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Spherically Symmetric Inhomogeneous Macro Model Filled With Perfect Fluid

S.N.Jena(1)
Retired Professor of Physics,
Berhampur University,
Berhampur,Odisha(India)

R.N.Patra(2)
P.G.Department of Mathematics,
Berhampur University, Odisha(India)
E.mail:raghunathpatra09@gmail.com

R.R.Swain(3)
Department of Physics,
U.P.Sc. College,
Sheragada, Odisha(India)

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Abstract

In this paper, we have studied spherically inhomogeneous cosmological model with perfect fluid in Einstein relativity. It is observed that, cosmic time (t) has no contribution towards the study of various physical and geometrical properties of the universe, but they are dependent on the parameter ‘r’ only. The universe is found to be non-rotating. Also Isotropy exists through out the evolution and the universe is expanding with respect to ‘r’ only. The model is not a space of constant curvature, but found to be symmetric.

1. Introduction:

The large-scale behaviour of the actual universe is simplified and described by studying spherically symmetric space time. Recently Mazumber[1] has obtained cosmological solutions for LRS Bianchi-I Space time filled with perfect fluid with arbitrary Cosmic Scale functions and Studied Kinematical Properties of the particular form of the solution. Haji-boutras and Sfeila [2] and Shriram [3] also obtained some solutions for the same field equation by using solution generation technique. Taub[4] and Tomimura [5] have studied in homogeneous cosmologies. In all thes models the material distribution is that of a perfect fluid.

In Section 2, we have studied the space time filled with perfect fluid in Einstein Theory of gravitation and obtained the exact cosmological solution. In Section 3 we have studied various physical and geometrical properties related to Section 2 and the concluding remarks are given in Section 4.

2. Field Equations and their Solutions:

We have considered the Spherically symmetric inhomogeneous space time of Bianchi type-1 Cosmological Model is of the form

$$ds^2 = -e^{\lambda}\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\mu}\, dt^2 , \qquad (1)$$

where λ and μ are functions of 'r' and 't'

i.e. $\lambda = \lambda(r,t)$

$\mu = \mu(r,t)$.

The field equation in Einstein Theory is given by

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -\frac{K}{4\pi} T_{ij} \quad . \tag{2}$$

In co-moving co-ordinate system, the velocity components are $v_1 = v_2 = v_3 = 0 \ and \ v_4^2 = g_{44}$. The energy momentum tensor for perfect fluid is

$$T_{ij} = (p + \rho) v_l \ v_j - p \ g_{ij}, \tag{3}$$

where $g_{ij} = 0 \ for \ i \neq j$

$\therefore T_{ij} = 0 \ for \ i \neq j$

The existing components are

$$T_{11} = e^{\lambda} p, T_{22} = r^2 p, T_{33} = r^2 p \sin^2 \theta, T_{44} = \rho e^{\mu}. \tag{4}$$

Using equation (4) in equation (2), the following equations are obtained,

$$-\frac{\mu_{11}}{2}\left(1+e^{\lambda}\right) + \frac{\lambda_1}{4}\left(e^{\lambda} - 1\right) + \frac{\mu_1^2}{2}\left(2 - e^{\lambda}\right) - \frac{\mu_1}{r} e^{\lambda} + \frac{k}{4\pi} e^{\lambda} P = 0 \tag{5}$$

$$\frac{re^{-\lambda}}{2}\left(\lambda_1 - \mu_1 + \frac{\lambda_1 \mu_1 r}{2} - \mu_{11} r - \frac{r\mu_1^2}{4}\right) + \frac{r^2}{2} e^{-\mu}\left(\lambda_{44} + \frac{\lambda_4^2}{2} - \frac{\lambda_4 \mu_4}{2}\right) + \frac{k}{4\pi} r^2 p = 0 \tag{6}$$

$$-\frac{\lambda_4}{r} = 0 \Rightarrow \lambda_4 = 0 \Rightarrow \lambda = f(r). \tag{7}$$

$$e^{\mu-\lambda}\left(-\frac{\lambda_1}{r} + \frac{\mu_1^2}{8} + \frac{1}{r^2}\right) - \frac{e^{-\mu}}{r^2} + \frac{k}{4\pi} \rho e^{\mu} = 0.$$

$$\Rightarrow e^{\mu}\left[e^{-\lambda}\left(\frac{-\lambda_1}{r} + \frac{\mu_1^2}{8} + \frac{1}{r^2}\right) - \frac{1}{r^2} + \frac{k\rho}{4\pi}\right] = 0 \tag{8}$$

By solving equations from (5) to (8), we get two cases.

Case-I:

If $e^{\mu} = 0$ in equation (8), then our metric reduces to three dimensional space i.e.,

$$ds^2 = -rdr^2 - r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right). \tag{9}$$

Hence we proceed to Case-II.

Case-II: In equation (8) if

$$e^{-\lambda}\left(\frac{-\lambda_1}{r} + \frac{\mu_1^2}{8} + \frac{1}{r^2}\right) - \frac{1}{r^2} + \frac{k\rho}{4\pi} = 0,$$

then by putting $\lambda = \ln r$ for equation(7), we get

$$\mu = A + B\sqrt{r} - 2\ln r + 4r\ln r .$$

Hence the metrics takes the form

$$ds^2 = -r\, dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\left(A + B\sqrt{r} - 2\ln r + 4r\ln r\right)} dt^2 \tag{10}$$

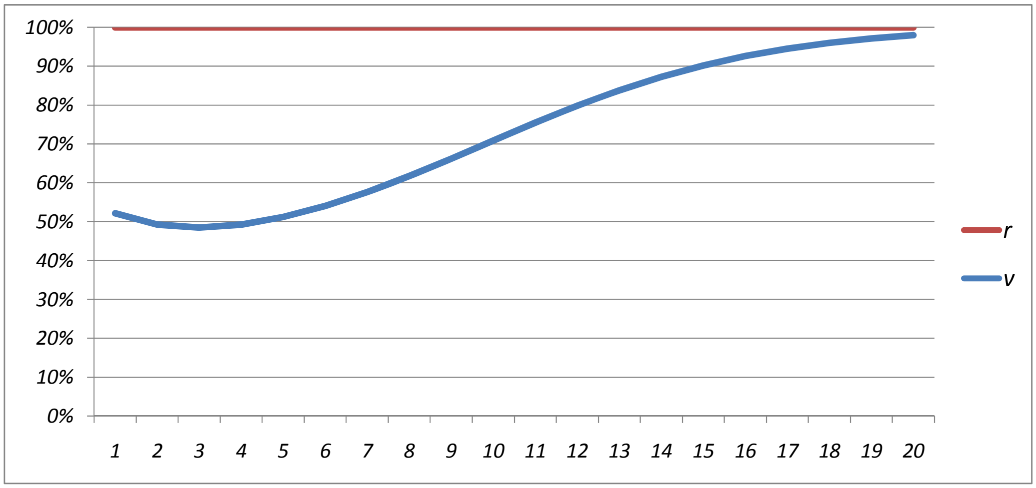
3. Some Physical and Geometrical Properties:

The following properties of model (10) have been discussed

(i) Volume (V) = $e^{\frac{1}{2}\left[A + B\sqrt{r} + (4r-2)\ln r\right]}\, r^2 \sin\theta$.

Here we observe that if '*r*' increases then 'V' increases. But expansion of the universe is independent of time.

Taking $A = B = 1, \theta = 90^{\circ}$, values of '*r*' along X-axis and the corresponding values of 'V' along Y-axis, we have plotted the graph which shows the sharp increase in volume upto a particular limit, after which there is no more expansion.



(ii) Scalar Expansion $(\theta) = V^i_{;i} = 0$.

Hence there is no scalar expansion of the Universe.

(iii) The anisotropy $|\sigma|$ defined as

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right]$$

$$= 0$$

$$\Rightarrow \sigma = 0.$$

Hence the model approaches to isotropy i.e. the isotropy exists through out the evolution.

(iv) $R_{ij} \neq \frac{1}{4} R g_{ij}$

Hence the model is not an Einstein's Space.

(v) $w_{ij} = 0 \; for \; i, j = 1,2,3,4$.

Hence the universe is non-rotating by nature.

(vi) the deceleration parameter 'q' given by Einstein as

$$q = -3\theta^2\left[\theta,_{\alpha} V^{\alpha} + \frac{1}{3}\theta^2\right]$$

Since $\theta = 0,\ q = 0$.

It shows that the model is neither accelerating nor decelerating, but the universe is expanding w.r.t ‘r’.

VII. Space of Constant Curvature:

A space satisfying the condition

$$R_{hijk} = \bar{k}\left(g_{hj}\ g_{ik} - g_{hk} g_{ij}\right) \qquad (11)$$

is called a space of constant curvature, where $\bar{k}$ is a constant, known as the curvature of Riemannian manifold and Riemannian curvature tensor R_{hijk} is given by the equation,

$$R_{hijk} = \frac{1}{2}\left[\frac{\partial^2 g_{hk}}{\partial x^i \partial x^j} + \frac{\partial^2 g_{ij}}{\partial x^h\ \partial x^k} - \frac{\partial^2 g_{hj}}{\partial x^i \partial x^k} - \frac{\partial g_{ik}}{\partial x^h\ \partial x^j}\right]$$

$$+\ g_{ab}\left[\Gamma^a_{ij}\ \Gamma^b_{hk} - g_{ab}\ \Gamma^a_{ik}\ \Gamma^b_{hj}\ \right] \qquad (12)$$

The existing components of equation (12) for metric (1) are R_{1212}, R_{1313}, R_{1414}, R_{2323}, R_{2424} and R_{3434}.

Matching the existing components of R_{hijk} with the corresponding components of right hand side of equation(11), we get the following forms of equations.

$$R_{1212} = \bar{K}\, r^2 e^{\lambda} = -\frac{r}{2}\ \lambda_1,$$

$$R_{1313} = \bar{K} r^2 e^{\lambda} \sin^2\theta = \frac{-\lambda_1}{2}\ r \sin^2\ \theta,$$

$$R_{1414} = -\overline{K}\ e^{\lambda+\mu} = \frac{e^{\mu}}{4}\left(-\mu_1^2 + \mu_1\ \lambda_1\ - 2\mu_{11}\right) + \frac{e^{\lambda}}{4}\left(2\lambda_{44} - \lambda_4\mu_4 + \lambda_4^2\right),$$

$$R_{2323} = \overline{K}\ r^4\ \sin^2\theta = r^2\sin^2\theta\left(e^{-\lambda} - 1\right),$$

$$R_{2424} = -\overline{K}r^2e^{\mu} = -\frac{\mu_1 r}{2}\ e^{(\mu-\lambda)}$$

and $$R_{3434} = -\overline{K}r^2e^{\mu}\sin^2\theta = \sin^2\ \theta\left[-\frac{\mu_1 r}{2}\ e^{(\mu-\lambda)}\right].$$

From the above equations, we get

$$\overline{K} = \frac{-e^{-\lambda}\lambda_1}{2r}$$

$$\overline{K} = \frac{-\lambda_1 e^{-\lambda}}{2r}$$

$$\overline{K} = \frac{-e^{-\lambda}}{4}\left(-\mu_1^2 + \mu_1\lambda_1 - 2\mu_{11}\right) - \frac{e^{-\mu}}{4}\left(2\lambda_{44} - \lambda_4\mu_4 + \lambda_4^2\right)$$

$$\overline{K} = \frac{-e^{-\lambda} - 1}{r^2}$$

$$\overline{K} = e^{-\mu}\left(-\frac{r_4}{r} - \frac{r_4^2}{r^2} + \frac{\mu_1}{2r}\ e^{\mu-\lambda}\right)$$

$$\overline{K} = e^{-\mu}\left(-\frac{r_{44}}{r} - \frac{r_4^2}{r^2} + \frac{\mu_1}{2r}\ e^{\mu-\lambda}\right)$$

The various values of $\overline{K}$ is not an unique constant. Hence the space time is not a space of constant curvature in general curved space.

VIII. Symmetric Space:

Co-variant derivative of Riemannian curvature tensor

$$R_{1212,1} = \frac{\partial R_{1212}}{\partial x^1} - R_{m212}\begin{Bmatrix} m \\ 11 \end{Bmatrix} - R_{im12}\begin{Bmatrix} m \\ 21 \end{Bmatrix} - R_{12m2}\begin{Bmatrix} m \\ 11 \end{Bmatrix} - R_{121m}\begin{Bmatrix} m \\ 21 \end{Bmatrix}$$

$$= \frac{\lambda_1}{2} + \frac{r\lambda_1^2}{2} - \frac{r\lambda_{11}}{2}.$$

Similarly the other existing components are

$$R_{1313,1} = \sin^2\theta(R_{1212,1}),$$

$$R_{1414,1} = e^{\mu}\left(\frac{-\mu_1^2}{2} + \frac{\mu_1\lambda_{11}}{4} + \frac{\lambda_1\mu_{11}}{4} - \frac{\mu_{111}}{2} + \frac{\lambda_1\mu_1^2}{4} - \frac{\lambda_1^2\mu_1}{4}\right),$$

$$+ e^{\lambda}\left(\frac{\lambda_{44,1}}{2} - \frac{\lambda_4\mu_{44}}{4} - \frac{\mu_4\lambda_{44}}{4} + \frac{\lambda_4\lambda_{4,1}}{2} - \frac{\mu_1\lambda_{44}}{2} + \frac{\mu_1\lambda_{44}}{4} - \frac{\lambda_4^2\mu_1}{4}\right),$$

$$R_{2323,1} = r\sin^2\theta\left(2 - r\lambda_1 e^{-\lambda} - 2e^{-\lambda}\right),$$

$$R_{2424,1} = \left(\frac{-\mu_1^2 r}{2} + \frac{\mu_1 r\lambda_1}{2} + \frac{\mu_1}{2} - \frac{\mu_{11}r}{2} + \frac{\mu_4\mu_1 r}{2}\right)e^{\mu-\lambda}$$

$$R_{3434,1} = \sin^2\theta(R_{2424,1})$$

$$R_{1212,4} = \frac{r}{2}\lambda_{1,4} - \frac{r}{2}\lambda_1\lambda_4$$

$$R_{1313,4} = \sin^2\theta(R_{1212,4})$$

$$R_{1414,4} = e^{\mu}\left(-\frac{\mu_1\mu_{1,4}}{2} + \frac{\mu_1\lambda_{1,4}}{4} + \frac{\lambda_1\mu_{1,4}}{4} + \frac{\mu_1\lambda_1\mu_4}{4} - \frac{\mu_{111}}{2} - \frac{\mu_{11}\mu_4}{2} + \frac{\mu_1^2}{2} + \frac{\mu_1\lambda_1}{2} + \mu_1\right)$$

$$+e^{\lambda}\left(\frac{\mu_{444}}{2}+\frac{\lambda_4\lambda_{44}}{2}-\frac{\lambda_4\mu_{44}}{4}-\frac{\lambda_{44}\mu_4}{4}-\frac{\mu_4\lambda_4^2}{4}-\lambda_{44}+\frac{\lambda_4\mu_4}{2}-\frac{\lambda_4^2}{2}\right),$$

$$R_{2323,4}=\sin^2\theta\left(-e^{-\lambda}r^2\lambda_4+2e^{-\lambda}r-2r\right),$$

$$R_{2424,4}=e^{\mu-\lambda}\left(\frac{r\mu_1\lambda_4}{2}-\frac{r\mu_{1,4}}{2}\right) \text{ and}$$

$$R_{3434,4}=\sin^2\theta\left(R_{2424,4}\right).$$

Equating the above existing values to zero and solving the equation we get the value of 'λ' and 'μ' as

$$\lambda=\frac{r}{t-C_1}+C_2$$

and

$$\mu=\left(t-C_1\right)e^{\frac{r}{t-C_1}}+C_3,$$

where C_1, C_2 and C_3 are constants of integration. Hence it is found that our space-time is symmetric with the above values of 'λ' and 'μ' which are functions of 'r' and 't' and the model takes the form

$$ds^2=-e^{\left(\frac{r}{t-C_1}+C_2\right)}dr^2-r^2d\theta^2-r^2\sin^2\theta\,d\phi^2$$

$$+e^{\left[\left(t-C_1\right)e^{\frac{r}{t-C_1}}+C_3\right]}dt^2.$$

Conclusion:

In this paper, we have investigated that the inhomogeneous cosmological model in presence of perfect fluid approaches isotropy through out the evolution. The volume expansion is independent of time ‘t’. The zero value of deceleration parameter shows that the model is neither accelerating nor decelerating, but expands w.r.t. ‘r’. There is no scalar expansion of the universe and the model is found to be non-rotating as the verticity tensor vanishes. Moreover, the model is not a space of constant curvature, but found to be symmetric.

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