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Solving the Transportation Problem Using Fuzzy Modified Distribution Method

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ABSTRACT

Transportation Problem is a special case of the linear programming problem which has been briefly studied in Operations Research. It has been often used to simulate different real life problems. In this Paper, we present the closed, bounded and non–empty feasible region of the transportation problem using fuzzy trapezoidal numbers which ensures the existence of an optimal solution to the balanced transportation problem. The multi-valued nature of Fuzzy Sets allows handling of uncertainty and vagueness involved in the cost values of each cells in the transportation table. For finding the initial solution of the transportation problem we use the Fuzzy Vogel's Approximation Method and for determining the optimality of the obtained solution Fuzzy Modified Distribution Method is used. The fuzzification of the cost of the transportation problem is discussed with the help of a numerical example. Finally, we discuss the computational complexity involved in the problem. To the best of our knowledge, this is the first work on obtaining the solution of the transportation problem using fuzzy trapezoidal numbers.

Keywords: Transportation Problem, Linear Programming Problem, Fuzzy trapezoidal numbers, Fuzzy Vogel's Approximation Method, Fuzzy Modified Distribution Method

1. Trapezoidal Membership Function

A membership function for a fuzzy set A on the universe of discourse X is defined μ_A:X → [0, 1], where each element of X is mapped to a value between 0 and 1. This value, called membership value or degree of membership, quantifies the grade of membership of the element in X to the fuzzy set A.

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Membership functions allow us to graphically represent a fuzzy set. The *x* axis represents the universe of discourse, whereas the *y* axis represents the degrees of membership in the [0, 1] interval.

The trapezoidal membership functionis specified by four parameters $\{x_1, x_2, x_3, x_4\}$ as follows and is defined by a lower limit x_1, an upper limit x_4, a lower support limit x_2, and an upper support limit x_3, where $x_1<x_2<x_3<x_4$.

$$trapezoid(x; x_1,x_2,x_3,x_4)=\begin{cases} 0, & x < x_1 \\ \frac{x-x_1}{x_2-x_1}, & x_1 \leq x < x_2 \\ 1, & x_2 \leq x < x_3 \\ \frac{x_4-x}{x_4-x_3}, & x_3 \leq x < x_4 \\ 0, & x \geq x_4 \end{cases} \quad \textbf{(1)}$$

2. Transportation Problem with Fuzzy TrapezoidalNumbers

Assume a situation having *m* origins or supply centers which contain various amounts of commodity that has to be allocated to *n* destinations or demand centers. Consider that the i^{th}origin must supply the fuzzy quantity

A_i=[$a_i^{(1)},a_i^{(2)},a_i^{(3)},a_i^{(4)}$]($>[-\delta, 0,0, \delta]$),whereas the j^{th} destination must receive the fuzzy quantity B_j=[$b_j^{(1)},b_j^{(2)},b_j^{(3)},b_j^{(4)}$]($>[-\delta, 0,0, \delta]$).

Let the fuzzy cost C_{ij} =[$c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)},c_{ij}^{(4)}$]of shipping a unit quantity from the origin *i* to the destination *j* be known for all the origins *i*and destinations *j*. As it is possible to transport from any one origin to any one destination,and the problem is to determine the number of units to be transported from origin *i* to thedestination *j* such that all requirements are satisfied at a total minimum transportation cost.This scenario holds for a *balanced transportation problem*.

Further in an *unbalancedtransportation problem*, the sum availabilities or supplies of the origins are not equal to thesum of the requirements or demands at the destinations. In order to solve this problem wefirst convert the unbalanced problem into a balanced one by artificially converting it to aproblem of equal demand and supply. For that, we introduce a fictitious or dummy origin ordestination that will provide the required supply or demand respectively. The costs oftransporting a unit from the fictitious origin as well as the costs of transporting a unit to

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thefictitious destination are taken as zero. This is equivalent to not transporting from a dummysource or to a dummy destination with zero transportation cost.

The mathematical formulation of the problem is as follows.
Let $Xij=[x_{ij}^{(1)},x_{ij}^{(2)},x_{ij}^{(3)},x_{ij}^{(4)}]$ be the fuzzy number of units supplied from the origin i to the destination j. Then the problem can be written as:

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}\, X_{ij} \quad where, Z = \left[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}\right] (\mathbf{2})$$

Subject to theconstraints:

$$\sum_{j=1}^{n} X_{ij} = A_i\,, i = 1,2, \dots \dots \dots \dots \dots \dots \dots \dots, m\ (Supply\ constraints)\ (\mathbf{3})$$

$$\sum_{i=1}^{m} X_{ij} = B_j\,, j = 1,2, \dots \dots \dots \dots \dots \dots, n\ \ (Destination\ constraints)\ (\mathbf{4})$$

$$\sum_{j=1}^{n} X_{ij} \ \geq [-\delta, 0,0, \delta]\ \ for\ all\ i\ and\ j \qquad (\mathbf{5})$$

Where A_i>$[\,-\delta, 0,0, \delta]$ *and* B_j>$[\,-\delta, 0,0, \delta]$ for all i and j. It should be noted that theTransportation Problem is a linear program. Suppose that there exists a feasible solution tothe problem. Then, it follows, from equations **(3)** and **(4),**so that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} = \sum_{i=1}^{m} A_i \ = \sum_{j=1}^{n} B_j$$

Thus, for the problem to be consistent, we must have the following consistency equation:

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$$\sum_{i=1}^{m} A_j = \sum_{j=1}^{n} B_j \qquad (6)$$

If the problem is inconsistent, the following equation holds:

$$\sum_{i=1}^{m} A_j \neq \sum_{j=1}^{n} B_j \qquad (7)$$

If the consistency condition (6) holds, then the transportation problem is *balanced*, such thatthe total supply is equal to the total demand otherwise the problem is *unbalanced*. It isobvious from the constraints (3), (4) and (5) that every component $X_{ij}$of a fuzzy feasiblesolution vector X is bounded, i.e.,

$[-\delta, 0, 0, \delta] \leq [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] \leq \min([a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}], [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}])$

Thus, the feasible region of the problem is closed, bounded and non-empty. Hence, therealways exists an optimal solution to the balanced transportation problem. Constraint equations (3) and (4) can be written in the matrix form as follows:

$$AX=B$$

with$X=(X_{11}, X_{12}, X_{13}, \ldots\ldots\ldots X_{1n}, X_{21}, X_{22}, X_{23}, \ldots, X_{2n}\ldots\ldots, X_{mn})^T$,

$B=(A_1,A_2,A_3,\ldots\ldots\ldots A_m, B_1,B_2,B_3,\ldots\ldots\ldots B_n)^T$and A as an $(m+n) \times mn$matrix given by

$$A=\begin{bmatrix} 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots 0 \\ 0..1 \ldots\ldots\ldots\ldots\ldots..0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.1 \\ I..I..I..I..I..I..I \end{bmatrix}$$

Here, 1 is the $1 \times n$ matrix with all the components as 1 and I is the $n \times n$identity matrix.Since the sum of m equations (3) equals the sum of n equations (4), the $(m + n)$ rows of A arelinearly dependent. This implies that $rank(A) \leq m + n - 1$.

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The transportation model has a special structure which enables us to represent it in the formof rectangular array called the *transportation table* as given in figure 2. In this table, each ofthe *mn*cells corresponds to a variable; each row corresponds to one of the *m* constraints (3)called row constraints and each column corresponds to one of the *n* constraints (4) calledcolumn constraints. The $(i, j)^{th}$cell at the intersection of the i^{th}row and j^{th}column containscost $C_{ij}$and decision variable X_{ij}. The cells in the transportation table can be classified asoccupied cells and unoccupied cells. The allocated cells in the transportation table are calledoccupied cells and empty cells are called unoccupied cells.

3.Fuzzy Modified Distribution Method*(FMODIM)*

Once the fuzzy initial basic feasible solution has been obtained, the next step is to determinewhether the solution obtained is fuzzy optimum or not. Optimality test can be conducted toany initial basic feasible solution of the transportation problem provided such allocationshave exactly $(m + n - 1)$ non–negative allocations, where m is the number of origins and n isthe number of destinations. Also these allocations must be in independent positions. Toperform the optimality test, we make use of the FMODIM using the fuzzy trapezoidalnumbers. The various steps involved in FMODIM for performing the optimality test aregiven below:

Step 1:

Find the fuzzy initial basic feasible solution of the fuzzy transportationproblem byusing FVAM.

Step 2:

Find a set of numbers $U_{i=}[u_i^{(1)},u_i^{(2)},u_i^{(3)},u_i^{(4)}]$and $V_{i=}[v_j^{(1)},v_j^{(2)},v_j^{(3)},v_j^{(4)}]$ foreach row and column satisfying $U_i(+)\ V_j= C_{ij}$for each occupied cell. We start by assigning anumber *fuzzy zero* (which may be [-0.05, 0, 0, 0.05] to any row or column having themaximum number of allocations. If the maximum number of allocation is more than one, wechoose any one arbitrarily.

Step 3:

For each empty or unoccupied cell, we find the sum of $U_i$and $V_j$and write it in eachcell.

Step 4:

Find the net evaluation value for each empty cell given as, $\Delta_{ij} = C_{ij}(-)(U_i(+)V_j)$ andalso write it in each cell. This gives the optimality conclusion which may be any of thefollowing:

a) If all $\Delta_{ij} > [-\delta, 0, 0, \delta]$, the solution is fuzzy optimum and a fuzzy unique solution exists.

b) If $\Delta_{ij} \geq [-\delta, 0, 0, \delta]$, then the solution is fuzzy optimum, but an alternate solution exists.

c) If at least one $\Delta_{ij} < [-\delta, 0, 0, \delta]$, the solution is not fuzzy optimum. In this case we go to the next step, to improve the total transportation cost.

Step 5:

Select the empty cell having the most negative value of Δ_{ij}. From this cell we draw aclosed path by drawing horizontal and vertical lines with the corner cells occupied. Assignpositive and negative signs alternately and find the minimum allocation from the cell havingthe negative sign. This allocation is to be added to the allocation having positive sign andsubtracted from the allocation having negative sign.

Step 6:

The **step 5** yields a better solution by making one or more occupied cell as emptyand one empty cell as occupied. For this new set of basic feasible allocations repeat from **steps 2** – **5** until an optimum basic feasible solution is obtained.

4. Numerical Example

In this section, we consider the fuzzy transportation problem and obtain the fuzzy initialbasic feasible solution of the problem by FVAM and determine the fuzzy membershipfunctions of costs and allocations. We then test the optimality of the solution obtained using FMODIM

4.1Initial Basic Feasible Solution by FVAM

The transportation problem consists of 3 origins and 4 destinations. The cost coefficients aredenoted by trapezoidal fuzzy numbers. The corresponding availability (supply) andrequirement (demand) vectors are also given in the figure below. In the above Table,

$$\sum_i A_i = [1009.85, 1010, 1010, 1010.15] (coloum\ sum)$$

$$and \quad \sum_j B_j = [1009.80, 1010, 1010, 1010.20] \quad (row\ sum)$$

$$Since, \sum_i A_i \ \ and \sum_j B_j \text{ are fuzzy equal, differing by the fuzzy zero viz., } [-0.35, 0, 0, 0.35]$$

the given problem is balanced and there exists a fuzzy feasible solution to the problem. We first find the row and column penalty as the difference betweenthe fuzzy least and next fuzzy least cost in the corresponding rows and columns respectively.

Consider the following Transportation problem with 3 Origins and 4 Destinations.

		DESTINATIONS				
ORIGINS		***D₁***	***D₂***	***D₃***	***D₄***	***Supply (A_i)***
	O_1	[12.95,13, 13,13.05]	[14.95,15, 15,15.05]	[15.95,16, 16,16.05]	[17.95,18, 8,18.05]	[279.95,280, 280,280.05]
	O_2	[19.95,20, 20,20.05]	[21.95,22, 22,22.05]	[10.95,11, 11,11.05]	[7.95,8, 8,8.05]	[329.95,330, 330,330.05]
	O_3	[18.95,19, 19,19.05]	[24.95,25, 25,25.05]	[16.95,17, 17,17.05]	[10.95,11, 11,11.05]	[399.95,400, 400,400.05]
	Demand (B_j)	[299.95,300, 300,300.25]	[249.95,250, 250,250.05]	[279.95,280, 280,280.05]	[179.95,180, 180,180.05]	

There are 6 positive independent allocations given by *m* + *n* -1 = 3 + 4 – 1. This ensures thatthe solution is a fuzzy non – degenerate basic feasible solution. The total transportation cost= {C_{11}(·) *X11*} (+) {C_{12}(·) X_{12}} (+) {C_{21}(·) X_{21}} (+) {C_{23}(·) X_{23}} (+){C_{31}(·) X_{31}} (+) {C_{34}(·) X_{34}} = {[12.95, 13, 13, 13.05] (·) [29.90, 30, 30, 30.10]} (+) {[14.95, 15, 15, 15.05] (·) [249.95, 250, 250, 250.05]} (+) {[19.95, 20, 20, 20.05] (·) [49.95, 50, 50, 50.05]} (+){[10.95,11, 11, 11.05] (·) [279.95, 280, 280, 280.05]} (+) {[18.95, 19, 19, 19.05] (·) [219.90, 220,220, 220.10]} (+) {[10.95, 11, 11, 11.05] (·) [179.95, 180, 180, 180.05]}= [14323.47, 14380, 14380, 14436.57]

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The final allocated matrix with the corresponding allocation values are given below:

		DESTINATIONS				
ORIGINS		***D_1***	***D_2***	***D_3***	***D_4***	***Supply (A_i)***
	O_1	**[12.95,13, 13,13.05] [29.90,30, 30,30.10]**	**[14.95,15, 15,15.05] [249.95,250, 250,250.05]**	[15.95,16, 16,16.05]	[17.95,18, 8,18.05]	[279.95,280, 280,280.05]
	O_2	**[19.95,20, 20,20.05] [49.95,50, 50,50.05]**	[21.95,22, 22,22.05]	**[10.95,11, 11,11.05] [279.95,280, 280,280.05]**	[7.95,8, 8,8.05]	[329.95,330, 330,330.05]
	O_3	**[18.95,19, 19,19.05] [219.90,220, 220,220.10]**	[24.95,25, 25,25.05]	[16.95,17, 17,17.05]	**[10.95,11, 11,11.05] [179.95,180, 180,180.05]**	[399.95,400, 400,400.05]
	Demand (B_j)	[299.95,300, 300,300.05]	[249.95,250, 250,250.05]	[279.95,280, 280,280.05]	[179.95,180, 180,180.05]	

Which is the final allocated matrix with the corresponding allocation values.

4.3 Fuzzy Membership Functions of the costs and allocations

Now, we present the fuzzy membership function of $C_{ij}$and $X_{ij}$and then the transportationcosts.

$$trapezoid(x; x_1, x_2, x_3, , x_4) = \begin{cases} 0, & x < x_1 \\ \frac{x - x_1}{x_2 - x_1}, & x_1 \leq x < x_2 \\ 1, & x_2 \leq x < x_3 \\ \frac{x_4 - x}{x_4 - x_3}, & x_3 \leq x < x_4 \\ 0, & x \geq x_4 \end{cases}$$

Now, the total transportation cost is given by:

$Cost_{\alpha} = \{C_{11_{\alpha}}(.)X_{11_{\alpha}}\}\ (+)\ \{C_{12_{\alpha}}(.)X_{12_{\alpha}}\}\ (+)\{C_{21_{\alpha}}(.)X_{21_{\alpha}}\}\ (+)\ \{C_{23_{\alpha}}(.)X_{23_{\alpha}}\}\ (+)$

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$\{C_{31_\alpha}(.)X_{31_\alpha}\}$ (+) $\{C_{34_\alpha}(.)X_{34_\alpha}\}$= [0.02α^2 + 56.51 α+ 14323.47, 0.02α^2 – 53.78 α+ 14436.57]

(9)

Hence the required fuzzy membership function of the transportation cost is given by:

$$\mu_{cost}(x) = \begin{cases} \left\{-56.51 + \sqrt{\left((56.51)^2 - 4\times 0.02 \times (14323.47 - x_1)\right)}\right\}/(2.0.02), & 14323.47 \le x \le 14380 \\ 1, & 14380 \le x \le 14380 \\ \left\{-53.78 + \sqrt{\left((53.78)^2 - 4\times 0.02 \times (14436.57 - x_2)\right)}\right\}/(2.0.02), & 14380, \le x \le 14436.57 \\ 0, & otherwise \end{cases}$$

(10)

5.Conclusion

The closed, bounded and non–empty feasible region of the transportationproblem using fuzzy trapezoidal numbers which ensures the existence of an optimal solutionto the balanced transportation problem. The multi-valued nature of Fuzzy Sets allowshanding of uncertainty and vagueness involved in the cost values of each cells in thetransportation table. For finding the initial solution of the transportation problem we use theFVAM and for determining the optimality of the obtained solution FMODIM is used. Thefuzzification of the cost of the transportation problem is discussed with the help of anumerical example. The computational complexity involved in the problem.The effectiveness of the solutions obtained for the problem can greatly be enhanced byincorporating the genetic algorithms along with the fuzzy trapezoidal numbers such that thecomputational complexity is greatly reduced.

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