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Optimization of Deteriorating EOQ Model with Shortages and Controllable Lead Time

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Abstract:

The main focus of the article is to analyze the optimization inventory system for deteriorating items with shortages are allowed. A proposed model is constructed and obtaining the optimal solutions of lot size optimum level of time which reduced the total cost. A numerical example is illustrated in this model.

Key words: *Demand, EOQ, deterioration, Shortages and Inventory cycle.*

1. Introduction:

In practical situations, electronic and electrical goods, costlier fashion clothes, road vehicles they fall with its category since they can become absolute overtime and their demand rate will decreases drastically. So Deterioration of items in inventory system has become an interesting feature for its practical importance. Deterioration refers to damages, spoilage, vaporization or obsolescence of the products. Example, deteriorating items include metal parts, which are prove to corrosion and rusting and some perishable food items which are subject to spoilage and decay. In this article an EOQ model is developed for deteriorating items and Shortages. This study is organized as section 2 is concerned with assumption and notations, in section 3 is mathematical model and in section 4 is conclusion.

2. Assumptions and Notations

(i) Assumptions: The following assumptions are used to formulate the problem.

a) A single period inventory cycle system of single type of products is considered..

b) Shortages are permissible.

c) The lead time is controllable.

d) Time horizon is finite.

(ii) Notations: The following notations are used in our analysis.

a) R – *Demand rate / year*

b) C_1 – *Setup Cost / Ordering Cost / set*

c) C_2 – *Holding Cost / unit / year*

d) C_3 – *Shoratge Cost / unit*

e) C_4 – *Deteriorating Cost / unit*

f) L – *Lead Time, where L is controllable*

g) Q_1 – *Maximum Inventory*

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$h)\ S\ -\ Shortages$
$i)\ \theta - Rate\ of\ \det eriorative$
$k)\ T_C\ -\ Total\ Cost$

3. Mathematical model

(a) EOQ Model for deteriorating items with Shortages

Let I(t) be the inventory level of the system at $t\,(0 \le t \le T)$.

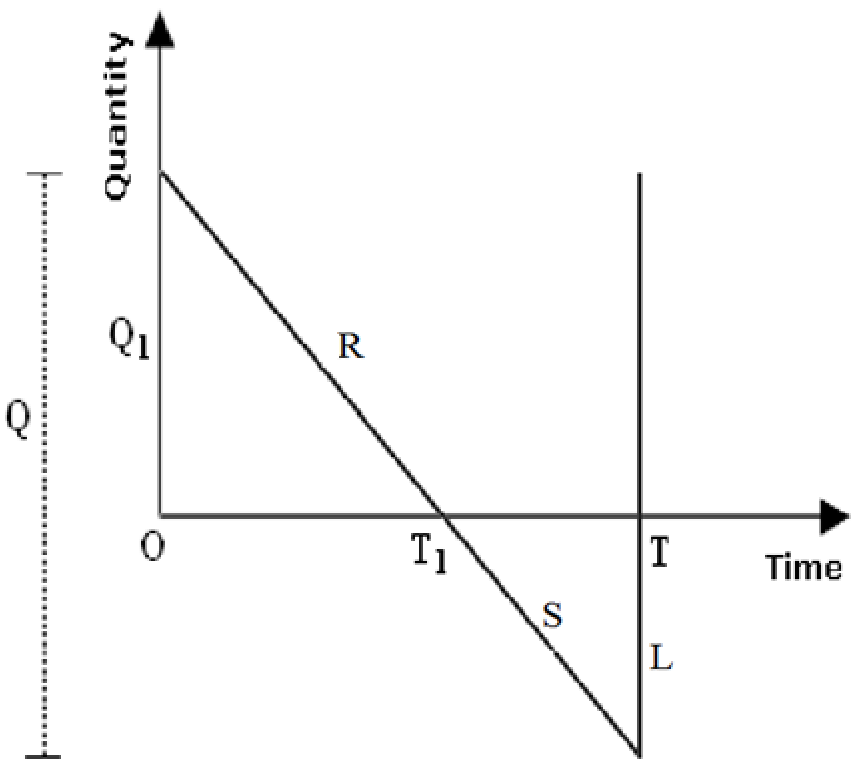

Figure: 1 EOQ model for deteriorating items with Shortages

The differential equations of I(t) over the period (0, T) is given by

$$\frac{d}{dt} I(t) + \theta I(t) = -R \quad 0 \le t \le T_1$$
(1)

$$\frac{d}{dt} I(t) = -R \qquad T_1 \le t \le T$$
(2)

with the boundary conditions are

$I(0) = Q_1 \qquad I(T_1) = 0 \qquad I(T) = S$

(3)

$$From(1), \quad I(t) = e^{-\theta t}\int_t^{T_1} R\, e^{\theta t}\, dt$$

$$= R e^{-\theta t}\left(\frac{e^{\theta t}}{\theta}\right)_t^{T_1}$$

$$= \frac{R}{\theta} e^{-\theta t}\left[e^{\theta T_1} - e^{\theta t}\right]$$

$$= \frac{R}{\theta}\left[e^{\theta(T_1 - t)} - 1\right]$$

(4)

$$From(2), \quad I(t) = \int_{T_1}^{t} R\, dt \quad = R(t - T_1)$$

(5) $From\ (3)\ and\ (4), \quad I(0) = Q_1$

$$Q_1 = \frac{R}{\theta}\left[e^{\theta T_1} - 1\right]$$

$$= \frac{R}{\theta}\left[1 + \theta T_1 + \frac{\theta^2 T_1^{\ 2}}{2} - 1\right]$$

$$= \left[R T_1 + \frac{R\theta T_1^{\ 2}}{2}\right]$$

(6)

Total cost (T_c): Total cost consists of ordering cost, holding cost, deteriorating cost and Shortage cost

$$(i)\ Ordering\ \ Cost = \frac{C_1}{T}$$

(7)

$$(ii)\ Holding\, Cost \ = \frac{C_2}{T}\int_0^{T_1} I(t)\, dt$$

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$$= \frac{C_2}{T} \int_0^{T_1} \frac{R}{\theta} \left[e^{\theta(T_1 - t)} - 1 \right] dt$$

$$= \frac{R C_2}{\theta T} \left[\frac{e^{\theta(T_1 - t)}}{-\theta} - t \right]_0^{T_1}$$

$$= \frac{-R C_2}{\theta^2 T} \left[e^{\theta(T_1 - t)} + \theta t \right]_0^{T_1}$$

$$= \frac{-R C_2}{\theta^2 T} \left[1 + \theta T_1 - e^{\theta T_1} \right]$$

$$= \frac{-R C_2}{\theta^2 T} \left[1 + \theta T_1 - 1 - \theta T_1 - \frac{\theta^2 T_1^2}{2} \right]$$

$$= \frac{-R C_2}{\theta^2 T} \left[-\frac{\theta^2 T_1^2}{2} \right]$$

$$= \frac{R C_2 T_1^2}{2T}$$

(8)

$$(iii)\ \textit{Deteriorating Cost} = \frac{\theta C_4}{T} \int_0^{T_1} I(t)\, dt$$

$$= \frac{\theta C_4}{T} \int_0^{T_1} \frac{R}{\theta} \left[e^{\theta(T_1 - t)} - 1 \right] dt$$

$$= \frac{R \theta C_4}{\theta T} \left[\frac{e^{\theta(T_1 - t)}}{-\theta} - t \right]_0^{T_1}$$

$$= \frac{-R C_4}{\theta T} \left[e^{\theta(T_1 - t)} + \theta t \right]_0^{T_1}$$

$$= \frac{-R C_4}{\theta T} \left[1 + \theta T_1 - e^{\theta T_1} \right]$$

$$= \frac{-R C_4}{\theta T} \left[1 + \theta T_1 - 1 - \theta T_1 - \frac{\theta^2 T_1^2}{2} \right]$$

$$= \frac{-R C_4}{\theta T} \left[-\frac{\theta^2 T_1^2}{2} \right]$$

$$= \frac{R \theta C_4 T_1^2}{2T}$$

(9)

$$(iv)\ \textit{Shortage Cost} = \frac{C_3}{T\ L} \int_{T_1}^{T} I(t)\, dt$$

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$$= \frac{C_3}{T\,L}\int_{T_1}^{T} R\,(t - T_1)\,dt$$

$$= \frac{R\,C_3}{T\,L}\left[\frac{t^2}{2} - T_1\,t\right]_{T_1}^{T}$$

$$= \frac{R\,C_3}{T\,L}\left[\frac{T^2}{2} - T_1\,T - \frac{T_1^{\,2}}{2} + T_1^{\,2}\right]$$

$$= \frac{R\,C_3}{T\,L}\left[\frac{T^2 - 2\,T\,T_1 + T_1^{\,2}}{2}\right]$$

$$= \frac{R\,C_3}{2T\,L}\,[T - T_1\,]^2$$

(10)

Total Cost = Ordering Cost + Holding Cost + Shortage Cost + Deteriorating Cost

$$T_C = \frac{C_1}{T} + \frac{R\,C_2\,T_1^{\,2}}{2T} + \frac{R\,C_3}{2T\,L}\,(T - T_1)^2 + \frac{R\,\theta\,C_4\,T_1^{\,2}}{2T}$$

$$T_C = \frac{C_1}{T} + \frac{R}{2T}(C_2 + \theta\,C_4)\,T_1^{\,2} + \frac{R\,C_3}{2T\,L}\,(T - T_1)^2$$

(11)

Differentiating (11) *Partially with respect to* T_1

$$\frac{\partial}{\partial T_1}(T_C) = \frac{2R}{2T}(C_2 + \theta\,C_4)\,T_1 + \frac{2R\,C_3}{2T\,L}\,(T - T_1)(-1) = 0 \quad and \quad \frac{\partial^2}{\partial T_1^{\,2}}(T_C) \rangle\, 0$$

$$(C_2 + \theta\,C_4)\,T_1 = \frac{C_3}{L}\,(T - T_1)$$

$$(C_2 + \theta\,C_4 + \frac{C_3}{L})\,T_1 = \frac{C_3}{L}\,T$$

$$T_1 = \frac{\frac{C_3}{L}\,T}{C_2 + \theta\,C_4 + \frac{C_3}{L}}$$

(12)

Differentiating (11) *Partially with respect to* T

$$\frac{\partial}{\partial T}(T_C) = \frac{-C_1}{T^2} - \frac{R\,(C_2 + \theta\,C_4)\,T_1^{\,2}}{2T^2} + \frac{R\,C_3}{2\,L}\left[\frac{T\,2(T - T_1) - (T - T_1)^2}{T^2}\right] = 0$$

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$$-2C_1 - R(C_2 + \theta C_4)T_1^2 + R\frac{C_3}{L}(2T^2 - 2TT_1 - T^2 + 2TT_1 - T_1^2) = 0$$

$$-2C_1 - R(C_2 + \theta C_4)T_1^2 + R\frac{C_3}{L}(T^2 - T_1^2) = 0$$

$$R\frac{C_3}{L}T^2 = 2C_1 + R\left(C_2 + \theta C_4 + \frac{C_3}{L}\right)T_1^2$$

$$T^2 = \frac{2C_1}{R\frac{C_3}{L}} + \frac{(C_2 + \theta C_4 + \frac{C_3}{L})}{\frac{C_3}{L}}\left[\frac{\frac{C_3^2}{L}T^2}{(C_2 + \theta C_4 + \frac{C_3}{L})^2}\right]$$

$$T^2\left[1 - \frac{\frac{C_3}{L}}{C_2 + \theta C_4 + \frac{C_3}{L}}\right] = \frac{2C_0}{R\frac{C_3}{L}}$$

$$T^2\left[\frac{C_2 + \theta C_4}{C_2 + \theta C_4 + \frac{C_3}{L}}\right] = \frac{2C_0}{R\frac{C_3}{L}}$$

$$T^2 = \left[\frac{2C_1(C_2 + \theta C_4 + \frac{C_3}{L})}{R\frac{C_3}{L}(C_2 + \theta C_4)}\right]$$

$$T = \sqrt{\left[\frac{2C_1(C_2 + \theta C_4 + \frac{C_3}{L})}{R\frac{C_3}{L}(C_2 + \theta C_4)}\right]} \quad and$$

$$Q = \sqrt{\left[\frac{2RC_1(C_2 + \theta C_4 + \frac{C_3}{L})}{\frac{C_3}{L}(C_2 + \theta C_4)}\right]}$$

Numerical Example

Let $R = 5000 \quad C_1 = 100 \quad C_2 = 10 \quad C_3 = 10 \quad C_4 = 150, \quad \theta = 0.01$ *and* $L = 2$

Optimal Solution:

T = 0.1071, T_1 =0.0325 Ordering cost = 933.71, Holding cost = 246.56, Deteriorating Cost = 36.98, Shortage Cost = 649.53, and Total cost = 1866.78

$$Note: \quad when\ \theta = 0\ then\ T = \sqrt{\frac{2C_1(C_2 + \frac{C_3}{L})}{R\ \frac{C_3}{L}\ C_2}}$$

$$and\ Q = \sqrt{\frac{2RC_1(C_2 + \frac{C_3}{L})}{\frac{C_3}{L}\ C_2}}$$

which are standard Inventory Models.

(b) EOQ model for deteriorating Items

Let I(t) be the inventory level of the system at $t\,(0 \le t \le T)$.

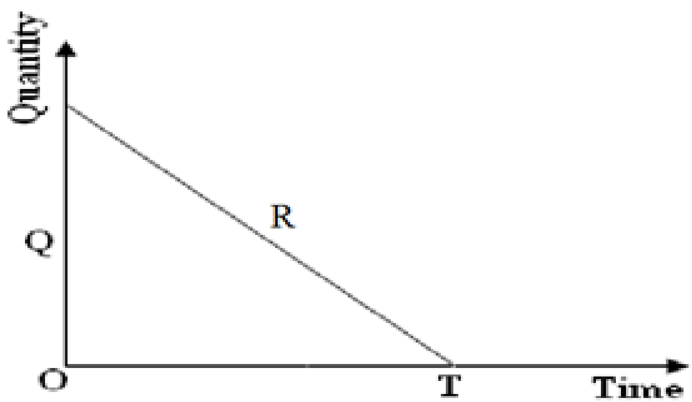

Figure: 2 EOQ model for Deteriorating items

The differential equations of I (t) over the period (0, T) is given by

$$\frac{d}{dt} I(t) + \theta I(t) = -R \quad 0 \le t \le T$$

with the boundary conditions are

$I(\theta) = Q\ and\ I(T) = 0$ (14)

From (14);

$$I(t) = e^{-\theta t} \int_t^T R\ e^{\theta t}\, dt$$

$$= e^{-\theta t} \frac{R}{\theta} \left(e^{\theta t}\right)_t^T$$

$$= \frac{R}{\theta} e^{-\theta t} \left(e^{\theta T} - e^{\theta t}\right)$$

$$I(t) = \frac{R}{\theta} \{e^{\theta (T-t)} - 1\}$$

(15)

Total cost (T_C): Total cost consists of setup cost, deteriorating cost and holding cost.

(i) Setup Cost = $\frac{C_1}{T}$

(16)

(ii) Holding Cost $= \frac{C_2}{T} \int_0^T I(t)\, dt$

$$= \frac{C_2}{T} \int_0^T \frac{R}{\theta} \{e^{\theta (T-t)} - 1\}\, dt$$

$$= \frac{RC_2}{\theta T} \int_0^T \{e^{\theta (T-t)} - 1\}\, dt$$

$$= \frac{RC_2}{\theta T} \left[\frac{e^{\theta (T-t)}}{-\theta} - t\right]_0^T$$

$$= \frac{-RC_2}{\theta^2 T} \left[e^{\theta (T-t)} + \theta t\right]_0^T$$

$$= \frac{-RC_2}{\theta^2 T} \left[e^{\theta (T-T)} + \theta T - e^{\theta T}\right]$$

$$= \frac{-RC_2}{\theta^2 T} \left[1 + \theta T - e^{\theta T}\right]$$

$$= \frac{RC_2}{\theta^2 T} \left[e^{\theta T} - \theta T - 1\right]$$

$$= \frac{RC_2}{\theta^2 T} \left[1 + \theta T + \frac{\theta^2 T^2}{2} - \theta T - 1\right]$$

$$= \frac{RC_2}{\theta^2 T}\left[\frac{\theta^2 T^2}{2}\right]$$

$$= \frac{RC_2 T}{2}$$

(17)

(iii) Deteriorating Cost = $\frac{\theta C_4}{T}\int_0^T I(t)\,dt$

$$= \frac{\theta C_4}{T}\int_0^T \frac{R}{\theta}\{e^{\theta(T-t)} - 1\}\,dt$$

$$= \frac{R\theta C_4}{\theta T}\int_0^T \{e^{\theta(T-t)} - 1\}\,dt$$

$$= \frac{RC_4}{T}\left[\frac{e^{\theta(T-t)}}{-\theta} - t\right]_0^T$$

$$= \frac{-RC_4}{\theta T}\left[e^{\theta(T-t)} + \theta t\right]_0^T$$

$$= \frac{-RC_4}{\theta T}\left[e^{\theta(T-T)} + \theta T - e^{\theta T}\right]$$

$$= \frac{-RC_4}{\theta T}\left[1 + \theta T - e^{\theta T}\right]$$

$$= \frac{RC_4}{\theta T}\left[e^{\theta T} - \theta T - 1\right]$$

$$= \frac{RC_4}{\theta T}\left[1 + \theta T + \frac{\theta^2 T^2}{2} - \theta T - 1\right]$$

$$= \frac{RC_4}{\theta T}\left[\frac{\theta^2 T^2}{2}\right]$$

$$= \frac{R\theta C_4 T}{2}$$

(18)

(iv) Total Cost = Setup Cost + Holding Cost + Deteriorating Cost

$$T_C = \frac{C_1}{T} + \frac{RC_2T}{2} + \frac{R\theta C_4 T}{2}$$

(19)

Differentiating the total Cost (19) with respect to T,

$$\frac{d}{dT}(T_C) = -\frac{C_1}{T^2} + \frac{RC_2}{2} + \frac{R\,\theta C_4}{2} = 0 \qquad and \quad \frac{d^2}{dt^2}(T_C) = \frac{2C_1}{T^3} \rangle\ 0$$

$$\frac{C_1}{T^2} = \frac{R[C_2 + \theta C_4]}{2} \qquad\qquad T^2 = \frac{2C_1}{R[C_2 + \theta C_4]}$$

$$T = \sqrt{\frac{2\,C_1}{R[C_2 + \theta C_4]}}$$

(20)

Numerical Example

$Let\ \ R = 5000\ \ C_1 = 100\ \ C_2 = 10\ \ \ C_4 = 150\ \ \ \theta = 0.01$

Optimal Solution:

T =0.059, Ordering Cost = 1694.92, Holding Cost = 1475 Deteriorating Cost = 221.25 and Total Cost = 3391.17

Which is the stationary inventory model.

$$Note: \quad If\ \ \theta = 0\ then\ \ T = \sqrt{\frac{2\,C_1}{RC_2}} \quad and \quad Q = \sqrt{\frac{2RC_1}{C_2 + \theta C_4}}$$

4. Conclusion:

This proposed model is developed the effect of deterioration items in the EOQ inventory model by two sections, one is the inventory model with shortages when lead time is controllable and another one describes the EOQ model for deteriorating items. We conclude that the lead time is inevitable even when shortages are fulfilled immediately. Infact, the lead time is controllable sense illustrated by suitable numerical examples. The main focus of this mathematical model to optimization of lot size and the time which minimizes the total cost in an inventory cycle.

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