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On Quasi Ideal Unique Sequential Spaces

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Abstract. In this paper, q-I-open sets are used to define q-I-US ideal topological spaces and study some of their basic properties.

1. INTRODUCTION

The subject of ideals in topological spaces has been introduced andstudied by Kuratowski [5] and Vaidyanathasamy [7]. An ideal Ion a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if P(X) is the set of all subsets of X, then the set operator $(.)^*$: $P(X) \rightarrow P(X)$, called the local function [7] of A with respect to τ and I, is defined as follows : for $A \subset X$, $A^*(\tau, I) = \{x \in X \mid U \cap A \notin I$, for every open set U containing x $\}$. A Kuratowski closure operator $Cl^*(.)$for a topology $\tau^*(\tau, I)$ called the $*$-topology , finer than τ is defined by $Cl^*(A) = A \cup A^*(\tau, I)$. When there is no chance for confusion, $A^*(\tau, I)$ is denoted by A^*. If I is an ideal on X, then (X, τ, I) is called an ideal topological space. In this paper, q-I-open sets are used to define q-I-US ideal topological spaces and study some of their basic properties.

2. PRELIMINARIES

Let A be a subset of a topological space (X, τ). We denote the closure of A and the interior of A by Cl(A) and Int(A), respectively. A subset S of an ideal topological space (X, τ, I) is called q-I-open [1] S $\subset Cl(Int(S^*))$. The complement of a q-I-open set is called a q-I-closed set [1]. The intersection of all q-I-closed sets containing S is called the q-I-closure of S and is denoted by qICl(S). The q-I-interior of S is defined by the union of all q-I-open sets contained in S and is denoted by q I Int(S). The set of all q-I-open sets of (X, τ, I) is denoted by QIO(X). The set of all q-I-open subsets of (X, τ, I) containing a point $x \in X$ is denoted by QIO(X,x)

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Definition 2.1. An ideal topological space$(X, \tau, I)$is said to be
(1)q-I-T_1[3] if for each pair of distinct points x and y of X, there exists q-I-open sets and U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.
(2)q-I-T_2 [3] if for each pair of distinct points x and y of X, there exists q-I-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.
(3)q-I-R_1 [4] if and only if for $x, y \in X$ with $q I Cl\{x\} \neq q I Cl(\{y\})$,there exist disjoint q-I-open sets U and V such that $qICl(\{x\}) \subset U$ and $qICl(\{y\}) \subset V$.
Theorem 2.2.[4] Let (X, τ, I) be an ideal topological space. Then (X, τ, I) is q-I-T_2 if and only if it is q-I-R_1 and q-I-T_0.

3. On q-I-US-spaces

Denition 3.1. A sequence (xn) is said to be q-I-convergence to a point x of X, denoted by (xn) $\overset{qI}{\rightarrow}$ x if (xn) is eventually in every q-I-open set containing x.

Denition 3.2. An ideal topological space (X ,τ ,I) is said to be a q-I-US space if every q-I-convergent sequence (xn) in X q-I-converges to a unique point.

Theorem 3.3. Every q-I-T_2 space is a q-I-US space.
Proof. Let (X, τ, I) be a q-I-T_2 space and (xn) be a sequence in X.
Suppose that (xn) q-I-converges to two distinct points x and y. That is, (xn) is eventually in every q-I-open set containing x and also in every q-I-open set containing y. This is a contradiction since (X, τ, I) is q-I-T_2. This shows that the space (X, τ, I) is a q-I-US space.

Theorem 3.4. Every q-I-US space is a q-I-T_1 space.
Proof. Let (X, τ, I) be a q-I-US space and x and y be two distinct points of
X. Consider the sequence (xn) where xn ≠ x for every n. Clearly (xn) q-I-converges to x. Also, since x ≠ y and (X, τ, I) is a q-I-US space, (xn) cannot q-I-converges to y, that is, there exists a q-I-open set V containing y but not x. Similarly, if we consider the sequence (yn) where yn ≠ y for all n, and proceeding as above we get a q-I-open set U containing x but not y. This shows that the space(X, τ, I) is a q-I-T_1 space.

Definition 3.5. A subset S of an ideal topological space (X, τ, I) is said to be:

(1) sequentially q-I-closed if every sequence in S q-I-converging in X q-I-converges to a point in S.
(2) sequentially q-I-compact if every sequence in S has a subse-quence which
q-I-converges to a point in S.

Theorem 3.6. In a q-I-US space, every sequentially q-I-compact set is sequentially q-I-closed.

Proof. Let(X, τ, I) be a q-I-Us space and Y be a sequentially q-I-compact subset of X. Let (xn) be a sequence in Y. Suppose that (xn) q-I-converges to a point in X−Y. Let (xnk) be a subsequence of (xn) which q-I-converges to a point y ∈ Y since Y is sequentially q-I-compact. Also, a subsequence (xnk) of (xn) q-I-converges to x ∈ X−Y. Since (xnk) is a sequence in the q-I-US space X, x = y. This shows that Y is sequentially q-I-closed set.

Denition 3.7. [2] Let A and X_0 be subsets of an ideal topological space (X, τ, I) such that A ⊂ X_0 ⊂ X. Then (X_0, $\mathcal{J}_{|xo}$, $I_{|xo}$) is an ideal topological space with an ideal I|X_0 = {I ∈ I : I ⊂ X_0} = {I ∩ X_0 : I ∈ I}

Lemma 3.8. [1] Let A and X_0 be subsets of an ideal topological space(X, τ, I)
If A ∈ QIO(X) and X_0 is open in (X, τ, I), then A∩X_0 ∈ QIO(X_0).

Theorem 3.9. Every open subset of a q-I-US space is a q-I-US space.
Proof. Let (X, τ, I)be a q-I-US space and Y an open subset of X. Let (xn) be a sequence in Y. Suppose that (xn) q-I-converge to x and y in Y. We shall prove that (xn) q-I-converges to x and y in X. Let U be any q-I-open subset of
X containing x and V be any q-I-open set of X containing y. Then by Lemma 3.8, U∩Y and V∩Y are q-I-open sets in Y. Therefore, (xn) is eventually in
U∩Y and V∩Y and so in U and V. Since X is q-I-US, this implies that x = y and hence the subspace Y is a q-I-US space.

Theorem 3.10. An ideal topological space(X, τ, I) Is q-I-T2 if and only if it is both q-I-R1 space and q-I-US space.
Proof. Let(X, τ, I) be a q-I-T2 space. Then (X, τ, I) is a q-I-R1 space by Theorem 2.2 and q-I-US space by Theorem 3.3. Conversely, let (X, τ, I)
be both q-I-R1 space and q-I-US space. By Theorem 3.4, we obtain that every
q-I-US space is q-I-T1 and X is both q-I-T1 and q-I-R1, it follows from Theorem 2.2 that (X, τ, I) is a q-I-T2 space

Definition 3.11. A function f : (X, τ, I) →(Y, σ, J) is said to be strongly q-I-open (resp. strongly q-I-closed) if f(A) ∈ QJO (Y) (resp. f(A) ∈ QJC(Y)) for every A ∈ QIO(X) (resp.A ∈ QIC(X)).

Lemma 3.12. Let a bijection f : (X, τ, I) → (Y, σ, J) is strongly q-I-open. Then for any A ∈ QIC(X) , f(A) ∈ QJC(Y)

Theorem 3.13. The image of a q-I-US space under a bijective strongly
q-I-closed is q-J-US spaces
Proof. Let f : (X, τ, I) →(Y, σ, J) be a strongly q-I-closed func-tion and let
(X, τ, I)) be a q-I-US space. Let (yn) be a sequence in Y. Suppose that (yn) q-J-converges to x and y. In that case, we shall prove that the sequence ($f^{-1}(y_n)$) q I converges to $f^{-1}(x)$ and $f^{-1}(y)$. Let U ∈ QIO(X, $f^{-1}(x)$). Then f(U) ∈ QJO(X,x) and hence (y_n) is eventually in f(U). Therefore $f^{-1}(y_n)$ eventually in U. Hence($f^{-1}(y_n)$) q J converges $f^{-1}(x)$).similarly, we can prove that ($f^{-1}(y_n)$) q I converges to $f^{-1}(y)$. This is not possible, since (X, τ, I) is a q I US spaces. Hence (Y, σ, J) is a q J US space

Denition 3.14. A function f: (X, τ, I) →(Y, σ, J) is said to be:

(1) sequentially q-I-continuous at x ∈ X if f(xn) q-J-converges to f(x) whenever(xn) is a sequence q-I-converging to x;

(2) sequentially q-I-continuous if f is sequentially q-I-continuous at all x ∈X; (3)sequentially nearly q-I-continuous if for each point x∈X and each sequence

(xn) in X q-I-converging to x, there exists subsequence (xnk) of (xn) such that f(xnk) $\overset{qJ}{\to}$ f(x);

(4) sequentially sub q-I-continuous if for each point x∈X and each sequence (xn) in X q I converging to x, there exist a subsequence (x_{nk}) of (x_n) and a point y∈Y such that f(xnk) $\overset{qJ}{\to}$ y;

(5) sequentially q-I-compact preserving if the image f(K) of ev-ery sequentially q-I-compact set K of X is sequentially q-J-compact in Y.

Theorem 3.15. Let f: (X, τ, I) (→(Y, σ, J) and g: (Y, σ, J) →(Z, η, K) be sequentially q-I-continuous functions and sequentially q-J-continuous functions, respectively. If (Y, σ, J) is a q-J-US space, then the set A = {x : f(x) = g(x)} is sequentially q-I-closed.

Proof. Let(Y, σ, J) be a q-J-US space and suppose that there exists a sequence (xn) in A q-I-converging to x ∈ X. By hypothesis, f(xn) $\overset{qJ}{\to}$ f(x) and g(xn)) $\overset{qK}{\to}$ g(x). Hence f(x) = g(x) and x ∈ A. Therefore, we obtain A is sequentially q-I-closed.

Theorem 3.16.Every function f(X, τ, I))→ (Y, σ, J) is sequen-tially sub-q-I-continuous if (Y, σ, J) is sequentially q-J-compact.
Proof. Let (xn) be a sequence in X q-I-converging to a point x of X. Then f(xn)) is a sequence in Y and as (Y, σ, J) is sequentially q-J-compact, there exists a subsequence (f(xnk)) of (f(xn)) q-J-converging to a point y∈Y. This shows that f: (X, τ, I) → (Y, σ, J) is sequentially sub q I continuous.

Theorem 3.17. Every sequentially nearly q-I-continuous function is sequentially q-I-compact preserving
Proof. Suppose that f: (X, τ, I) → (Y, σ, J) is sequentially nearly q-I-continuous function and let A be any sequentially q-J-compact set of Y. Let (yn) be any sequence in f(A). Then for each positive integer n, there exists a point xn ∈ A such that f(xn) = yn. Since (xn) is a sequence in the sequentially q-I-compact set A, there exists a subsequence (xnk) of (xn) q-I-converging to a point x ∈ A. Since f is sequentially nearly q-I-continuous, then there exists a subsequence (xj) of (xnk) such that f(xj)) $\overset{qJ}{\to}$ f(x). Therefore, there exists a subsequence (yj) of (yn) q-J-converging to f(x) ∈ f(A). This shows that f (M) is sequentially q J compact in (Y,,J).

Theorem 3.18. Every sequentially q-I-compact preserving function is sequentially sub-q-I-continuous.

Proof. Suppose f: (X, τ ,I)→(Y,σ, J) is a sequentially q-I-compact preserving function. Let x be any point of X and (xn) any sequence in X q-I-converging to x. We shall denoted the set{xn : n= 1,2,....}by A and B = A∪{x}. Then B is sequentially q-I-compact since xn $\overset{qI}{\to}$x. Since f is sequentially q

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-I-compact set preserving, it follows that f(B) is a sequentially q-J compact set of (Y,σ ,J). Since (f(xn))is a sequence in f(B), there exists a subsequence (f(xnk)) of (f(xn)) q-J-converging to a point y ∈ f(B). This implies that f is sequentially sub-q-I-continuous.

Theorem 3.19. A function f: (X, τ, I) → (Y, σ, J) is sequentially q-I-compact preserving if and only if f $_{|M}$: (M , $\mathcal{J}$ |M,I |M)→ f(M) is se-quentially sub-q-I |M-continuous for each sequentially q-I-compact sub-set M of X.

Proof.Suppose that f: ((X, τ, I)→(Y, σ, J) is a sequentially q-I-compact preserving function. Then f(M) is sequentially q-J-compact set M of X. Therefore, by Theorem 3.16, f$_|$ M: M→ f(M) is se-quentially sub-q-I $_{|M}$-continuous function. Conversely, let M be any sequentially q-J-compact set inY. We shall show that f(M) is se-quentially q-J-compact set in Y. Let (yn) be any sequence in f(M).
Then for each positive integer n, there exists a point xn ∈ M such that
f(xn) = yn. Since (xn) is a sequence in a sequentiallyq-I-compactset M, there exists a subsequence (xnk) of (xn) q-I-converging to a point x ∈ M. Since f |M : M→ f(M) is sequentially sub-semi-I |M-continuous, there exists a subsequence (ynk) of (yn) q-I |M-converging to a point y ∈f (M). This implies that f(M) is sequentially q-J-compactset inY. Thus, f is sequentially q-I-compact preserving . The following theorem gives a sufficient condition for a sequentially sub-q-I-continuous function to be a sequentially q-I-compact preserving.

Theorem 3.20. If a function f: (X, τ, I) →(Y , σ, I) is sequentially sub-q-I-continuous and f(M) is sequentially q-J-closed set in Y for each sequentially q-I-compact set M of X, then f is sequentially q-I-compact preserving function.

Proof. We use the previous Theorem. It suffices to prove that f |M : M→ f(M) is sequentially sub-q-I |M-continuous for each sequentially
q-I-compact subset M of X. Let (xn) be any sequence in M q-I-converging to a point x∈M. Then since f is sequentially sub-q-I |M-continuous, there exists a subsequence (xnk) of (xn) and a point y ∈ Ysuch that f(xnk) q-I |M-converges to
y. Since f(xnk) is a sequence in the sequentially q-J-closed set f(M) of
Y, we obtain y ∈ f(M). This implies that f $|_M$: M→ f(M) is sequentially sub-
q-I |M-continuous

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