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# An algorithm for a discretized confined flow problem near Lamu Basin Reservoir, Kenya, for a Study of Petroleum Reservoir Characterization in Fully Confined Aquifer

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**Abstract**

***We present a mathematical algorithm for a discretized confined flow problem near Lamu basin reservoir for petroleum reservoir characterization, of the Kenyan coast using the equations governing single phase fluid flow.***

**Keywords**:DiscretizedFlow Algorithm(DFA), integrated finite difference method (IFDM), confined particle flow.

## 1. Introduction

The confined flow near Lamu basinreservoir is pressure dependent, decreasing with decreasing pressure in time. The partial differential equation was discretised using integrated finite difference method (IFDM) and approximated by an algebraic equation which was valid in a small sub-region for a particular time. A number of such equations were written for all the sub-regions into which the entire domain of study was suitably divided. The discretised mathematical problem was translated into an algorithm for computer program using finite difference numerical scheme,

## 2 A Simple Finite-Volume Method

Finite-volume method as a conservative finite-difference scheme treats the grid cells as control volumes. To derive a set of finite-volume mass balance equations we denoted by $\Omega_i$ a grid cell in $\Omega$ and consider the following integral over $\Omega_i$

$$\int_{\Omega_i}\left(\frac{q}{\rho}-\nabla.v\right)dx=0 \quad (1)$$

By the divergence theorem, taking $v$ as sufficiently smooth, equation (4.1) becomes a mass-balance equation:

$$\int_{\partial\Omega_i}V.n\,dv=\int_{\Omega_i}\frac{q}{\rho}dx \quad (2)$$

Here $n$ denotes the outward-pointing unit normal on$\Omega_i$. Corresponding finite-volume methods were now obtained by approximating the pressure $p$.

## 3Finite Difference Approximation of the Incompressible Fluid Flow Equation

The single phase flow model equation in the 3 dimesion with $i,j,k$ components is, written as,

$$\frac{\partial}{\partial x}\left[\frac{A_xk_x}{\mu B}\left(\frac{\partial p}{\partial x}-\rho g\frac{\partial Z}{\partial x}\right)\right]\Delta x+\frac{\partial}{\partial y}\left[\frac{A_yk_y}{\mu B}\left(\frac{\partial p}{\partial y}-\rho g\frac{\partial Z}{\partial y}\right)\right]\Delta y+\frac{\partial}{\partial z}\left[\frac{A_zk_z}{\mu B}\left(\frac{\partial p}{\partial z}-\rho g\frac{\partial Z}{\partial z}\right)\right]\Delta z+\frac{q}{\rho}=\frac{\partial\phi}{\partial t} \quad (3)$$

where $\Delta x,\Delta y$ and $\Delta z$ denote the difference along the $x,y$ and $z$ axes respectively and $A_x$ represents the area element normal to the $x-$ axis, and similarly to $A_y$ and $A_z$, while $k_x,k_y$ and $k_z$ are the $x,y$ and $z$ direction permiabilities and $B$ is the volume element.

For incompressible fluid flow in a heterogeneous and anisotropic formulation, density $\rho$ is constant, viscosity $\mu$ and bulk volume $B$ are constant, (with $B=1$ for isothermal conditions).

The flow equation across a unit area element $A_x=A_y=A_z=1$, equation (3) reduced to

$$\frac{\partial}{\partial x}\left[\frac{k_x}{\mu}\left(\frac{\partial p}{\partial x}-\rho g\frac{\partial Z}{\partial x}\right)\right]\Delta x+\frac{\partial}{\partial y}\left[\frac{k_y}{\mu}\left(\frac{\partial p}{\partial y}-\rho g\frac{\partial Z}{\partial y}\right)\right]\Delta y+\frac{\partial}{\partial z}\left[\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z}-\rho g\frac{\partial Z}{\partial z}\right)\right]\Delta z+\frac{q}{\rho}=\frac{\partial\phi}{\partial t} \quad (4)$$

Also, the porous medium is incompressible, which implies that porosity $\phi$ is constant. These assumptions imply that the right hand side of our equation contains only constants and the temporal derivative vanished. Setting the partial derivatives with respect to time equal to zero implied that steady state conditions exist.

The resulting equation is

$$\frac{\partial}{\partial x}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial x}-\rho g\frac{\partial Z}{\partial x}\right)\right]\Delta x+\frac{\partial}{\partial y}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial y}-\rho g\frac{\partial Z}{\partial y}\right)\right]\Delta y+\frac{\partial}{\partial z}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial z}-\rho g\frac{\partial Z}{\partial z}\right)\right]\Delta z+\frac{q}{\rho}=0 \quad (5)$$

where$k_x = k_y = k_z = k$ (constant), for a horizontal reservoir with negligible fluid gravity, (as evidenced in the Lamu basin). Thus Equation (5) was written for every unit volume element as

$$\frac{\partial}{\partial x}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial x}\right)\right]\Delta x+\frac{\partial}{\partial y}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial y}\right)\right]\Delta y+\frac{\partial}{\partial z}\left[\frac{k}{\mu}\left(\frac{\partial p}{\partial z}\right)\right]\Delta z+\frac{q}{\rho}=0 \quad (6)$$

in the process of finding a finite difference solution we replaced Equation (6) with its finite difference approximation, expressed as

$$\left(\frac{k}{\mu\Delta x}\right)_{i+\frac{1}{2},j,k}(p_{i+1,j,k}-p_{i,j,k})-\left(\frac{k}{\mu\Delta x}\right)_{i-\frac{1}{2},j,k}(p_{i,j,k}-p_{i-1,j,k})+\left(\frac{k}{\mu\Delta y}\right)_{i,j+\frac{1}{2},k}(p_{i,j+1,k}-p_{i,j,k})-\left(\frac{k}{\mu\Delta y}\right)_{i-\frac{1}{2},j,k}(p_{i,j,k}-p_{i,j-1,k})+\left(\frac{k}{\mu\Delta z}\right)_{i,j,k+\frac{1}{2}}(p_{i,j,k+1}-p_{i,j,k})-\left(\frac{k}{\mu\Delta z}\right)_{i,j,k-\frac{1}{2}}(p_{i,j,k}-p_{i,j,k-1})+q_{i,j,k}=0 \quad (7)$$

The coefficients $\left(\frac{k}{\mu\Delta x}\right)_{i+\frac{1}{2},j,k}$, $\left(\frac{k}{\mu\Delta y}\right)_{i,j+\frac{1}{2},k}$ and $\left(\frac{k}{\mu\Delta z}\right)_{i,j,k+\frac{1}{2}}$ are the transmissibilities at the $x, y$ and $z$ directions denoted by, $T_{x|i+\frac{1}{2},j,k}$, $T_{x|i,j+\frac{1}{2},k}$ and $T_{x|i,j,k+\frac{1}{2}}$ which are required at the grid block boundaries.

The calculation of these transmissibilities was calculated individually for each boundary since the dimensions, porosity, thickness, permiabilities and overrun of each block may differ. The technique for obtaining averages that we employed is the use of harmonic mean $\overline{H}$ defined as

$$\frac{1}{\overline{H}}=\frac{1}{N}\left(\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_N}\right) \quad (8)$$

Using the notations above, and assuming homogeinity the entire transmissibility term in the $x$ −direction was defined as $T_x=\frac{k}{\mu\Delta x}$ and the same applied to $y$ and $z$ directions.

The transmissibilities at the cell boundaries were defined as follows.

Let

$$T_{x|i-\frac{1}{2},j,k}=W_{i,j,k}=\left(\frac{k_x}{\mu\Delta x}\right)_{i-\frac{1}{2},j,k} \quad (9)$$

$$T_{x|i+\frac{1}{2},j,k}=E_{i,j,k}=\left(\frac{k_x}{\mu\Delta x}\right)_{i+\frac{1}{2},j,k} \quad (10)$$

$$T_{y|i,j-\frac{1}{2},k}=S_{i,j,k}=\left(\frac{k_y}{\mu\Delta y}\right)_{i,j-\frac{1}{2},k} \quad (11)$$

$$T_{y|i,j+\frac{1}{2},k}=N_{i,j,k}=\left(\frac{k_y}{\mu\Delta y}\right)_{i,j+\frac{1}{2},k} \quad (12)$$

$$T_{z|i,j,k-\frac{1}{2}}=B_{i,j,k}=\left(\frac{k_z}{\mu\Delta z}\right)_{i,j,k-\frac{1}{2}} \quad (13)$$

$$T_{z|i,j,k+\frac{1}{2}}=M_{i,j,k}=\left(\frac{z}{\mu\Delta z}\right)_{i,j,k+\frac{1}{2}} \quad (14)$$

and the sum of transmissibilities and the production rate or source term defined as,

$$C_{i,j,k}=-(M_{i,j,k}+B_{i,j,k}+N_{i,j,k}+S_{i,j,k}+E_{i,j,k}+W_{i,j,k}) \quad (15)$$

and

$$Q_{i,j,k}=-q_{i,j,k} \quad (16)$$

The finite difference system was given by

$$M_{i,j,k}p_{i,j,k+1}+B_{i,j,k}p_{i,j,k-1}+N_{i,j,k}p_{i,j+1,k}+S_{i,j,k}p_{i,j-1,k}+E_{i,j,k}p_{i+1,j,k}+W_{i,j,k}p_{i-1,j,k}+C_{i,j,k}p_{i,j,k} \quad (17)$$

The coefficients defined describe the interaction of the central block to the neighbouring 6 blocks surrounding the central block. The transmissibility rates specified apply to both cells with wells and those without wells. The wells containing pressure specified wells have the source term $q_{i,j,k}$ unknown.

## 4 Boundary Conditions

two dimensional incompressible flow required that we set the transmissibilities parallel to the $z$ −axis namely, $M_{i,j,k}p_{i,j,k+1}$ and $B_{i,j,k}p_{i,j,k-1}$,$T_z$ equal to zero and drop the third subscript $k$ to obtain;

$$N_{i,j}p_{i,j+1}+S_{i,j}p_{i,j-1}+E_{i,j}p_{i+1,j}+W_{i,j}p_{i-1,j}+C_{i,j}p_{i,j}=Q_{i,j} \quad (18)$$

In the case of Lamu basin, we illustrate a 2-dimesnsional flow of petroleum in a reservoir that is confined such that, apart from the top and bottom boundaries on a 3-dimensional grid block, the top and left boundaries on a 2-dimensional flow (representing the main land) was a no flow zone. These are regions with non-porous rocks. We therefore set the coefficient $W = 0$ for the grid blocks located at $i = 1$ on the left boundary. Note that the value of $C_{i,j,k}$ in equation (15) must also be adjusted to have $W_{1,j} = 0$, for $j = 1, 2, 3, 4$. Similarly, we set $N_{i,4} = 0$ and for $i = 1, 2, 3, 4$. For the lower and right boundaries, we used the reflection grid-blocks. In summary, the transmissibility coefficient for each gridblock yielded the following matrix.

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$$\begin{bmatrix} CW_{1,1} & E_{1,1} & 0 & 0 & N_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_{2,1} & C_{2,1} & E_{2,1} & 0 & 0 & N_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & W_{3,1} & C_{3,1} & E_{3,1} & 0 & 0 & N_{3,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & W_{4,1} & C_{4,1} & 0 & 0 & 0 & N_{4,1} & 0 & 0 & 0 & 0 \\ S_{1,2} & 0 & 0 & 0 & CW_{1,2} & E_{1,2} & 0 & 0 & N_{1,2} & 0 & 0 & 0 \\ 0 & S_{2,2} & 0 & 0 & W_{2,2} & C_{2,2} & E_{2,2} & 0 & 0 & N_{2,2} & 0 & 0 \\ 0 & 0 & S_{3,2} & 0 & 0 & W_{3,2} & C_{3,2} & E_{3,2} & 0 & 0 & N_{3,2} & 0 \\ 0 & 0 & 0 & S_{4,2} & 0 & 0 & W_{4,2} & C_{4,2} & 0 & 0 & 0 & N_{4,2} \\ 0 & 0 & 0 & 0 & S_{1,3} & 0 & 0 & 0 & CW_{1,3} & E_{1,3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{2,3} & 0 & 0 & W_{2,3} & C_{2,3} & E_{2,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{3,3} & 0 & 0 & W_{3,3} & C_{3,3} & E_{3,3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{4,3} & 0 & 0 & W_{4,3} & C_{4,3} \end{bmatrix} \begin{pmatrix} p_{1,1} \\ p_{2,1} \\ p_{3,1} \\ p_{4,1} \\ p_{1,2} \\ p_{2,2} \\ p_{3,2} \\ p_{4,3} \\ p_{1,3} \\ p_{2,3} \\ p_{3,3} \\ p_{4,3} \end{pmatrix} = \begin{pmatrix} g_x W_{1,2}\Delta x \\ 0 \\ 0 \\ 0 \\ g_x W_{1,2}\Delta x \\ 0 \\ 0 \\ 0 \\ g_x W_{1,3}\Delta x \\ 0 \\ -q \\ 0 \end{pmatrix} \quad (19)$$

where $CW_{ij} = C_{ij} + W_{ij}$ for $i, j = 1,2, \ldots$ for

## 5 Treatment of Wells, Sources/Sinks

The ultimate goal of reservoir in our study was to forecast well flow rates and/or flowing bottom hole pressure for the purpose of accurately estimating the pressure and saturation distribution for the purpose of effective extraction of hydrocarbons. Wells are considered to be internal boundaries of the system. As such, a boundary condition was specified at the well to develop a properly posed problem. This internal boundary condition was in the form of flowing sand face pressure specification (Dirichlet type boundary condition) or Neumann-type boundary condition.

In our case, for the purpose of numerical simulation, we allowed no more than one well to penetrate a grid block and also created at least one or two empty grid blocks between wells to model pressure interface effects adequately. In this study, we also placed the position of the well so that it is approximately at the centre of the grid block.

The internal boundary conditions created by the wells in various grid blocks were taken care of by entering the specified flow rate on the right side of the finite difference equation representing the row or column containing the well.

For example, suppose the well is located at grid block with coordinates $(i, j) = (3, 3)$ the finite difference equation is obtained from equation (18) or from the matrix in (19) to be

$$S_{3,3}p_{3,2} + W_{3,3}p_{2,3} + C_{3,3}p_{3,3} + E_{3,3}p_{4,3} = -q \quad (20)$$

Notice that the value of $N_{3,3}p_{3,4} = 0$ because of Neumann boundary condition in the upper boundary.

We now solved the matrix equation (19) using Crank – Nicholson iterative numerical scheme with Neumann boundary conditions. We define the finite difference representation from the definitions.

$$\frac{\partial u}{\partial t} = \frac{u_{i,\ j+1} - u_{i,\ j}}{\Delta t} \quad (21)$$

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,\ j} - \ u_{i,\ j}}{\Delta x} \quad (22)$$

and

$$\frac{\partial^2 u}{\partial x^2} = \frac{(u_{i+1,\ j} - 2u_{i,\ j} + u_{i-1,j})}{(\Delta x)^2} \quad (23)$$

The Crank Nicholson model was written as,

$$-\alpha u_{i-1,j+1} + (2 + 2\alpha)u_{i,j+1} - \alpha u_{i+1,j+1} = \alpha u_{i-1,j} + (2 - 2\alpha)u_{i,j} + \alpha u_{i+1,j} \quad (24)$$

or in matrix form as

$$\begin{bmatrix} -\alpha & 2+2\alpha & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & 2+2\alpha & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 2+2\alpha & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & 2+2\alpha & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha & 2+2\alpha & -\alpha \end{bmatrix} \begin{pmatrix} u_{0,j+1} \\ u_{1,j+1} \\ u_{2,j+1} \\ . \\ . \\ . \\ u_{N-1,j+1} \\ u_{N,j+1} \end{pmatrix} = \begin{bmatrix} \alpha & 2-2\alpha & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 2-2\alpha & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 2-2\alpha & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 2-2\alpha & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 2-2\alpha & \alpha \end{bmatrix} \begin{pmatrix} u_{0,j} \\ u_{1,j} \\ u_{2,j} \\ . \\ . \\ . \\ u_{N-1,j} \\ u_{N,j} \end{pmatrix} \quad (25)$$

Using Dirichlet boundary conditions with the values of $u_{0,j+1}$ and $u_{N,j+1}$ being known, our system of equation (25) became;

$$\begin{bmatrix} 2+2\alpha & -\alpha & 0 & 0 & 0 & 0 \\ -\alpha & 2+2\alpha & -\alpha & 0 & 0 & 0 \\ 0 & -\alpha & 2+2\alpha & -\alpha & 0 & 0 \\ 0 & 0 & . & . & . & 0 \\ 0 & 0 & 0 & -\alpha & 2+2\alpha & -\alpha \\ 0 & 0 & 0 & 0 & -\alpha & 2+2\alpha \end{bmatrix} \begin{pmatrix} u_{1,j+1} \\ u_{2,j+1} \\ . \\ . \\ . \\ u_{N-1,j+1} \end{pmatrix} + \begin{pmatrix} -\alpha u_{0,j+1} \\ 0 \\ . \\ . \\ . \\ -\alpha u_{N,j+1} \end{pmatrix} = \begin{bmatrix} 2-2\alpha & \alpha & 0 & 0 & 0 & 0 \\ \alpha & 2-2\alpha & \alpha & 0 & 0 & 0 \\ 0 & \alpha & 2-2\alpha & \alpha & 0 & 0 \\ 0 & 0 & . & . & . & 0 \\ 0 & 0 & 0 & \alpha & 2-2\alpha & \alpha \\ 0 & 0 & 0 & 0 & \alpha & 2-2\alpha \end{bmatrix} \begin{pmatrix} u_{1,j} \\ u_{2,j} \\ . \\ . \\ . \\ u_{N-1,j} \end{pmatrix} + \begin{pmatrix} \alpha u_{0,j} \\ 0 \\ . \\ . \\ . \\ \alpha u_{N,j} \end{pmatrix} \quad (26)$$

This is in the form of

$$\boldsymbol{M}_{j+1}\boldsymbol{u}_{j+1} + \boldsymbol{r}_{j+1} = \boldsymbol{M}_j\boldsymbol{u}_j + \boldsymbol{r}_j$$

where, $\boldsymbol{M}_{j+1}$ and $\boldsymbol{M}_j$ are the coefficient matrices and $\boldsymbol{r}_{j+1}$ and $\boldsymbol{r}_j$ are the known vectors. The only unknown vector here was $\boldsymbol{u}_{j+1}$, which was solved from;

$$\boldsymbol{u}_{j+1} = (\boldsymbol{M}_{j+1})^{-1}(\boldsymbol{M}_j\boldsymbol{u}_j + \boldsymbol{r}_j - \boldsymbol{r}_{j+1}) \quad (27)$$

The matrix of coefficients in equation (19) is sparse, consisting of a tri-diagonal part corresponding to the x-derivative, and two off-diagonal bands corresponding to

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the $y$ −derivatives.

**6 Results**

Thethree Dimensional finite difference Discretization Matrixyielded a system of equation briefly denoted by

$$A\hat{h} = \hat{q} \qquad (28)$$

where $A$ is the coefficient matrix representing conductances, $\hat{h}$ is a vector representing the unknown hydraulic heads into the gridblock and $\hat{q}$ denotes the known flux sources/sinks in the model.

The coefficient matrix $A$ of the discretization equation for the illustrated grid block in was given by a $36 \times 36$ square matrix given as

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

where$(A_{ij})$, $i,j = 1,2,3$ each is a $12 \times 12$ matrix defined by

$$A_{11} = \begin{bmatrix} \begin{matrix} C_1 & E_1 & 0 & 0 \\ W_2 & C_2 & E_2 & 0 \\ 0 & W_3 & C_3 & E_3 \\ 0 & 0 & W_4 & C_4 \end{matrix} & \begin{matrix} N_1 & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 \\ 0 & 0 & N_3 & 0 \\ 0 & 0 & 0 & N_4 \end{matrix} & 0_{4\times4} \\ \begin{matrix} S_5 & 0 & 0 & 0 \\ 0 & S_6 & 0 & 0 \\ 0 & 0 & S_7 & 0 \\ 0 & 0 & 0 & S_8 \end{matrix} & \begin{matrix} C_5 & E_5 & 0 & 0 \\ W_6 & C_6 & E_6 & 0 \\ 0 & W_7 & C_7 & E_7 \\ 0 & 0 & W_8 & C_8 \end{matrix} & \begin{matrix} N_5 & 0 & 0 & 0 \\ 0 & N_6 & 0 & 0 \\ 0 & 0 & N_7 & 0 \\ 0 & 0 & 0 & N_8 \end{matrix} \\ 0_{4\times4} & \begin{matrix} S_9 & 0 & 0 & 0 \\ 0 & S_{10} & 0 & 0 \\ 0 & 0 & S_{11} & 0 \\ 0 & 0 & 0 & S_{12} \end{matrix} & \begin{matrix} C_9 & E_9 & 0 & 0 \\ W_{10} & C_{10} & E_{10} & 0 \\ 0 & W_{11} & C_{11} & E_{11} \\ 0 & 0 & W_{12} & C_{12} \end{matrix} \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} \begin{matrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{matrix} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & \begin{matrix} B_5 & 0 & 0 & 0 \\ 0 & B_6 & 0 & 0 \\ 0 & 0 & B_7 & 0 \\ 0 & 0 & 0 & B_8 \end{matrix} & 0_{4\times4} \\ 0_{4\times4} & 0_{4\times4} & \begin{matrix} B_9 & 0 & 0 & 0 \\ 0 & B_{10} & 0 & 0 \\ 0 & 0 & B_{11} & 0 \\ 0 & 0 & 0 & B_{12} \end{matrix} \end{bmatrix},$$

$$A_{13} = 0_{4\times4},$$

$$A_{21} = \begin{bmatrix} \begin{matrix} T_1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_3 & 0 \\ 0 & 0 & 0 & T_4 \end{matrix} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & \begin{matrix} T_5 & 0 & 0 & 0 \\ 0 & T_6 & 0 & 0 \\ 0 & 0 & T_7 & 0 \\ 0 & 0 & 0 & T_8 \end{matrix} & 0_{4\times4} \\ 0_{4\times4} & 0_{4\times4} & \begin{matrix} T_9 & 0 & 0 & 0 \\ 0 & T_{10} & 0 & 0 \\ 0 & 0 & T_{11} & 0 \\ 0 & 0 & 0 & T_{12} \end{matrix} \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} \begin{matrix} C_{13} & E_{13} & 0 & 0 \\ W_{14} & C_{14} & E_{14} & 0 \\ 0 & W_{15} & C_{15} & E_{15} \\ 0 & 0 & W_{16} & C_{16} \end{matrix} & \begin{matrix} N_{13} & 0 & 0 & 0 \\ 0 & N_{14} & 0 & 0 \\ 0 & 0 & N_{15} & 0 \\ 0 & 0 & 0 & N_{16} \end{matrix} & 0_{4\times4} \\ \begin{matrix} S_{17} & 0 & 0 & 0 \\ 0 & S_{18} & 0 & 0 \\ 0 & 0 & S_{19} & 0 \\ 0 & 0 & 0 & S_{20} \end{matrix} & \begin{matrix} C_{17} & E_{17} & 0 & 0 \\ W_{18} & C_{18} & E_{18} & 0 \\ 0 & W_{19} & C_{19} & E_{19} \\ 0 & 0 & W_{20} & C_{20} \end{matrix} & \begin{matrix} N_{17} & 0 & 0 & 0 \\ 0 & N_{18} & 0 & 0 \\ 0 & 0 & N_{19} & 0 \\ 0 & 0 & 0 & N_{20} \end{matrix} \\ 0_{4\times4} & \begin{matrix} S_{21} & 0 & 0 & 0 \\ 0 & S_{22} & 0 & 0 \\ 0 & 0 & S_{23} & 0 \\ 0 & 0 & 0 & S_{24} \end{matrix} & \begin{matrix} C_{21} & E_{21} & 0 & 0 \\ W_{22} & C_{22} & E_{22} & 0 \\ 0 & W_{23} & C_{23} & E_{23} \\ 0 & 0 & W_{24} & C_{24} \end{matrix} \end{bmatrix},$$

$$A_{23} = \begin{bmatrix} \begin{matrix} B_{13} & 0 & 0 & 0 \\ 0 & B_{14} & 0 & 0 \\ 0 & 0 & B_{15} & 0 \\ 0 & 0 & 0 & B_{16} \end{matrix} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & \begin{matrix} B_{17} & 0 & 0 & 0 \\ 0 & B_{18} & 0 & 0 \\ 0 & 0 & B_{19} & 0 \\ 0 & 0 & 0 & B_{20} \end{matrix} & 0_{4\times4} \\ 0_{4\times4} & 0_{4\times4} & \begin{matrix} B_{21} & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 \\ 0 & 0 & B_{23} & 0 \\ 0 & 0 & 0 & B_{24} \end{matrix} \end{bmatrix},$$

$$A_{31} = 0_{12\times12}$$

$$A_{32} = \begin{bmatrix} \begin{matrix} T_{25} & 0 & 0 & 0 \\ 0 & T_{26} & 0 & 0 \\ 0 & 0 & T_{27} & 0 \\ 0 & 0 & 0 & T_{28} \end{matrix} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & \begin{matrix} T_{29} & 0 & 0 & 0 \\ 0 & T_{30} & 0 & 0 \\ 0 & 0 & T_{31} & 0 \\ 0 & 0 & 0 & T_{32} \end{matrix} & 0_{4\times4} \\ 0_{4\times4} & 0_{4\times4} & \begin{matrix} T_{33} & 0 & 0 & 0 \\ 0 & T_{34} & 0 & 0 \\ 0 & 0 & T_{35} & 0 \\ 0 & 0 & 0 & T_{36} \end{matrix} \end{bmatrix} \quad and$$

$$A_{33} = \begin{bmatrix} \begin{matrix} C_{25} & E_{25} & 0 & 0 \\ W_{26} & C_{26} & E_{26} & 0 \\ 0 & W_{27} & C_{27} & E_{27} \\ 0 & 0 & W_{28} & C_{28} \end{matrix} & \begin{matrix} N_{25} & 0 & 0 & 0 \\ 0 & N_{26} & 0 & 0 \\ 0 & 0 & N_{27} & 0 \\ 0 & 0 & 0 & N_{28} \end{matrix} & 0_{4\times4} \\ \begin{matrix} S_{29} & 0 & 0 & 0 \\ 0 & S_{30} & 0 & 0 \\ 0 & 0 & S_{31} & 0 \\ 0 & 0 & 0 & S_{32} \end{matrix} & \begin{matrix} C_{29} & E_{29} & 0 & 0 \\ W_{30} & C_{30} & E_{30} & 0 \\ 0 & W_{31} & C_{31} & E_{31} \\ 0 & 0 & W_{32} & C_{32} \end{matrix} & \begin{matrix} N_{29} & 0 & 0 & 0 \\ 0 & N_{30} & 0 & 0 \\ 0 & 0 & N_{31} & 0 \\ 0 & 0 & 0 & N_{32} \end{matrix} \\ 0_{4\times4} & \begin{matrix} S_{33} & 0 & 0 & 0 \\ 0 & S_{34} & 0 & 0 \\ 0 & 0 & S_{35} & 0 \\ 0 & 0 & 0 & S_{36} \end{matrix} & \begin{matrix} C_{33} & E_{33} & 0 & 0 \\ W_{34} & C_{34} & E_{34} & 0 \\ 0 & W_{35} & C_{35} & E_{35} \\ 0 & 0 & W_{36} & C_{36} \end{matrix} \end{bmatrix}$$

while vector $\hat{h}$ is a $36 \times 1$ matrix represented by

$\hat{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$, where the sub-vectors $h_1, h_2$ and $h_3$ are $12 \times 1$ matrices given by

$$h_1 = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \\ H_7 \\ H_8 \\ H_9 \\ H_{10} \\ H_{11} \\ H_{12} \end{bmatrix}, \quad h_2 = \begin{bmatrix} H_{13} \\ H_{14} \\ H_{15} \\ H_{16} \\ H_{17} \\ H_{18} \\ H_{19} \\ H_{20} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{24} \end{bmatrix}, \quad h_3 = \begin{bmatrix} H_{25} \\ H_{26} \\ H_{27} \\ H_{28} \\ H_{29} \\ H_{30} \\ H_{31} \\ H_{32} \\ H_{33} \\ H_{34} \\ H_{35} \\ H_{36} \end{bmatrix}$$

and the solution vector $\hat{q}$ is a $36 \times 1$ vector representing known heads of sources/sinks. The matrix $\hat{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$ where the sub-vectors $q_i,\ i = 1, 2, 3$ is represented by

$$q_1 = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \end{bmatrix}, \quad q_2 = \begin{bmatrix} Q_{13} \\ Q_{14} \\ Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \\ Q_{20} \\ Q_{21} \\ Q_{22} \\ Q_{23} \\ Q_{24} \end{bmatrix}, \quad q_3 = \begin{bmatrix} Q_{25} \\ Q_{26} \\ Q_{27} \\ Q_{28} \\ Q_{29} \\ Q_{30} \\ Q_{31} \\ Q_{32} \\ Q_{33} \\ Q_{34} \\ Q_{35} \\ Q_{36} \end{bmatrix}$$

Here, the symbols used, that is $N, E, W, S, C, T, B$ represented the position of the cells in the block in relation to the other cells, and they denoted North, East, West, South, Center, Top and Bottom respectively while the notations in the matrixClearly, the size of matrix is enormous and as we intend to obtain fine details by splitting the Lamu Region into fine grid matrix, we obtain a very large matrix which can only be solved using a computer software.

## 7.Conclusion

For a finite confined flow, the DFA shows superiority over other space algorithms such as Functional Space algorithm. A comprehensive discretization flow algorithm for the flow of petroleum in a fully confined aquifer has been discussed. Various characteristics of the acquifer including fluid and rock characteristics, can be solved over a region through the series of algebraic equations. The formulation with interface conditions was found to give qualitatively and quantitatively better results on finer meshes and lead to linear and nonlinear systems that were easier to solve.

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