

# MHD free convective boundary layer flow of a viscous fluid at a vertical surface through porous media with non-uniform heat source

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## Abstract

Magnetohydrodynamic unsteady convective boundary layer flow past a vertical stretching sheet through a porous medium in the presence of non-uniform heat source / sink has been investigated in this present paper. The governing nonlinear, non-homogenous and coupled partial differential equations are transformed to ordinary by using similarity transformation. The solutions of these differential equations are obtained by Runge-Kutta fourth order method following shooting technique. Numerical results for skin friction coefficient and Nusselt number are thus obtained and analysed for different pertinent parameters on the flow and heat transfer phenomena. It noteworthy that, magnetic field and porous matrix on boundary layer flow passed a stretching sheet is to decelerate the velocity.

**Keywords:** Heat transfer, Porous medium, Magnetic field, Heat source, Stretching sheet, Runge-Kutta method, Shooting technique.

## 1. Introduction

Studies of heat transfer and flow over moving smooth surfaces impact many technological processes. Examples include aerodynamic extrusion of plastic sheeting and the purification of molten metal to remove non-metallic inclusions. For example, continuous casting, also called strand casting, is the process whereby molten metal is solidified into a

"semi finished" billet, bloom, or slab for subsequent rolling in the finishing mills. In this process, molten metal is poured at a controlled rate into a short vertical metal die or mould that is open at both ends. The melt is cooled rapidly by means of water circulation around the mould, and the solidified product is withdrawn in a continuous length from the bottom of the mould at a rate consistent with the pouring rate. It is employed mainly for copper, brass, bronze and aluminum and increasingly for cast iron and steel.

Sakiadis [1] first examined the boundary-layer flow of a viscous fluid in the context of plate motion in its own plane. Erickson *et al.* [2] and Gupta and Gupta [3] extended this problem to the case in which suction or blowing existed at the moving surface. Crane [4] and Mc Cormack and Crane [5] studied the boundary-layer flow of a Newtonian fluid caused by the stretching of an elastic flat sheet which moves in its own plane with the velocity varying linearly with the distance from a fixed point by the application of a uniform stress. Afzal and Varshney [6], Kuiken [7] and Banks [8] considered the power law stretching of the plate Effects of variable thermal conductivity on MHD flow near a stagnation point on a linearly stretching sheet is studied by Sharma and Singh [9] and study of chemically reactive solute distribution in a steady

### Nomenclature

$a, b, c, B_2$	Constants
$A$	Unsteady parameter
$C_f$	Skin friction
$C_p$	Specific heat at constant pressure
$f$	Dimensionless stream function
$f_w$	Surface mass transfer parameter
$g$	Acceleration due to gravity
$k(T)$	Thermal conductivity
$k_w$	Thermal conductivity at the sheet
$k_\infty$	Thermal conductivity far away from the sheet
$Nu_x$	Nusselt number
$N_r$	Thermal radiation parameter
$P_r$	Prandtl number
$q_w$	Local heat flux at the sheet
$q_r$	Radioactive heat flux
$q'''$	Non uniform heat source/sink
$t$	Time
$\alpha$	space dependent heat source/sink
$\beta$	temperature dependent heat source/sink

$T$	Fluid temperature
$T_w$	Given temperature at the sheet
$T_\infty$	Constant temperature of the fluid far away from the sheet
$x$	Horizontal distance
$y$	Vertical distance
$u$	Velocity in $x$ -direction
$U_w$	Velocity of the stretching surface
$v_w$	Suction/blowing velocity
$v$	Velocity in $y$ -direction
$\alpha_\infty$	Thermal diffusivity
$\varepsilon$	Thermal conductivity parameter
$\eta$	Similarity variable
$\nu$	Kinematic viscosity
$\beta^*$	Thermal expansion coefficient
$\mu$	Dynamic viscosity
$\psi$	Stream function
$\rho$	Density
$\sigma^*$	Stephan-Boltzmann constant
$k^*$	Mean absorption coefficient
$\tau_{xy}$	Shear stress
$\theta$	Dimensionless temperature variable
$\lambda$	Free convection or buoyancy parameter

MHD boundary layer flow over a stretching surface is presented by Uddin *et al.* [10]. All these authors have neglected the importance of porous medium; however the analysis of flow through a porous medium has become the core of several scientific and engineering applications. This type of flow is important to a wide range of technical problems, such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles and blood rheology. Veena *et al.* [11] found the non-similar solution for heat and mass transfer flow in an electrically conducting visco-elastic fluid over a stretching sheet embedded in a porous medium.

The momentum boundary layer for linear stretching of sheet was first studied by Crane [12]. The temperature field in the flow over stretching surface subject to a uniform heat flux was studied by Grubka and Bobba [13], Elbashbeshy [14] considered the case of stretching surface with a variable surface heat flux. Elbashbeshy and Bazid [15] have presented similarity solutions of the boundary layer equations, which describe the unsteady flow and heat transfer over a stretching sheet. Sharidan *et al.* [16] investigated the unsteady flow and heat transfer over a stretching sheet in viscous and incompressible fluid. Nazar *et al.* [17] have studied unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. Y'ur'usoy [18]

investigated unsteady boundary layer flow of power-law fluid on stretching sheet surface.

Chen [19] has studied effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet. Dandapat *et al.* [20] investigated the effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet. Chiam [21] considered the effect of a variable thermal conductivity on the flow and heat transfer from a linearly stretching sheet. Seddeek *et al.* [22] studied the effects of variable viscosity and thermal conductivity on an unsteady two dimensional laminar flow of a viscous incompressible conducting fluid past a semi-infinite vertical porous moving plate taking into account the effect of a magnetic field in the presence of variable suction. Odda and Farhan [23] have considered the effects of variable viscosity and variable thermal conductivity on heat transfer from a stretching sheet. The fluid viscosity and the thermal conductivity are assumed to vary as inverse linear functions of temperature.

Recently, Vajravelu *et al.* [24] have studied the unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. They have considered the fluid to be electrically non-conducting. Apart from that they have not considered the effect of heat source/sink in the heat transfer phenomena. Therefore, the present study is intended to study the flow and heat transfer effect in a conducting fluid in the presence of non-uniform heat source/sink.

## 2. Mathematical Formulation

The unsteady laminar two-dimensional magnetohydrodynamics boundary layer flow of an incompressible fluid past a semi-infinite porous stretching sheet coinciding with the plane  $y = 0$  through porous medium has been considered (Fig.1). The positive  $x$ -axis

extending along the sheet in the upward direction, while the  $y$ -axis is measured normal to the surface of the sheet and is positive in the direction from the sheet to the fluid. We assume that, the fluid and heat flows are steady for time  $t < 0$  and the unsteady flows start at  $t = 0$ ,  $U_w(x, t)$  be the stretching velocity of the sheet along the  $x$ -axis, keeping the origin fixed. The temperature of the sheet  $T_w(x, t)$  is assumed to be a linear function of  $x$ . The thermo-physical properties of the sheet and the ambient fluid are assumed to be constant except density variations and the thermal conductivity which are assumed to vary linearly with temperature. The governing equations for the convective flow and heat transfer of the viscous fluid with the boundary conditions are:

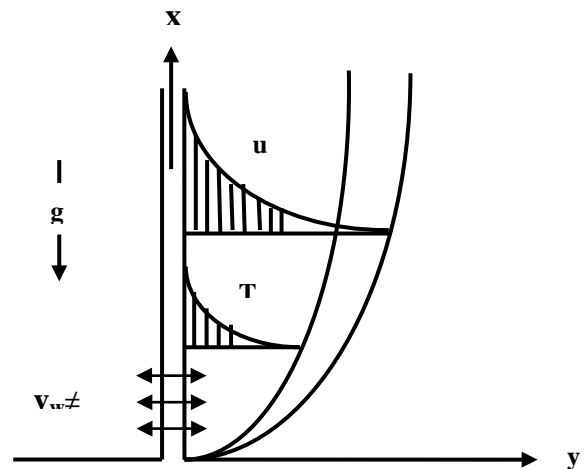


Fig.1 Flow geometry

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \pm g \beta^* (T - T_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu u}{K_p^*} \tag{2}$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k(T) \frac{\partial T}{\partial y} \right) - \frac{\partial q_r}{\partial y} + q''' \quad (3)$$

$$\begin{aligned} u = U_w, v = v_w, T = T_w \quad \text{at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (4)$$

where  $v_w = v_0 / \sqrt{1 - ct}$ . The '±' sign in equation (2) refer to the buoyancy assisting and opposing flow situations, respectively. Here, the thermal conductivity is assumed to vary linearly with temperature [25] as:

$$k(T) = k_\infty \left( 1 + \varepsilon \frac{(T - T_\infty)}{(T_w - T_\infty)} \right) \quad (5)$$

The radiative heat flux can be expressed by using Roseland approximation [26] as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad \text{and the non-uniform heat source/sink, } q''' \quad [27] \quad \text{is modeled as}$$

$$q''' = \frac{\rho k u_0}{x k^*} \left[ A^* (T_w - T_\infty) f' + B^* (T - T_\infty) \right] \quad (6)$$

We assume that the temperature difference within the flow is such that the term  $T^4$  can be expressed as a linear function of temperature. Hence expanding  $T^4$  by Taylor series about  $T_\infty$  and neglecting higher order terms, we obtain

$$T^4 \approx 4T_\infty^3 T - T_\infty^4 \quad (7)$$

Following Ishak *et al.* [28], the stretching velocity is assumed as  $U_w(x, t) = ax/(1 - ct)$  (with  $a \geq 0$  and  $c \geq 0$  where  $ct < 1$ ), and both  $a$  and  $c$  have dimension of  $t^{-1}$ , we have  $a$  as the initial stretching rate  $a/(1 - ct)$  and it is increasing with time. In the context of polymer extrusion, the material properties, in particular the elasticity of the extruded sheet may vary with time even

though the sheet is being stretched by a constant force. With unsteady stretching, however,  $a^{-1}$  becomes the representative time scale of the resulting unsteady boundary layer problem. We assume the surface temperature  $T_w(x, t)$  of the stretching sheet to vary with the distance  $x$  and an inverse square law for its decrease with time in the following form:

$$T_w(x, t) = T_\infty + \frac{bx}{(1 - ct)^2} \quad (8)$$

here ' $b$ ' has dimension temperature/length, with  $b > 0$  and  $b < 0$  corresponding to the assisting and opposing flows, respectively and  $b = 0$  is for the forced convection limit (absence of buoyancy force). These particular forms of  $U_w(x, t)$  and  $T_w(x, t)$  have been chosen in order to obtain a new similarity transformation, which transforms the governing partial differential equations (1)-(3) into a set of coupled ordinary differential equations with variable coefficients. Defining the following dimensionless functions  $f$  and  $\theta$  and the similarity variable  $\eta$  as

$$\eta = \left( \frac{a}{v(1 - ct)} \right)^{\frac{1}{2}} y, \quad \psi = \left( \frac{va}{(1 - ct)} \right)^{\frac{1}{2}} x f(\eta), \quad \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)} \quad (9)$$

Here  $\psi(x, y, t)$  is defined as  $(u, v) = (\partial \psi / \partial y, -\partial \psi / \partial x)$  which identically satisfies the mass conservation Eq.(1). Substituting (9) into (2) and (4) and making use of (2) and (7) we obtain:

$$f''' + ff'' - (f')^2 - A \left( f' + \frac{1}{2} \eta f'' \right) - (M + K_p) f' + \lambda \theta = 0 \quad (10)$$

$$((1 + \varepsilon \theta + Nr)\theta') - Pr(f'\theta - f\theta') - A Pr \left( 2\theta + \frac{1}{2} \eta \theta' \right) + (\alpha^* e^{-\eta} + \beta^* \theta) = 0 \quad (11)$$

$$\begin{aligned}
 f'(\eta) = 1, f(\eta) = f_w, \theta(\eta) = 1 \text{ at } \eta = 0 \\
 f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 1 \quad \text{as } \eta \rightarrow \infty
 \end{aligned}
 \tag{12}$$

where prime denotes differentiation with respect to  $\eta$ ,

$$A = c/a, \quad Nr = \frac{16\sigma^* T_\infty^3}{3k_\infty k^*}, \quad P_r = \frac{\nu}{\alpha_\infty}, \quad \alpha_\infty = \frac{k_\infty}{\rho C_p},$$

$$f_w = -\frac{\nu_w}{\sqrt{\nu a}}, \quad \alpha^* = \frac{k_\infty A^*}{k(T)C_p}, \quad \beta = \frac{k_\infty B^*}{k(T)C_p}. \quad \text{The}$$

suction / injection parameter is used to control the strength and direction of the normal flow at the boundary. Further,  $\lambda = g\beta^* b/a^2$  is a dimensionless constant where  $\lambda > 0$ , assisting and  $\lambda < 0$ , opposing flows,  $\lambda = 0$ , forced convection flow respectively. It is to note that, the absence of the unsteady parameter, magnetic parameter, porosity parameter, variable thermal conductivity parameter, impermeability of the boundary wall, space and time dependent heat source/sink, equations (10) and (11) reduce to those of Vajravelu [29], while in the absence of the free convection parameter, magnetic parameter, porosity parameter and space and time dependent heat source/sink, the equations (10) and (11) reduce to those of Grubka and Bobba [13]. Further, when the thermal radiation and thermal conductivity parameter are absent the equations reduce to those of Ishak *et al.* [28].

The important characteristics of the flow are Skin-friction coefficient and the Nusselt number, respectively defined as:

$$C_f = \frac{\tau_w}{\rho U_w^2 / 2}, \quad Nu_x = \frac{xq_w}{k_\infty (T_w - T_\infty)}$$

where the skin friction  $\tau_w$  and the heat transfer  $q_w$  from the sheet are given by:

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}; \quad q_w = -k_\infty \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

### 3. Results and discussion

The non-linear self-similar governing equations (2) & (3) represent a two point BVP with boundary conditions (4) are solved by the R-K method with shooting technique. To convert BVP into IVP, we assign some guess values to unknown initial condition and initiate the process of computation. The step size  $\Delta\eta = 0.001$  is taken during the computation. The process is repeated till the accuracy of  $10^{-5}$  is attained. For conformity the result, we have compared our result with Ishak *et al.* [28], Vajravelu [29] and Grubka and Bobba [13] as particular cases in respect of skin friction and rate of heat transfer at the surface and observed that both the results are in good agreement.

Another aspect of the numerical computation in the present study is addition of additional parameter i.e. magnetic and porosity in momentum equation and source/sink parameter in the energy equation. The numerical results are obtained for several values of the pertinent physical parameters for three cases, suction ( $f_w < 0$ ), injection ( $f_w > 0$ ) and impermeable ( $f_w = 0$ ) stretching sheet. The solution is obtained both for steady case ( $A = 0$ ) and unsteady case ( $A \neq 0$ ).

Figs. 2(a)-(c) exhibit the effects of magnetic parameter ( $M$ ) and porous matrix ( $K_p$ ) in the velocity profile for three different cases such as suction ( $f_w < 0$ ), impermeable ( $f_w = 0$ ) and injection ( $f_w > 0$ ) respectively. From these figs., it is clear that the velocity profile decreases monotonically to zero as  $\eta$  increases from the boundary. In case of steady i.e. when the unsteady parameter ( $A = 0$ ) or in case of unsteady the profiles are initiated from the unity satisfying the condition on

bounding surface and then tends to zero asymptotically. It is interesting to note that in the absence of both  $M$  and  $K_p$  ( $M = 0$  and  $K_p = 0$ ) the velocity attain its maximum value in the velocity boundary layer and in the presence of  $M$  and  $K_p$  ( $M = 1$  and  $K_p = 1$ ) velocity gradually decreases. Thus, it is remarked that the effect of magnetic field and porous matrix on boundary layer flow passed a stretching sheet is to decelerate the velocity. There is no remarkable change occurs to the velocity profile in these three cases of suction, impermeable and injection.

The variation of free convection parameter  $\lambda$  in the absence of  $M$  and  $K_p$  ( $M = 0$  and  $K_p = 0$ ) and presence of  $M$  and  $K_p$  ( $M = 1$  and  $K_p = 1$ ) is shown from figs.3(a) -(c) for the three distinct cases such as suction ( $f_w < 0$ ), impermeable ( $f_w = 0$ ) and injection ( $f_w > 0$ ) respectively. All the cases are exhibited in the absence of uniform heat source/sink coefficient. Free convective parameter  $\lambda > 0$  stands for heating of the fluid or cooling of the surface i.e. assisting flow  $\lambda < 0$  means cooling of the fluid or heating of the surface i.e. opposing flow and  $\lambda = 0$ , the absence of free convection current i.e. forced convective flow. It is clear to note that the increase in the values of  $\lambda$  can lead to an increase in the velocity profile in the absence or presence of  $M$  and  $K_p$ . But the reverse trend is observed for the increasing value of the unsteady parameter from '0' to '1', i.e. the effect of increasing value of the unsteady parameter  $A$  is to decrease the velocity profile in each of the case of figs. 3(a)-(c).

Figs. 4(a)-(c) illustrates the effect of free convective parameter  $\lambda$  on the temperature profile in the presence of uniform heat source/sink parameter ( $\alpha = \beta = 0.1$ ). The analysis is made for both the presence and absence of  $M$  and  $K_p$  for  $f_w < 0$ ,  $f_w = 0$  and  $f_w > 0$  respectively.

From figs 3(a)-(c) it is observed that for different values of  $f_w$ , an increase in free convective parameter  $\lambda$  results in a decrease in the thermal boundary layer thickness. In the steady case ( $A = 0$ ) the increase in both  $M$  and  $K_p$  the thermal boundary layer. The increasing value of unsteady parameter  $A$  from '0' to '1' enhance the temperature profile in both absence or presence of  $M$  and  $K_p$  shown in figs. 5(a)-(c) in the presence of uniform heat generation/absorption coefficients.

Figs. 6(a)-(c) depicts the effect of prandtl number on the temperature distribution in the absence or presence of  $M$  and  $K_p$ . In these cases the value of the uniform heat generation/ absorption is taken to be ( $\alpha = \beta = 0.1$ ) with  $f_w < 0$ ,  $f_w = 0$  and  $f_w > 0$  respectively. The higher Prandtl number fluid causes a fall in temperature due to low thermal diffusivity i.e. when  $P_r$  increases, the thermal boundary layer thickness become thinner and thinner where as the permeability parameter and magnetic parameter enhance the profile at all points. Thus, it may be inferred that due to resistance offered by the porous matrix and electromagnetic force temperature increases.

Figs. 7(a)-(c) and figs. 8(a)-(c) exhibit the effect of temperature dependent heat source/sink for

( $M = 0$  and  $K_p = 0$ ) the absence of  $M$  and  $K_p$  as well as ( $M = 1$  and  $K_p = 1$ ) the presence of  $M$  and  $K_p$ . It is interesting to note that temperature increases with space dependent heat source/sink for  $K_p = 0$  and  $K_p = 1$  in the absence or presence of magnetic parameter. Moreover, permeability  $K_p$  also increases the temperature in respect of source/sink. It is also pointed out that temperature dependent heat source/sink has more significant effect in increasing the temperature than space dependent heat source/sink.

Therefore, we concluded that in steady case, when the unsteady parameter ( $A = 0$ ) throughout the problem for different pertinent parameter in the velocity, temperature profile the result is in good agreement with that of Ishak [28], Grubka and Bobba [13]. Further, for unsteady case, when the unsteady parameter ( $A \neq 0$ ) ( $A = 1$ ). Our result is in good agreement with the result of Vajravelu [29].

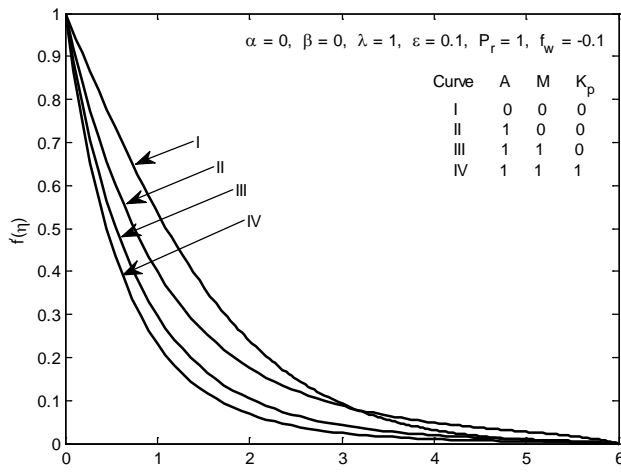


Fig.2(a) Variation of A, M and  $K_p$  on velocity profile

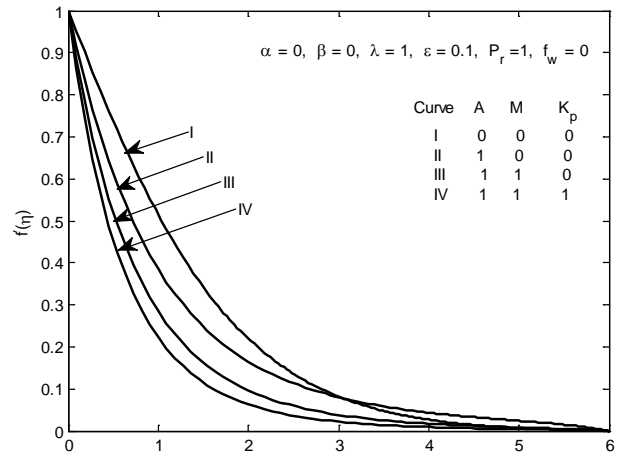


Fig.2(b) Variation of A, M and  $K_p$  on velocity profile

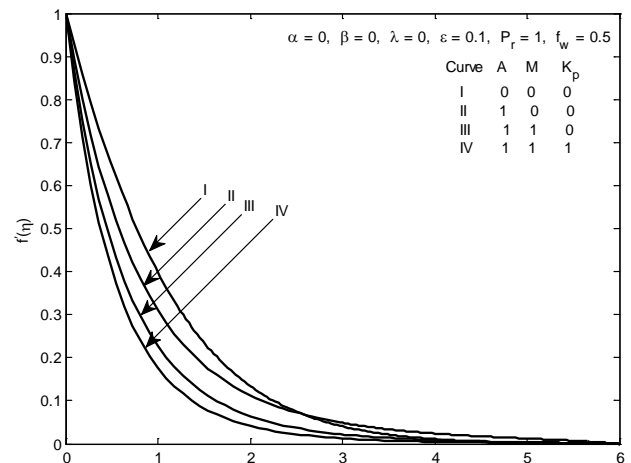


Fig.2(c) Variation of A, M and  $K_p$  on velocity profile

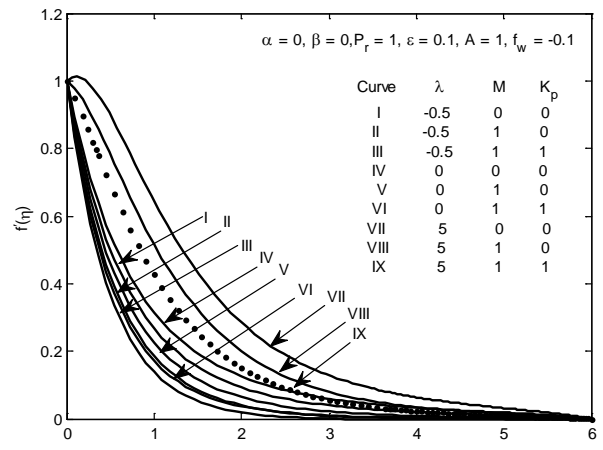


Fig.3(a) Variation of  $\lambda$ , M and  $K_p$  on velocity profile

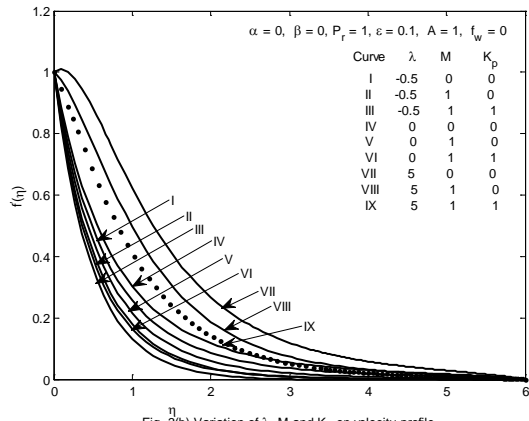


Fig. 3(b) Variation of  $\lambda, M$  and  $K_p$  on velocity profile

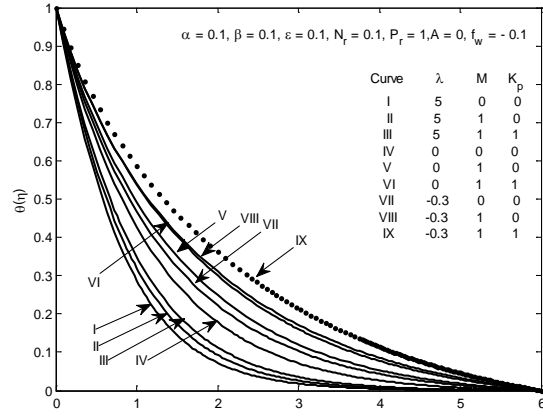


Fig.4(a) Variation of  $\lambda$  on temperature profile

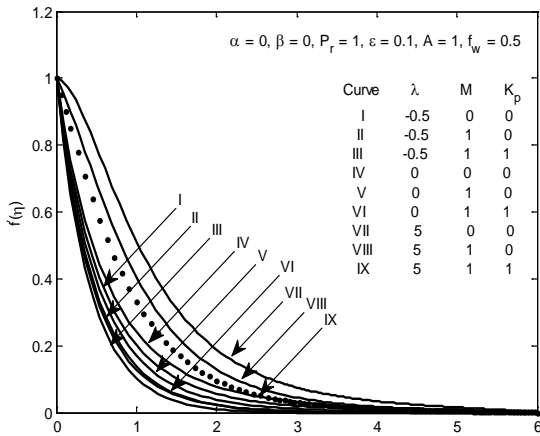


Fig.3(c) Variation of  $\lambda, M$  and  $K_p$  on velocity profile

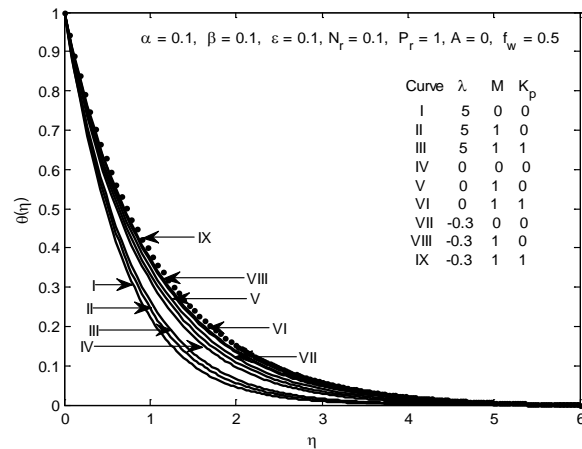


Fig. 4(c) Variation of  $\lambda$  on temperature profile

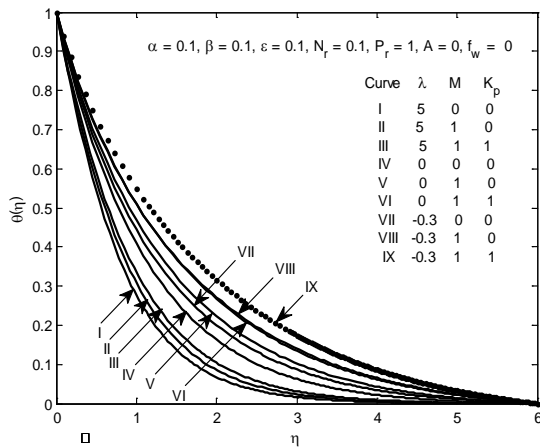


Fig.4(b) Variation of  $\lambda$  on temperature profile

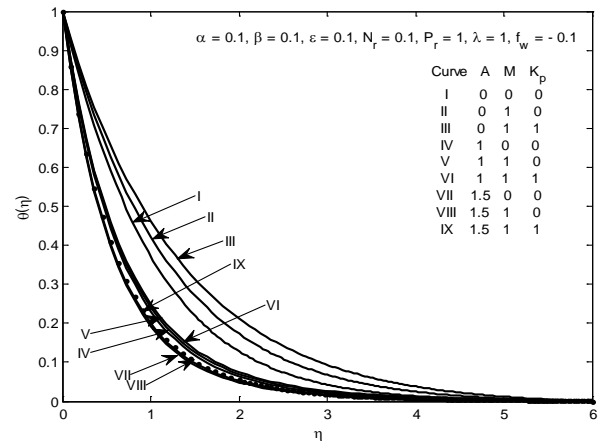


Fig.5(a) Variation of  $A$  on temperature profile



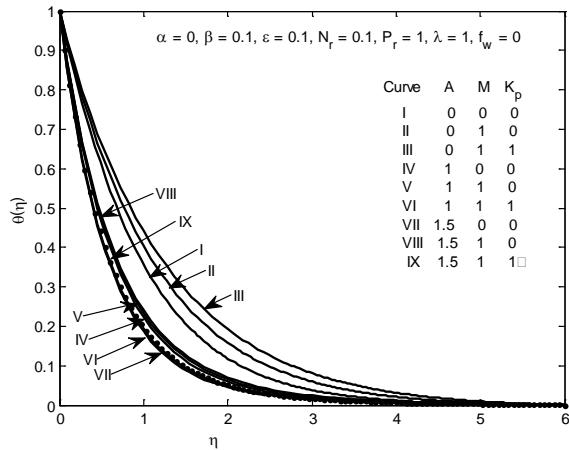


Fig.5(b) Variation of A on temperature profile

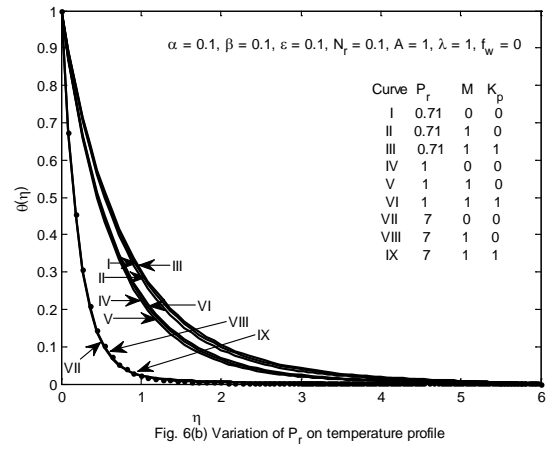


Fig. 6(b) Variation of P<sub>r</sub> on temperature profile

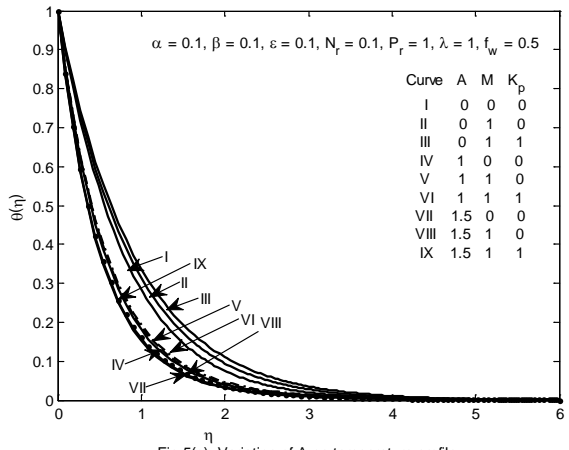


Fig.5(c) Variation of A on temperature profile

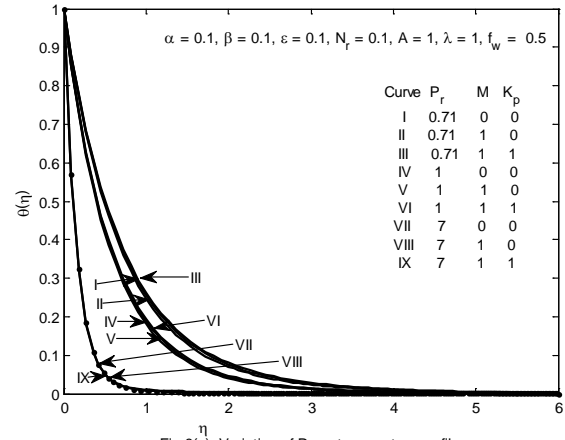


Fig.6(c) Variation of P<sub>r</sub> on temperature profile

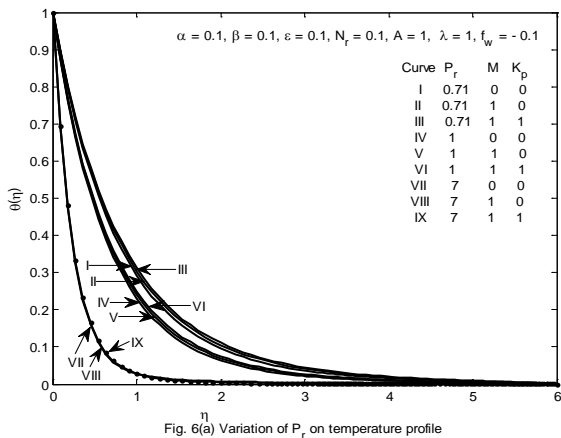


Fig. 6(a) Variation of P<sub>r</sub> on temperature profile

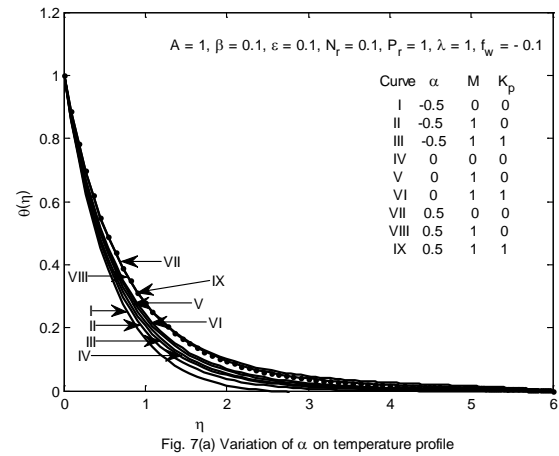


Fig. 7(a) Variation of alpha on temperature profile

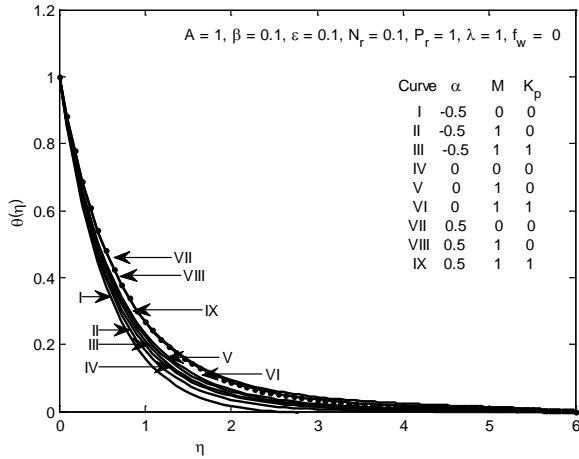


Fig. 7 (b) Variation of  $\alpha$  on temperature profile

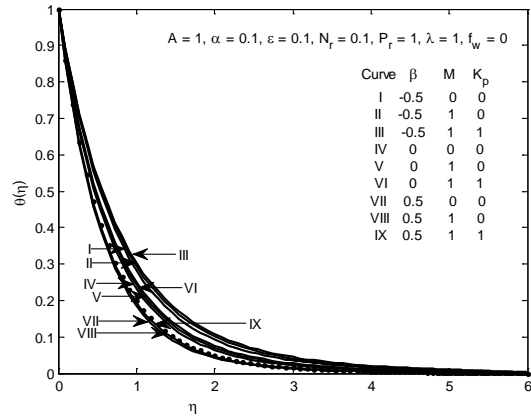


Fig.8(b) Variation of  $\beta$  on temperature profile

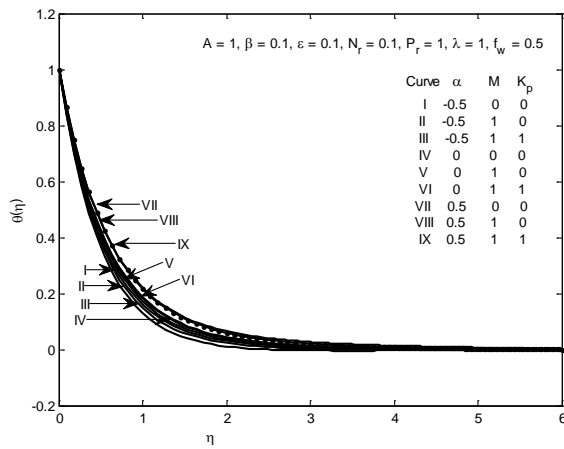


Fig.7(c) Variation of  $\alpha$  on temperature profile

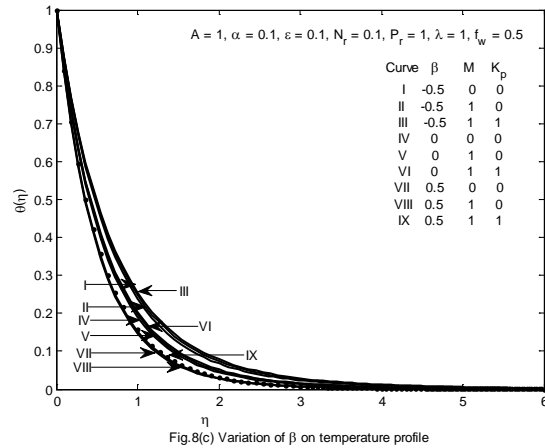


Fig.8(c) Variation of  $\beta$  on temperature profile

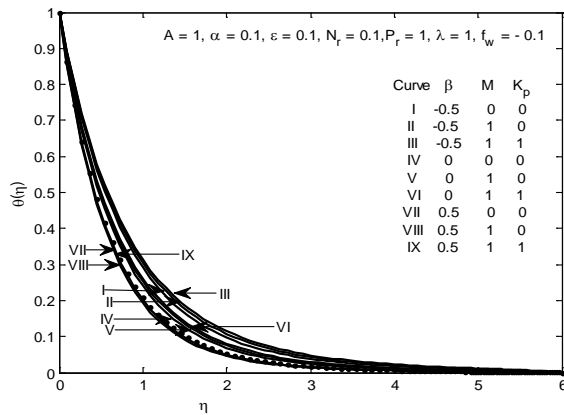


Fig 8(a) Variation of  $\beta$  on temperature profile

Table: Skin friction and wall temperature gradient

$\lambda$		$f_w = -0.1$		$f_w = 0$		$f_w = 0.5$	
		$\varepsilon = 0.1, A = 0, P_r = 1, N_r = 0.1, M = 0, K_p = 0, \alpha = 0, \beta = 0$					
		$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0		-0.95179	0.844375	-1.00048	0.883688	-1.28098	1.110334
1		-0.49306	0.933988	-0.53825	0.972562	-0.81159	1.188729
5		0.947077	1.094528	0.910492	1.133057	0.668019	1.34299
$M$	$K_p$	$A = 1, \lambda = 1, \varepsilon = 0.1, P_r = 1, N_r = 0.1, \alpha = 0, \beta = 0$					
0	0	-0.94224	1.49127	-0.98517	1.52971	-1.23194	1.737705
1	0	-1.30151	1.466116	-1.34664	1.504513	-1.59846	1.713352
1	1	-1.59927	1.44738	-1.64551	1.485799	-1.89915	1.695419
$M$	$K_p$	$A = 1, \lambda = 1, \varepsilon = 0.1, P_r = 1, N_r = 0.1, \alpha = 0.1, \beta = 0.1$					
0	0	-0.94174	1.498008	-0.98476	1.537504	-1.23218	1.751456
1	0	-1.30218	1.476353	-1.34744	1.515895	-1.59997	1.730985
1	1	-1.60035	1.460063	-1.64671	1.499675	-1.90101	1.715698
$N_r$	$M$	$A = 1, K_p = 1, \lambda = 1, \varepsilon = 0.1, P_r = 1, \alpha = 0.1, \beta = 0.1$					
0	1	-1.60595	1.526389	-1.65269	1.569764	-1.90904	1.807161
0.1		-1.60035	1.460063	-1.64671	1.499675	-1.90101	1.715698
2		-1.5438	0.893825	-1.58645	0.908392	-1.81987	0.985415
$\varepsilon$	$M$	$A = 1, K_p = 1, \lambda = 1, N_r = 0.1, P_r = 1, \alpha = 0.1, \beta = 0.1$					
0	1	-1.60539	1.548095	-1.65199	1.591841	-1.90744	1.830574
0.1		-1.60035	1.460063	-1.64671	1.499675	-1.90101	1.715698
0.5		-1.56652	1.018806	-1.61109	1.03966	-1.85608	1.15255
$\alpha$	$M$	$A = 1, K_p = 1, \lambda = 1, N_r = 0.1, P_r = 1, \varepsilon = 0.1, \beta = 0.1$					
-0.5	1	-1.61512	1.627635	-1.66164	1.667368	-1.9162	1.88176
0		-1.60286	1.488247	-1.64925	1.527882	-1.90359	1.743636
0.5		-1.59012	1.346249	-1.63636	1.38575	-1.89044	1.602835
$\beta$	$M$	$A = 1, K_p = 1, \lambda = 1, N_r = 0.1, P_r = 1, \varepsilon = 0.1, \alpha = 0.1$					
-0.5	1	-1.57938	1.235516	-1.62477	1.26795	-1.87428	1.447001
0		-1.59676	1.419563	-1.64297	1.457945	-1.89655	1.667755

#### 4. Conclusion

The flow and heat transfer effect in a conducting fluid in the presence of non-uniform heat source/sink has been discussed in this chapter. The nonlinear non-homogeneous partial differential equations are transformed into a set of linear ordinary differential equations and then solved by using Runge-kutta method along with shooting technique. The behavior of pertinent physical parameters are obtained through graphs and table and then discussed. The conclusions are listed below

- Magnetic field and porous matrix on boundary layer flow is to decelerate the velocity profile.
- Increase in the values of  $\lambda$  lead to an increase in the velocity profile in the absence / presence of  $M$  and  $K_p$ .
- The higher value of Prandtl number causes a fall in temperature due to low thermal diffusivity.
- Temperature profile increases with the increase in space dependent heat source/sink.
- Skin friction coefficient and rate of heat transfer increase with increase in  $\lambda$ .
- The rate of heat transfer decreases in the absence/presence of space and time dependent source/sink.
- Increase in thermal radiation and thermal conduction parameter in the presence of magnetic parameter the rate of heat transfer decreases.

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