IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 4, April 2015.

www.ijiset.com

ISSN 2348 – 7968

Concepts on Ordered Ternary Semirings

Dr.D.Madhusudanarao [1], G. Srinivasarao [2], P. Siva Prasad[3]

[1] **Head, Department of Mathematics,**
V.S.R. & N.V.R. College, Tenali, A.P., India.

[2] **Assistant Professor of Mathematics,**
Tirumala Engineering College, Narasaraopet, A.P., India

[3]**Asstistant Professor of Mathematics**
Universal College of Engineering & Technology, Perecherla,
Guntur, A.P. India.

ABSTRACT

In this paper, we study the properties of ordered Ternary semi-rings satisfying the identity $a + ab^2 = a$. It is proved that , let (T , + , . , $\leq$) be a totally ordered ternary semiring satisfying the condition $a + ab^2 = a$, $\forall$ a , $b \in T$. If (T , + , $\leq$) is positively totally ordered (negatively totally ordered) , then (T , **. ,** $\leq$ **)** is non-positively ordered (non-negatively ordered).

Key Words: *Ordered ternary semiring, positively totally ordered, non-positively ordered.*

1. **Introduction**

Algebraic structures play a prominent role in Mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the like. This provides sufficient motivation to researchers to review various concepts and results.

The theory of ternary algebraic systems was studied by LEHMER [9] in 1932, but earlier such structures were investigated and studied by PRUFER in 1924, BAER in 1929.

Generalizing the notion of ternary ring introduced by Lister [10], Dutta and Kar [6] introduced the notion of ternary semiring. Ternary semiring arises naturally as follows, consider the ring of integers Z which plays a vital role in the theory of ring. The subset Z+ of all positive integers of Z is an additive semigroup which is closed under the ring product, i.e. Z^+ is a semiring. Now, if we consider the subset $Z-$ of all negative integers of Z, then we see that Z is an additive semigroup which is closed under the triple ring product (however, $Z-$ is not closed under the binary ring product), i.e. $Z-$ forms a ternary semiring. Thus, we see that in the ring of integers Z, Z+ forms a semiring whereas $Z-$ forms a ternary semiring.

2. Preliminaries

Definition 2.1 : A non-empty set T together with a binary addition and a ternary multiplication denoted by justaposition, is said to be **ternary semiring** if T is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)de = a(bcd)e = ab(cde)$
(ii) $(a+b)cd = acd + bcd$
(iii) $a(b+c)d = abd + acd$
(iv) $ab(c + d) = abc + abd,$

for all $a , b , c , d , e \in T.$

Example 2.2 : Let T = { 0 , 1 , 2 , 3 , 4 } is a ternary semiring with respect to addition modulo 5 and multiplication modulo 5 as ternary operation is defined as follows:

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

x_5	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Definition 2.3 : A ternary semigroup (T , **.**) is said to be

(i) ***left regular***, if it satisfies the identity $ab^2 = a$ $\forall$ a, $b \in$ T

(ii) ***right regular***, if it satisfies the identity $b^2 a = a$ $\forall$ a, $b \in$ T

(iii) ***lateral regular***, if it satisfies the identity $aba = a$ $\forall$ a , $b \in$ T

(iv) ***two-sided regular*,** if it is both left as well as right regular.

(v) ***regular***, it is left, lateral and right regular.

Definition 2.4 : An element a of a ternary multiplicative semigroup ' T ' is called an ***E-inverse*** if there exist an element x such that $(axa)(axa)(axa) = axa$, *i.e.*, $axa \in E$ (**.**), where E(**.**) is the set of all ternary multiplicative idempotent elements of T.

Definition 2.5 : A ternary semigroup T is called an ***E-inverse ternary semigroup*** if every element of T is an E-inverse.

Definition 2.6 : An element a of an additive semigroup ' T ' is called an ***E-inverse*** if there is an element x in T such that $axa + axa = axa$ i.e. $axa \in E(+)$, where E (+) is the set of all additive idempotent elements of T.

Definition 2.7 : A semigroup T is called an ***E-inverse semigroup*** if every element of T is an E-inverse.

Definition 2.8 : A ternary semigroup $(T, +)$ is said to satisfy ***quasi separative*** if $x^3 = xyx = yxy = y^3$ implies $x = y \; \forall \, x, y \in T$.

3. Ordering on Ternary Semiring Satisfying the Identity $ab^2 + a = a$

Definition 3.1 : A ternary semiring $(T, +, .)$ is said to be ***totally ordered ternary semiring*** if there exist a partially order " $\leq$ " on T such that

(i) $(T, +, \leq)$ is a totally ordered semigroup

(ii) $(T, ., \leq)$ is a totally ordered ternary semiring.

It is denoted by $(T, +, ., \leq)$.

Example 3.2 : Consider the set $T = \{1, 2, 3, 4\}$ with the order $1 < 2 < 3 < 4$ and with the following addition and multiplication.

+	1	2	3	4
1	2	3	4	4
2	3	4	4	4
3	4	4	4	4
4	4	4	4	4

.	1	2	3	4
1	2	4	4	4
2	4	4	4	4
3	4	4	4	4
4	4	4	4	4

Hence $(T, +, ., \leq)$ is a totally ordered ternary semiring.

Definition 3.3 : An element ' x ' in a partially ordered ternary semigroup $(T, ., \leq)$ is ***non-negative*** **(*non-positive*)** if $x^3 \geq x$ $(x^3 \leq x)$.

Definition 3.4 : A partial ordered ternary semi-group $(T, ., \leq)$ is ***non-negatively*** **(*non-positively*)** ***ordered*** if every element in T is non-negative (non-positive).

Definition 3.5 : An element x in a partial ordered semigroup $(T, +, \leq)$ is ***non-negative*** **(*non-positive*)** if $x + x \geq x$ $(x + x \leq x)$.

Definition 3.6: A partial ordered semi-group $(T, +, \leq)$ is ***non-negatively*** **(*non-positively*)** ordered if every element in T is non-negative (non-positive).

Definition 3.7 : A ternary semiring $(T, +, .)$ is said to be a ***positive rational domain*** **(PRD)** if and only if $(T, .)$ is an ternary abelian group.

Theorem 3.8 : Let $(T, +, ., \leq)$ be a totally ordered ternary semiring and satisfying the identity $ab^2 + a = a \; \forall \, a, b \in T$. If $(T, +, \leq)$ is non-negatively ordered (non-positively ordered), then $(T, ., \leq)$ is non-positively (non-negatively) ordered.

***Proof* :** We know that $a + a^3 = a \; \forall \, a \in T$. Since $(T, +, \leq)$ is non-negatively ordered, we have $a^3 = a + a^3 \geq a$, $a^3 \geq a \;\; \forall \, a \in T$. Hence $(T, ., \leq)$ is non-negatively ordered. Suppose $(T, +, \leq)$ is non-positively ordered, $a^3 = a + a^3 \leq a \Rightarrow a^3 \leq a \; \forall \, a \in T$. Therefore $(T, ., \leq)$ is non-positively ordered.

Definition 3.9: In a totally ordered ternary semiring $(T, +, ., \leq)$.

(i) $(T, +, \leq)$ is ***negatively totally ordered*** if $a + b \leq a$ and b, $\forall \, a, b \in T$ and

(ii) $(T, ., \leq)$ is ***negatively totally ordered*** if $ab^2 \leq a$ and b, $\forall \, a, b \in T$.

Definition 3.10: An element x in totally ordered ternary semi-ring is ***minimal*** **(*maximal*)** if $x \leq a$ $(x \geq a)$, $\forall \, a \in T$.

Theorem 3.11: Let $(T, +, ., \leq)$ be a totally ordered positive rational domain ternary semiring satisfying the identity $ab^2 + a = a \; \forall \, a, b \in T$. If $(T, +, \leq)$ is positively totally ordered (negatively totally ordered), then 1 is minimum (maximum) element.

Proof: We know that $1 + a = a \;\; \forall \, a \in T$. Suppose $(T, +, \leq)$ is positively totally ordered $\Rightarrow a = 1 + a \geq a$ and $1 \Rightarrow a \geq 1 \Rightarrow \Rightarrow$ ' 1 ' is the minimal element. Suppose $(T, +, \leq)$ negatively totally ordered $\Rightarrow a = a + 1 \leq a$ and 1 , $\Rightarrow a \leq 1 \Rightarrow$ 1 is the maximal element.

4. Ordered Ternary Semiring

Definition 4.1 : A totally ordered ternary semigroup $(T, ., \leq)$ is said to be ***non-negatively*** **(*non–positively*)** ***ordered*** if every one of its element is non-negative (non – positive).

Definition 4.2 : A ternary semigroup $(T, .)$ is ***positively*** **(*negatively*)** ***ordered in strict sense*** if $ab^2 \geq a$ and $a^2b \geq b$ ($ab^2 \leq a$ and $a^2b \leq b$) $\forall\, a, b \in T$.

Theorem 4.3 : Suppose $(T, +, ., \leq)$ is a totally ordered ternary semiring satisfying the identity $a + ab^2 = a$, $\forall\, a, b \in T$ and $(T, +, \leq)$ is positively totally ordered (negatively totally ordered), then $(T, ., \leq)$ is negatively totally ordered (positively totally ordered).

Proof **:** Suppose $a + ab^2 = a\ \forall\, a, b \in T$.
$\Rightarrow a = a + ab^2 \geq ab^2, \Rightarrow a \geq ab^2$
Suppose $b \leq a^2b$, $\Rightarrow a + b \leq a + a^2b$. Since $(T, +)$ is positively totally ordered $\Rightarrow a + b \Rightarrow b$. It is a contradiction to the fact that $(T, .)$ is positively totally ordered. Then $b \geq a^2b$. Therefore $ab^2 \leq a$ & $a^2b \leq b$. Hence $(T, .)$ is negatively totally ordered.

Similarly we can prove that $(T, .)$ is positively totally ordered.

Example 4.4 [12] : $1 < b < a$

+	1	a	b
1	1	a	b
a	a	a	a
b	b	a	b

×	1	a	b
1	1	a	b
a	a	a	a
b	b	a	b

Theorem 4.5 : Let $(T, +, ., \leq)$ be a totally ordered ternary semiring satisfying the condition $a + ab^2 = a$ $\forall\, a, b \in t$. If $(T, +, \leq)$ is positively totally ordered (negatively totally ordered), then $(T, ., \leq)$ is non-positively (non-negatively) ordered.

Proof **:** Given that $a + ab^2 = a\ \forall\, a, b \subseteq T$.
Since $(T, ., \leq)$ is positively totally ordered , we have $a + a^3 \geq a$ & $a^3 \Rightarrow a = a + a^3 \geq a^3, \Rightarrow a \geq a^3$
$\Rightarrow a^3 \leq a$. Therefore $(T, ., \leq)$ is non-positively ordered. Similarly if $(T, +, \leq)$ is negatively totally ordered, then $(T, ., \leq)$ is non-negatively ordered.

5. Conclusions

In this paper mainly we studies about some properties of ordered ternary semi rings.

Acknowledgments

The authors would like to thank the referee(s) for careful reading of the manuscript.

References

[1] Arif Kaya and Satyanarayana M. *Semirings satisfying properties of distributive type,* Proceeding of the American Mathematical Society, Volume 82, Number 3, July 1981.

[2] Chinaram, R., *A note on quasi-ideal in ¡¡semirings*, Int. Math. Forum, 3 (2008), 1253{1259.

[3] Daddi. V. R and Pawar. Y. S. *Ideal Theory in Commutative Ternary A-semirings*, International Mathematical Forum, Vol. 7, 2012, no. 42, 2085 – 2091.

[4] Dixit, V.N. and Dewan, S., *A note on quasi and bi-ideals in ternary semigroups*, Int. J. Math. Math. Sci. 18, no. 3 (1995), 501{508.

[5] Dutta, T.K. and Kar, S., *On regular ternary semirings*, Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scienti¯c, New Jersey, 2003, 343-355.

[6] Dutta, T.K. and Kar, S., *A note on regular ternary semirings*, Kyung-pook Math. J., 46 (2006), 357{365.

[7] Jonathan S. Golan. *Semirings and Affine Equations over Them*: *Theory and Applications*, Kluwer Academic.

[8] Kar, S., *On quasi-ideals and bi-ideals in ternary semirings*, Int. J. Math. Math. Sc., 18 (2005), 3015-3023.

[9] Lehmer. D. H., *A ternary analogue of abelian groups,* Amer. J. Math., 59(1932), 329-338.

[10] Lister, W.G., *Ternary rings*, Trans Amer. Math.Soc., 154 (1971), 37-55.

[11] Madhusudhana Rao. D**.,** *Primary Ideals in Quasi-Commutative Ternary Semigroups* International Research Journal of Pure Algebra – 3(7), 2013, 254-258.

[12] M.Amala and T.Vasanthi, *Some Studies on Semirings and Ordered Semirings*, International Journal of Engineering Inventions, Vol.3, Issue 3(October, 2013), PP: 12-14.

Dr D. MadhusudhanaRao completed his M.Sc. from Osmania University, Hyderabad, Telangana, India. M. Phil. from M. K. University, Madurai, Tamil Nadu, India. Ph. D. from AcharyaNagarjuna University, Andhra Pradesh, India. He joined as Lecturer in Mathematics, in the department of Mathematics, VSR & NVR College, Tenali, A. P. India in the year 1997, after that he promoted as Head, Department of Mathematics, VSR & NVR College, Tenali. He helped more than 5 Ph.D's and at present he guided 5 Ph. D. Scalars and 3 M. Phil., Scalars in the department of Mathematics, AcharyaNagarjuna University, Nagarjuna Nagar, Guntur, A. P.A major part of his research work has been devoted to the use of semigroups, Gamma semigroups, duo gamma semigroups, partially ordered gamma semigroups and ternary semigroups, Gamma semirings and ternary semirings, Near rings ect. He acting as peer review member to the "***British Journal of Mathematics & Computer Science".*** He published more than **45 research papers** in different International Journals in the last three academic years.

G. Srinivasarao: He is working as an Assistant Professor in the Department of Applied Sciences & Humanities, Tirumala Engineering College. He completed his M.Phil. inMadhuraiKamaraj University . He was submitted his Ph.D. under the guidance of Dr. D.Madhusudanarao in Acharya Nagarjuna University, Guntur, A. P. India. He published more than 15 research papers in popular international Journals to his credit. His area of interests are ternary semirings, ordered ternary semirings, semirings and topology. Presently he is working on Ternary semirings.

P. Siva Prasad : He is working as Assistant Professor in the Department of Mathematics, Universal College of Engineering & Technology, Perecharla, Guntur, A. P. India. He completed his M.Phil. inMadhuraiKamaraj University . He is pursuing Ph.D. under the guidance of Dr. D. Madhusudanarao in Acharya Nagarjuna University.

* * * * *