,τ), $\tau_{[\gamma,\gamma']}$-$Cl(A)$ denotes the intersection of all $[\gamma,\gamma']$-closed sets containing A, that is, $\tau_{[\gamma,\gamma']}\text{-}Cl(A)=\bigcap\{F : A\subset F, X\setminus F\in\tau_{[\gamma,\gamma']}\}$.

Definition 2.5. Let A be any subset of X. The $\tau_{[\gamma,\gamma']}$-$Int(A)$ is defined as $\tau_{[\gamma,\gamma']}$-$Int(A)=\cup\{U : U$ is a $[\gamma,\gamma']$-open set and $U\subset A\}$.

Definition 2.6. A subset A of a topological space (X,τ) is said to be $[\gamma,\gamma']$-semiopen [2] if $A\subset\tau_{[\gamma,\gamma']}\text{-}Cl(\tau_{[\gamma,\gamma']}\text{-}Int(A))$.

Theorem 2.7. A subset A of a bioperation-topological space (X,τ,γ,γ') is $[\gamma,\gamma']$-semi open if,and only if for each $x\in X$ there exists a $[\gamma,\gamma']$-semiopen set U suchthat $x\in U\subset A$.

Definition 2.8. [2] A function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is said to be $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous if the inverse image of every $[\beta,\beta']$-open set in (Y,σ,β,β') is a $[\gamma,\gamma']$-semiopen set in (X,τ,γ,γ').

3 WEAK BIOPERATION-SEMICONTINUOUS FUNCTIONS

In this section, we define weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous function in bioperation-topological space and study some of their properties and theorems on them.

Definition 3.1. A function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is said to be weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous if for each $x\in X$ and each $[\beta,\beta']$-open set V of Y containing $f(x)$, there exists a $[\gamma,\gamma']$-semiopen set U containing x such that $f(U)\subset[\beta,\beta']$-$Cl(V)$.

Proposition 3.2. Every almost $[\gamma,\gamma']$-$[\beta,\beta']$-semi continuous function is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

Proof. Let U be any $[\beta,\beta']$-regular open subset in Y. Then, since f is almost $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous, $f^{-1}(U)$ is a $[\gamma,\gamma']$-semiopen set in X. From the fact $U\subset[\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(U))$, it follows $f^{-1}(U)=[\gamma,\gamma']\text{-}sInt(f^{-1}(U))\subset[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(U))))\subset[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']Cl(U)))$. Thus, f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

The following example shows that the converse of Proposition 3.2 is not true, in general.

Example 3.3 Let $X=\{a,b,c\}$, $\tau=\{\varnothing,\{b\},\{c\},\{a,b\},\{b,c\},X\}$ and take the $\sigma=\{\varnothing,\{a\},\{b\},\{a,b\},\{b,c\},X\}$. Let $\gamma,\gamma':\tau\to P(X)$ and $\beta,\beta':\sigma\to P(X)$ be operations defined as follows:

$$A^{\gamma}=\begin{cases}A & \text{if } A=\{b\},\\ A\cup\{c\} & \text{if } A\neq\{b\},\end{cases} \quad A^{\gamma'}=\begin{cases}A & \text{if } A=\{c\},\\ A\cup\{b\} & \text{if } A\neq\{c\},\end{cases}$$

$$A^{\beta}=\begin{cases}A & \text{if } A=\{a\},\\ A\cup\{b\} & \text{if } A\neq\{a\},\end{cases} \text{and } A^{\beta'}=\begin{cases}A & \text{if } A=\{b\},\\ A\cup\{a\} & \text{if } A\neq\{b\},\end{cases}$$

Then the identity function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is weak $[\gamma,\gamma']$-$[\beta,\beta']$-semi continuous but not almost $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

Corollary 3.4. Every $[\gamma,\gamma']$-$[\beta,\beta']$-semi continuous function is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

Theorem 3.5. For a function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$, the following properties are equivalent:

(1). f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous;

(2). $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))))\subset f^{-1}([\beta,\beta']\text{-}Cl(B))$ for every subset B of Y;

(3). $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(F)))\subset f^{-1}(F)$ for every $[\beta,\beta']$-regular closed set F of Y;

(4). $[\gamma,\gamma']\text{-}sCl(f^{-1}(V))\subset f^{-1}([\beta,\beta']\text{-}Cl(V))$ for every $[\beta,\beta']$-open set V of Y;

(5). $f^{-1}(V)\subset[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$ for every $[\beta,\beta']$-open set V of Y.

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Suppose that $x\in X\setminus f^{-1}([\beta,\beta']\text{-}Cl(B))$. Then $f(x)\in Y\setminus[\beta,\beta']\text{-}Cl(B)$ and there exists a $[\beta,\beta']$-open set V of Y containing $f(x)$ such that $V\cap B=\varnothing$. Hence, $V\cap[\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))=\varnothing$ and then $[\beta,\beta']\text{-}Cl(V)\cap[\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))=\varnothing$. By assumption, there exists a $[\gamma,\gamma']$-semiopen set U containing x such that $f(U)\subset[\beta,\beta']\text{-}Cl(V)$. Hence, we have $U\cap f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B)))=\varnothing$ and $x\in X\setminus[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))))$. Thus, we obtain $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))))\subset f^{-1}([\beta,\beta']\text{-}Cl(B))$.

(2) $\Rightarrow$ (3): Let F be a $[\beta,\beta']$-regular closed subset of Y. Then $F=[\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int(F))$ and we have $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(F)))=[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int(F)))))\subset f^{-1}([\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int(F)))=f^{-1}(F)$.

(3) $\Rightarrow$ (4): Let V be a $[\beta,\beta']$-open subset of Y. Then $[\beta,\beta']\text{-}Cl(V)$ is $[\beta,\beta']$-regular closed. Then we obtain $[\gamma,\gamma']\text{-}sCl(f^{-1}(V)) \subset [\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))) \subset f^{-1}([\beta,\beta']\text{-}Cl(V))$.

(4) $\Rightarrow$ (5): Let V be a $[\beta,\beta']$-open subset of Y. Then $Y\setminus[\beta,\beta']\text{-}Cl(V)$ is $[\beta,\beta']$-open and we have $[\gamma,\gamma']\text{-}sCl(f^{-1}(Y\setminus[\beta,\beta']\text{-}Cl(V))) \subset f^{-1}([\beta,\beta']\text{-}Cl(Y\setminus[\beta,\beta']\text{-}Cl(V)))$ and hence $X\setminus[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V))) \subset X\setminus f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V))) \subset X\setminus f^{-1}(V)$.There fore, we obtain $f^{-1}(V) \subset [\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$.

(5) $\Rightarrow$ (1): Let $x \in X$ and V be a $[\beta,\beta']$-open set containing $f(x)$. We have $x \in f^{-1}(V) \subset [\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$. Put $U = [\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$. Then U is a $[\gamma,\gamma']$-semiopen set containing x and $f(U) \subset [\beta,\beta']\text{-}Cl(V)$. This shows that f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

Theorem 3.6. For a function $f:(X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$, the following properties are equivalent:

1) f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.
2) $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(K))) \subset f^{-1}(K)$ for every $[\beta,\beta']$-regular closed subset K of Y.
3) $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))) \subset f^{-1}([\beta,\beta']\text{-}Cl(V))$ for every $[\beta,\beta']$-semi-preopen subset V of Y.
4) $[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(U)))) \subset f^{-1}([\beta,\beta']\text{-}Cl(U))$ for every $[\beta,\beta']$-semiopen subset U of Y.

Proof. (1)$\Rightarrow$(2): Let K be any $[\beta,\beta']$-regular clsoed set of Y. Then since $[\beta,\beta']\text{-}Int(K)$ is a $[\beta,\beta']$-open set in Y, from Theorem 3.5 (4) and $K = [\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int(K))$, we have $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(K))) \subset f^{-1}([\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int(K))) = f^{-1}(K)$. Thus $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(K))) \subset f^{-1}(K)$.

(2)$\Rightarrow$(3): Let V be any $[\beta,\beta']$-semiopen set. Then $[\beta,\beta']\text{-}Cl(V) = [\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))$, so we know that $[\beta,\beta']\text{-}Cl(V)$ is $[\beta,\beta']$-regular closed. By (2), $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))) \subset f^{-1}([\beta,\beta']\text{-}Cl(V))$.

(3)$\Rightarrow$(4): Since every $[\beta,\beta']$-semiopen set is $[\beta,\beta']$-semi-preopen, it is obvious.

(4)$\Rightarrow$(1): Let V be any $[\beta,\beta']$-open subset of Y. Then V is also a $[\beta,\beta']$-semiopen set and so from (4), $[\gamma,\gamma']\text{-}sCl(f^{-1}(V)) \subset [\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))) \subset f^{-1}([\beta,\beta']\text{-}Cl(V))$. Hence by Theorem 3.5 (4), f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

Theorem 3.7. For a function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$, the following properties are equivalent:

1) f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous;
2) $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(G))))\subset f^{-1}([\beta,\beta']\text{-}Cl(G))$ for every $[\beta,\beta']$-preopen set G of Y;
3) $[\gamma,\gamma']\text{-}sCl(f^{-1}(V))\subset f^{-1}([\beta,\beta']\text{-}Cl(V))$ for every $[\beta,\beta']$-preopen set V of Y;
4) $f^{-1}(V)\subset[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$ for every $[\beta,\beta']$-preopen set V of Y;
5) $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int(G)))\subset f^{-1}(G)$ for every $[\beta,\beta']$-preclosed set G of Y.

Proof. (1) $\Rightarrow$ (2): Let G be any $[\beta,\beta']$-preopen subset of Y. Then $[\beta,\beta']\text{-}Cl(G)=[\beta,\beta']\text{-}Cl([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(G)))$, so $[\beta,\beta']\text{-}Cl(G)$ is $[\beta,\beta']$-regular closed. From Theorem 3.6 (2), we have $[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(G))))\subset f^{-1}([\beta,\beta']\text{-}Cl(G))$.

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (4): Let V be any $[\beta,\beta']$-preopen subset of Y. By (3), $[\beta,\beta']$-preopenness of $Y\setminus[\beta,\beta']\text{-}Cl(V)$ we have $f^{-1}(V)\subset f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(V)))=X\setminus f^{-1}([\beta,\beta']\text{-}Cl(Y\setminus[\beta,\beta']\text{-}Cl(V)))\subset X\setminus[\gamma,\gamma']\text{-}sCl(f^{-1}(Y\setminus[\beta,\beta']\text{-}Cl(V)))=[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$.

(4) $\Rightarrow$ (1): Let V be any $[\beta,\beta']$-open set of Y. Then V is $[\beta,\beta']$-preopen set in Y and by (4) $f^{-1}(V)\subset[\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$. By Theorem 3.5 (5), f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

(4) $\Leftrightarrow$ (5): The proof is obvious.

Theorem 3.8. The set of all points x of X at which a function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is not weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous is identical with the union of all of the $[\gamma,\gamma']$-$[\beta,\beta']$-semifrontiers of the inverse images of the $[\beta,\beta']$-closure of $[\beta,\beta']$-open sets of Y containing $f(x)$.

Proof. Let x be a point of X at which $f(x)$ is not weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous. Then, there exists a $[\beta,\beta']$-open set V of Y containing $f(x)$ such that $U\cap(X\setminus f^{-1}([\beta,\beta']\text{-}Cl(V)))\neq\varnothing$ for every $[\gamma,\gamma']$-semiopen set U of X containing x. Then $x\in[\gamma,\gamma']\text{-}sCl(X\setminus f^{-1}([\beta,\beta']\text{-}Cl(V)))$. Simultaneously, since $x\in f^{-1}([\beta,\beta']\text{-}Cl(V))$, we have $x\in[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Cl(V)))$ and hence

$x \in [\gamma,\gamma']\text{-}sFr(f^{-1}([\beta,\beta']\text{-}Cl(V)))$. Conversely, if f is weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous at x, then for each $[\beta,\beta']$-open set V of Y containing $f(x)$, there exists a $[\gamma,\gamma']$-semiopen set U containing x such that $f(U) \subset [\beta,\beta']\text{-}Cl(V)$ and hence $x \in U \subset f^{-1}([\beta,\beta']\text{-}Cl(V))$. Therefore, we obtain that $x \in [\gamma,\gamma']\text{-}sInt(f^{-1}([\beta,\beta']\text{-}Cl(V)))$. This contradicts that $x \in [\gamma,\gamma']\text{-}sFr(f^{-1}([\beta,\beta']\text{-}Cl(V))$.

Now we characterize the set of points at which a function $f:(X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$ is not weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous.

For a function $f:(X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$, we define $D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)$ as follows:
$D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f) = \{x \in X \ : \ f$ is not weakly $[\gamma,\gamma']$-$[\beta,\beta']$-semicontinuous at $x\}$.

Theorem 3.9 For a function $f:(X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$, the following properties hold:

$$
\begin{aligned}
&D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)\\
&= \bigcup_{B \in \mathrm{P}(Y)} \{[\gamma,\gamma'] - sCl(f^{-1}([\beta,\beta'] - Int([\beta,\beta'] - Cl(B)))) \setminus f^{-1}([\beta,\beta'] - Cl(B))\}\\
&= \bigcup_{F \in [\beta,\beta'] - RC(Y,\sigma)} \{[\gamma,\gamma'] - sCl(f^{-1}([\beta,\beta'] - Int(F))) \setminus f^{-1}(F)\}\\
&= \bigcup_{V \in \sigma_{[\beta,\beta']}} \{[\gamma,\gamma'] - sCl(f^{-1}(V)) \setminus f^{-1}([\beta,\beta'] - Cl(V))\}\\
&= \bigcup_{V \in \sigma_{[\beta,\beta']}} \{f^{-1}(V) \setminus [\gamma,\gamma'] - sInt(f^{-1}([\beta,\beta'] - Cl(V)))\}.
\end{aligned}
$$

Proof. We shall show only the equality since the proof of the other are similar to the first. Let $x \in D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)$. By Theorem 3.5, for every subset B of Y such that $x \in [\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B))))$ and $x \notin f^{-1}([\beta,\beta']\text{-}Cl(B))$. Therefore, we have $x \in [\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B)))) \setminus f^{-1}([\beta,\beta']\text{-}Cl(B)) \subset \bigcup_{B \in \mathrm{P}(Y)} \{[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B)))) \setminus f^{-1}([\beta,\beta']\text{-}Cl(B))\}$. Conversely, let $x \in \bigcup_{B \in \mathrm{P}(Y)} \{[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B)))) \setminus f^{-1}([\beta,\beta']\text{-}Cl(B))\}$. Then $B \in P(Y)$ such that $x \in \{[\gamma,\gamma']\text{-}sCl(f^{-1}([\beta,\beta']\text{-}Int([\beta,\beta']\text{-}Cl(B)))) \setminus f^{-1}([\beta,\beta']\text{-}Cl(B))\}$. By Theorem 3.5, we obtain $x \in D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)$.

Similarly, by Theorems 3.6 and 3.7, we obtain the following theorems

Theorem 3.10 For a function $f:(X,\tau,\gamma,\gamma')\rightarrow(Y,\sigma,\beta,\beta')$, the following properties hold:

$$D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)$$
$$=\bigcup_{K\in[\beta,\beta']-RC(Y,\sigma)}\{[\gamma,\gamma']-sCl(f^{-1}([\beta,\beta']-Int(K)))\setminus f^{-1}(K)\}$$
$$=\bigcup_{V\in[\beta,\beta']-SPO(Y,\sigma)}\{[\gamma,\gamma']-sCl(f^{-1}([\beta,\beta']-Int([\beta,\beta']-Cl(V))))\setminus f^{-1}([\beta,\beta']-Cl(V))\}$$
$$=\bigcup_{U\in[\beta,\beta']-SO(Y,\sigma)}\{[\gamma,\gamma']-sInt(f^{-1}([\beta,\beta']-Int([\beta,\beta']-Cl(U))))\setminus f^{-1}([\beta,\beta']-Cl(U))\}.$$

Theorem 3.11 For a function $f:(X,\tau,\gamma,\gamma')\rightarrow(Y,\sigma,\beta,\beta')$, the following properties hold:

$$D_{wsc}^{[\gamma,\gamma']-[\beta,\beta']}(f)$$
$$=\bigcup_{G\in[\beta,\beta']-PO(Y,\sigma)}\{[\gamma,\gamma']-sCl(f^{-1}([\beta,\beta']-Int([\beta,\beta']-Cl(G))))\setminus f^{-1}([\beta,\beta']-Cl(G))\}$$
$$=\bigcup_{V\in[\beta,\beta']-PO(Y,\sigma)}\{[\gamma,\gamma']-sCl(f^{-1}(V))\setminus f^{-1}([\beta,\beta']-Cl(V))\}$$
$$=\bigcup_{V\in[\beta,\beta']-PO(Y,\sigma)}\{f^{-1}(V)\setminus[\gamma,\gamma']-sInt(f^{-1}([\beta,\beta']-Cl(V)))\}$$
$$=\bigcup_{G\in[\beta,\beta']-PC(Y,\sigma)}\{[\gamma,\gamma']-sCl(f^{-1}([\beta,\beta']-Int(G)))\setminus f^{-1}(G)\}.$$

REFERENCES

[1] S.Kasahara, Operation-compact spaces, Math. Japonica 24 (1979), 97 -105
[2] C. Carpintero, N. Rajesh and E. Rosas, On $[\gamma,\gamma']$-semiopen, Boletin de Matematicas Colombia, 17(2), 125-136, (2010).
[3] H. Maki and T. Noiri, Bioperations and some separation axioms, Scientiae Math. Japonicae, 53(1)(2001), 165-180.

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