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# Steady Flow in Pipes of Rectangular Cross-Section in Magnetic Field

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**ABSTRACT:** In this paper we have investigated the steady flow in pipes of rectangular cross-section in magnetic field. We have investigated the velocity, flux and vortex line.

**KEY WORDS:** Steady flow, rectangular Cross section incompressible fluid and magnetic field.

**NOMENCLATURE**

u = velocity component along x – axis
v = velocity component along y – axis
w (x,y) = velocity in x-y plane
t = the time
$\rho$ = the density of fluid
P = the fluid pressure
K= the thermal conductivity of the fluid
$\mu$ = Coefficient of viscosity
$\upsilon$ = Kinematic viscosity
Q = the volumetric flow
$\Omega_x$ = Vorticity component in x – direction
$\Omega_y$ = Vorticity component in y – direction
$\Omega_z$ = Vorticity component in z - direction

**INTRODUCTION**

We have investigated the steady flow in pipes of rectangular cross-section through porous medium. Attempts have been made by several researchers i.e. D. M. Fontana, M. L. Corcione, A. Monaco & L. Santarpia [1] laminar free convection from vertical plates in partly dissociated gases. L. K. Forbes [2] on the effects of non-linearity in free-surface flow about a submerged point vortex. B. Fornberg [3] a numerical study of steady viscous flow past a circular cylinder. B. Fornberg [4] Steady viscous flow past a circular cylinder up to Reynolds number 600. A. Fortin, M. Jardak, J. J.Gervais & R. Pierre [5] localization of hoaf bifurcations in fluid flow problems. Y. B. Fu & N. H. Scott [6] propagation of simple waves and shock wanes in a rod of slowly varying cross sectional area. S. Ganesh & S. Krishnambal [7] unsteady MHD stokes flow of viscous fluid between two parallel porous plates.J. P. Garandet & T. Alboussiere and R. Moreau [8] Buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. P. H. Gaskell, H. M. Thompson & M. D. Savage [9] A finite element analysis of steady viscous flow in triangular cavities. J. J. Gervais, D. Lemelin & R. Pierre [10] some experiments with stability analysis of discrete incompressible flows in the lid-driven cavity. U. Ghia, K. N. Ghia & C. T. Shin [11] High-Reynolds solutions for incompressible flow using the Navier-stokes equations and a multi grid method. S. K. Ghosh [12] Hall effects on hydromagnetic convective flow in a rotating channel. G. Giannakidis [13] Prandtl-Batchelor flow on a circular cylinder and on aerofoil sections. In this paper we have investigated the velocity, flux and vortex line.

**FORMULATION OF THE PROBLEM**

Let **z -** axis be taken the direction of flow along the incompressible fluid the velocity component is axis of the pipe. Then u = 0, v = 0 for steady and independent of **z**.

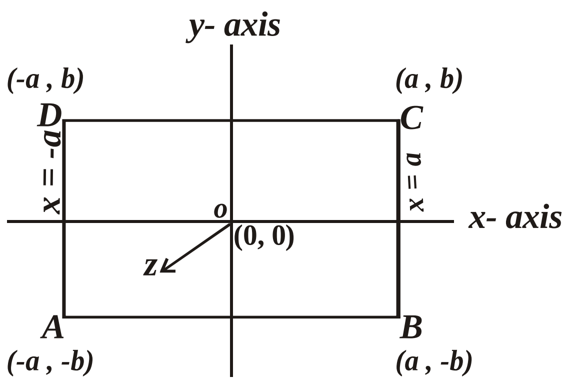
The equation of continuity.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \text{..................(1)}$$

$$But \quad u=0,\; v=0,\; \frac{\partial w}{\partial z}=0 \;\Rightarrow\; w=w(x,y) \text{.......(2)}$$

i.e. w is independent of **z**

The Navier-Stokes equations of motion in the absence of body forces.




$$-\frac{\partial P}{\partial x}=0 \text{ .............(3)}$$

$$-\frac{\partial P}{\partial y}=0\text{.........(4)}$$

$$\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}w=0$$

$$\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho\mu}\mu w=0\text{...........(5)}$$

It is clear from (3) & (4) **P** is independent of **x** & **y** i.e. **p** is the Function of **z**

**SOLUTION OF THE PROBLEM** p = p (z) $\frac{\partial p}{\partial z} = \frac{dp}{dz} = \text{Constant} = -P$

$$let \quad \frac{\sigma B_0^2}{\rho\mu}=B^2 \;\Rightarrow\; \mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w\right]=\frac{dp}{dz} \;\Rightarrow\; \frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w=-\frac{P}{\mu}\text{............(6)}$$

$\left(D^2+D'^2-B^2\right)w=-\frac{P}{\mu} \;\therefore\; C.F. = \sum a_n\, e^{h_n x+h'_n y}$ Where $h_n$ & $h'_n$ are related by $h_n^2+h'^2_n-B^2=0$

and $P.I. = \frac{1}{D^2+D'^2-B^2}\left(-\frac{P}{\mu}\right) = \frac{P}{B^2\mu} \Rightarrow w(x,y)=\sum_{n=1}^{\infty} a_n\, e^{h_n x+h'_n y}+\frac{1}{B^2\mu}P$ Where $h_n^2+h'^2_n=B^2$

**Case -1** $w\,(x,\,y)\;=\;0\;\;at\;\;(a,\,b),\;\;w\,(x,\,y)\;=\;0\;\;at\;\;(a,-b)$

$$\sum_{n=1}^{\infty} a_n\;\; e^{h_n a + h_n^{'} b} + \frac{P}{\mu B^2} = 0 \qquad and \qquad \sum_{n=1}^{\infty} a_n\;\; e^{h_n a - h_n^{'} b} + \frac{P}{\mu B^2} = 0$$

$$\Rightarrow\; -\frac{P}{\mu B^2} = \sum_{n=1}^{\infty} a_n\;\; e^{h_n a + h_n^{'} b} \;................(a) \qquad \& \qquad -\frac{P}{\mu B^2} = \sum_{n=1}^{\infty} a_n\;\; e^{h_n a - h_n^{'} b} \;................(b)$$

$$h_n^{'} = 0 \qquad \Rightarrow \qquad h_n = -B \;\Rightarrow\; -\frac{P}{\mu B^2} = e^{-aB} \sum_{n=1}^{\infty} a_n \qquad \Rightarrow \sum_{n=1}^{\infty} a_n = e^{-\frac{P}{\mu B^2}} e^{-aB}$$

$$w_1(x,y) = -\frac{P}{\mu B^2}\;\; e^{aB}\;\; e^{-xB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2}\;\; e^{B(-x+a)} + \frac{P}{\mu B^2}$$

**Case -2** $w\,(x,\,y)\;=\;0\;\;\;at\;\;(-a,\,b)\;\;\&\;\;(-a,-b)$

$$w_2(x,y) = -\frac{P}{\mu B^2}\;\; e^{aB}\;\; e^{xB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2}\;\; e^{B(x+a)} + \frac{P}{\mu B^2}$$

**Case - 3** $w\,(x,\,y)\;=\;0\;\;\;at\;\;(-a,\,b)\;\;\&\;\;(a,\,b)$

$$w_3(x,y) = -\frac{P}{\mu B^2}\;\; e^{bB} e^{-yB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2}\;\; e^{B(-y+b)} + \frac{P}{\mu B^2}$$

**Case -4** $w\,(x,\,y)\;=\;0\;\;\;at\;\;(-a,\,-b)\;\;\&\;\;(a,\,-b)\;\;,\;\;w_4(x,y) = -\frac{P}{\mu B^2}\;\; e^{B(y+b)} + \frac{P}{\mu B^2}$

$$w(x,y) = \frac{P}{\mu B^2}\left[1 - 2\;\; e^{aB}\;\; Cosh\; x\,B - 2\; e^{bB}\;\; Cosh\; y\,B\right] ............. (7)$$

**In particular case:** In the case of square i.e. $a = b$

$$w(x,y) = \frac{P}{\mu B^2}\left[1 - 2\;\; e^{aB}\left(Cosh\, x\,B + Cosh\; y\,B\right)\right] ............ (8)$$

**Flux Q of the fluid over an area of rectangular cross-section:**

$$Q = \int_{x=-a}^{a}\int_{y=-b}^{b} w(x,y)\; dx\; dy \;= \int_{-a}^{a}\int_{-b}^{b} \frac{P}{\mu B^2}\left\{1 - 2\;\; e^{aB}\;\; Cosh\, x\,B - 2\;\; e^{bB} Cosh\; y\,B\right\} dx\; dy$$

$$= \frac{2P}{\mu B^2}\int_{-a}^{a}\int_{0}^{b}\left\{1 - 2\;\; e^{aB} Cosh\, x\,B - 2\;\; e^{bB} Cosh\; y\,B\right\}\; dy\; dx \;= \frac{2P}{\mu B^2}\int_{-a}^{a}\left\{\left(1 - 2\;\; e^{aB} Cosh\, x\,B\right) b - 2\;\; e^{bB}\left(\frac{1}{B}\;\; Sinh\, b\,B\right)\right\} dx$$

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$$=\frac{4P}{\mu B^2}\int_0^a \left\{b\left(1-2\ e^{aB}\ Cosh\,x\,B\right)-\frac{2}{B}\ e^{bB}\,Sinh\,b\,B\right\}dx\ =\frac{4P}{\mu B^2}\left[b\left\{x-\frac{2}{B}\ e^{aB}\ Sinh\,x\,B\right\}_0^a-\frac{2a}{B}\ e^{bB}\ Sinh\,b\,B\right]$$

$$=\frac{4P}{\mu B^2}\left[b\left\{a-\frac{2}{B}\ e^{aB}\ Sinh\,a\,B\right\}-\frac{2a}{B}\ e^{bB}\ Sinh\,b\,B\right]$$

$$Q=\frac{4P}{\mu B^2}\left[ab-\frac{2}{B}\left(b\ e^{aB}\ Sinh\,a\,B+a\ \ e^{bB}\ Sinh\,b\,B\right)\right]\ ............\ (9)$$

**In particular case:** In the case of square $a = b$

$$Q=\frac{4P}{\mu B^2}\left[a^2-\frac{4a}{B}\ e^{aB}\ Sinh\,a\,B\right]\ ............\ (10)$$

$$\bar{q}=ui+vj+wk\ =\frac{P}{\mu B^2}\left[1-2\ e^{aB}\ Cosh\,x\,B-2\ e^{bB}\ Cosh\,y\,B\right]\hat{k}$$

**The equation of vortex line** $\frac{dx}{\Omega_x}=\frac{dy}{\Omega_y}=\frac{dz}{\Omega_z}$ **where** $\Omega_x,\ \Omega_y\ \&\ \Omega_z$ **are vorticity components**

$$\Omega_x=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=\frac{P}{\mu B^2}\left[-2\,B\ e^{bB}\ Sinh\,y\,B\right]\ =-\frac{2P}{\mu B}\ e^{bB}\ Sinh\,y\,B$$

$$\Omega_y=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=-\frac{P}{\mu B^2}\left[-2\,B\ \ e^{aB}\ Sinh\,x\,B\right]=\frac{2P}{\mu B}\ \ e^{aB}\ Sinh\,x\,B\quad \&\quad \Omega_z=0$$

$$\Rightarrow\ \frac{dx}{-\frac{2P}{\mu B}\ e^{bB}\ Sinh\,y\,B}=\frac{dy}{\frac{2P}{\mu B}\ e^{aB}\ Sinh\,x\,B}=\frac{dz}{0}\qquad \&\quad dz=0\ \ \Rightarrow\ z=B$$

$$\frac{dx}{-e^{bB}\ Sinh\,y\,B}=\frac{dy}{e^{aB}\ Sinh\,x\,B}\ \Rightarrow\ e^{aB}\int Sinh\,x\,B\,dx\ +\ e^{bB}\int Sinh\,y\,B\,dy=C_1$$

$$\frac{1}{B}\ e^{aB}\ Cosh\,x\,B+\frac{1}{B}\ e^{bB}\ Cosh\,y\,B=C_1\quad ,\quad e^{aB}\ Cosh\,x\,B+e^{bB}\ Cosh\,y\,B=C_1\,B=A$$

$$\therefore\ \text{The vortex lines:}\ \ e^{aB}\ Cosh\,x\,B+\ e^{bB}\ Cosh\,y\,B=A\qquad \&\qquad z=B\ .............\ (11)$$

**Clearly the flow is Rotational in pipe**

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**In particular case:** In the case of square $a = b$

$$e^{aB}\left[Cosh\,x\,B + Cosh\,y\,B\right] = A \qquad \& \qquad Z = B \;.............\; (12)$$

**Table for velocity:** $Let\ \ P = \frac{1}{4},\ \ \mu = .5,\ \ a = b = 1,\ are\ same\ and\ \ B = \sqrt{\frac{\sigma B_0^2}{\rho\,\mu}},\ (x,y)\,are\,change$

**Table-1 (for velocity)**

| | $(x,y)$ | **(.1, .1)** | **(.2, .3)** | **(.3, .4)** | **(.4, .5)** | **(.5, .6)** | **(.6, .7)** | **(.7, .8)** |
|---|---|---|---|---|---|---|---|---|
| $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}} = 1$ | $w(x,y)$ | **-4.964** | **-5.114** | **-5.28** | **-5.504** | **-5.788** | **-6.134** | **-6.547** |
| $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}} = \frac{1}{2}$ | $w(x,y)$ | **-11.21** | **-11.297** | **-11.396** | **-11.529** | **-11.696** | **-11.897** | **-12.133** |
| $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}} = \frac{1}{3}$ | $w(x,y)$ | **-20.635** | **-20.712** | **-20.796** | **-20.908** | **-21.048** | **-21.216** | **-21.414** |
| $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}} = \frac{1}{4}$ | $w(x,y)$ | **-33.102** | **-33.172** | **-33.249** | **-33.352** | **-33.481** | **-33.636** | **-33.816** |
| $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}} = \frac{1}{6}$ | $w(x,y)$ | **-67.07** | **-67.135** | **-67.21** | **-67.3** | **-67.419** | **-67.56** | **-67.73** |

## CONCLUSION AND DISCUSSION

In this paper we have investigated the velocity by the **table-1** of equations (7) between velocity and point $(x,y)$. It is clear that the velocity of fluid increases uniformly with negative sign in the interval $(.1,.1) \le (x,y) \le (.7,.8)$ at different values of $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}}$ again velocity increases uniformly in the interval $(.1,.1) \le (x,y) \le (.7,.8)$ when $\sqrt{\frac{\sigma B_0^2}{\rho\,\mu}}$ decreases from 1 to $\frac{1}{6}$. Negative sign of velocity shows that direction of flow is opposite to the direction of motion of fluid. We have investigated vortex lines and the volumetric flow of elliptic and circle by equations. (8), (9), (10), (11) and (12) respectively.

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