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Multi-level Multi-objective Linear plus Linear Fractional Programming Problem Based on FGP Approach

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Abstract

In the paper, multi- level multi-objective linear plus linear fractional programming problem is presented. The objective functions of level decision makers are characterized by linear plus linear fractional form of decision variables. Linear system constraints are considered. Each level decision maker possesses more than one objective functions. Membership function for each objective function is constructed by taking individual best solution of each objective function as aspiration level. The non-linear membership functions are transformed into linear membership functions by using first order Taylor's series. Three FGP models are developed to solve the converted multi-objective multi- level problems with linear constraints. Euclidean distance function is used to select the best compromise solution. A numerical example is solved to illustrate the proposed approach.

Keywords: *Multi- level Programming, Fuzzy Goal Programming, Multi- level Multi – objective Programming, Linear Plus Linear Fractional Programming, Fractional Programming*

1. Introduction

Multi-level programming problem (MLPP) is used to deal hierarchical decision making problems. Burton [1], Bard and Falk [2], Anandalingam [3] established different models to solve multi-level systems. Lai [4] presented hierarchical optimization method and obtained a satisfactory solution based on fuzzy set theory. Shih et al. [5] solved MLP using fuzzy approach. Interactive fuzzy programming for multi-level linear programming problem was investigated by Sakawa et al. [6]. Sakawa et al. [7] applied genetic algorithm to solve multi- level 0 – 1 programming based on interactive fuzzy approach. Sinha [8, 9] presented fuzzy mathematical approach to solve MLPP. Pramanik and Roy [10] developed fuzzy goal programming (FGP) approach to solve MLPP. In 2010, Baky [11] proposed FGP approach to solve multi objective MLPP (MOMLPP). Additive FGP model for solving MOMLPP was presented by Arbaiy and Watada [12].

Multi-level linear fractional programming problem (MLLFPP) where objective functions are linear fraction of decision variables is a special type of non-linear MLPP. Lachhwani and Poonia [13] solved MLLFPP using FGP approach. They defined separate membership functions for numerator and denominator of each level fractional objective function. Recently, Dey et al. [14] proposed FGP models for MLLFPP and MOMLLFPP.

Pramanik and Banerjee [15] studied chance constrained multi-objective linear plus linear fractional programming problem based on first order Taylor's series approximation. Their concept has been further extended to chance constrained linear plus linear fractional bi-level programming problem [16]. The main objective of the paper is to present FGP models to solve multi-objective multi- level linear plus linear fractional programming problem (MOMLLPLFPP) with linear set of constraints. The proposed models are used to solve a numerical example.

The rest of the paper is designed in the following way. In Section 2, MOMLLPLFPP with linear set of constraints is formulated. In Section 3, non-linear membership functions are constructed for each objective function. Linearization technique for non- linear membership function and selection of compromise solution for each level are calculated in section 4. In Section 5, preference bounds on the decision variables are described. Three FGP models are developed for solving MOMLLPLFPP in the subsequent section 6. The Euclidean distance function is described in order to select the best compromise solution in the next section 7. In the section 8, the process for solving MOMLLPLFPP is step wise presented. Section 9 presents illustrative numerical example of MOMLLPLFPP. Conclusion and future work is discussed in the section 10.

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2. MOMLLPLFPP Formulation

General q (>2)-level MOMLLPLFPP can be formulated as follows:

$\underset{\bar{x}_1}{\text{Max}}\ Z_1(\bar{x}) = \underset{\bar{x}_1}{\text{Max}}\ (Z_{11}(\bar{x}), Z_{12}(\bar{x}), ..., Z_{1v_1}(\bar{x}))$ [1st level] (1)

$\underset{\bar{x}_2}{\text{Max}}\ Z_2(\bar{x}) = \underset{\bar{x}_2}{\text{Max}}\ (Z_{21}(\bar{x}), Z_{22}(\bar{x}), ..., Z_{2v_2}(\bar{x}))$ [2nd level] (2)

.
.
.

$\underset{x_q}{\text{Max}}\ Z_q(\bar{x}) = \underset{x_q}{\text{Max}}\ (Z_{q1}(\bar{x}), Z_{q2}(\bar{x}), ..., Z_{qv_q}(\bar{x}))$

[qth level] (3)

Subject to

$\bar{x} \in S = (\bar{x} : A\bar{x} \le B \text{ and } \bar{x} \ge 0).$ (4)

Here

$$A = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{13} \dots & \bar{a}_{1q} \\ \bar{a}_{21} & \bar{a}_{22} & \bar{a}_{23} \dots & \bar{a}_{2q} \\ \dots & \dots & \dots & \dots \\ \bar{a}_{p1} & \bar{a}_{p2} & \bar{a}_{p3} \dots & \bar{a}_{pq} \end{bmatrix}_{p\times q} \quad B = (b_1, b_2, ..., b_p)_{p\times 1}$$

$\bar{x} = \bar{x}_1 \cup \bar{x}_2 \cup ... \cup \bar{x}_q,\ \bar{x}_k = (x_{k1}, x_{k2}, .., x_{kn_k})$ (k = 1, 2, …, q). The k-th level DM controls $\bar{x}_k$ vector which consists of n_k variables. The total number of variables involved in the problem is $n = n_1 + n_2 + ... + n_i + ... + n_q$. $\bar{a}_{mk}$ (m = 1, 2, …, p; k = 1, 2, …, q) is a vector of components n_k and b_m (m = 1, 2, …, p) is scalar.

$$Z_{ij}(\bar{x}) = \sum_{k=1}^{q} c_{ij}^k \bar{x}_k + \frac{\sum_{k=1}^{q} g_{ij}^k \bar{x}_k + s_{ij}}{\sum_{k=1}^{q} h_{ij}^k \bar{x}_k + r_{ij}},\ (i = 1, 2, ..., q;\ j = 1, 2, ..., v_i) \quad (5)$$

Here, v_i, (i = 1, 2, ..., q) is the number of objective functions for i-th level DM, $c_{ij}^k, g_{ij}^k, h_{ij}^k$ (i = 1, 2, ..., q; j = 1, 2, ..., v_i, k = 1, 2,..., q) are vectors of n_k components and s_{ij}, t_{ij} are scalars. It is also assumed that $\sum_{k=1}^{q} h_{ij}^k \bar{x}_k + r_{ij} > 0$ for all $\bar{x} \in S$ and $S \ne \Phi$.

3. Construction of Membership Function for MOMLLPLFPP

Let $Z_{ij}^B = \underset{x \in S}{\text{Max}}\ Z_{ij}(\bar{x})$ and $Z_{ij}^W = \underset{x \in S}{\text{Min}}\ Z_{ij}(\bar{x})$, (i = 1, 2, ..., q; j = 1, 2, ..., v_i) be the individual best and worst solutions of the ij-th objective function. Then the fuzzy goals can be presented as follows:

$Z_{ij}(\bar{x}) \gtrsim Z_{ij}^B$, (i = 1, 2, ..., q; j = 1, 2, ..., v_i). (6)

The non-linear membership function $\mu_{ij}(Z_{ij}(\bar{x}))$ for the fuzzy objective goal defined in (6) can be formulated as follows:

$$\mu_{ij}(Z_{ij}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{ij}(\bar{x}) \ge Z_{ij}^B \\ \dfrac{Z_{ij}(\bar{x}) - Z_{ij}^W}{Z_{ij}^B - Z_{ij}^W}, & \text{if } Z_{ij}^W \le Z_{ij}(\bar{x}) \le Z_{ij}^B \\ 0, & \text{if } Z_{ij}(\bar{x}) \le Z_{ij}^W \end{cases},$$

(i = 1, 2, ... , q; j = 1, 2, ..., v_i) (7)

4. Selection of Compromise Solution for Each Level

The individual best solution points are generally different for different objective functions in the same level of the hierarchical organization. For the i-th level there are v_i numbers of objective functions. Selection of compromise solution for each level is described below.

4.1 Compromise solution for the first level

Let $\underset{\bar{x} \in S}{\text{Max}}\ \mu_{1j}$ occur at the point $\bar{x}^{1j} = (\bar{x}_1^{1j}, \bar{x}_2^{1j}, ..., \bar{x}_q^{1j})$ (j = 1, 2,..., v_1). Linearizing μ_{1j} about the point $\bar{x}^{1j} = (\bar{x}_1^{1j}, \bar{x}_2^{1j}, ..., \bar{x}_q^{1j})$ using first order Taylor series, we obtain

$$\mu_{1j}(Z_{1j}(\bar{x})) \approx \mu_{1j}(Z_{1j}(\bar{x}^{1j})) + \sum_{k=1}^{q} (\bar{x}_k - \bar{x}_k^{1j}) \left(\frac{\partial}{\partial x_k} \mu_{1j}(Z_{1j}(\bar{x}))\right)_{\bar{x}=\bar{x}^{1j}}$$

$$= \mu_{1j}^*(Z_{1j}(\bar{x})) \quad (8)$$

The FGP model is presented below in order to solve compromise solution for the 1st. level.

$\text{Min}\ \lambda_1$

$\mu_{1j}^*(Z_{1j}(\bar{x})) + d_{1j}^- = 1$ (j = 1, 2, ..., v_1) ,

$\lambda_1 \ge d_{1j}^-$,

$0 \le d_{1j}^- \le 1$

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$\bar{x} \in S$

The above model gives the compromise solution for the first level as $\bar{x}^{1*} = (\bar{x}_1^1, \bar{x}_2^1, ..., \bar{x}_q^1)$

4.2 Compromise solution for the second level

Let $\underset{\bar{x}\in S}{\text{Max}}\, \mu_{2j}$ occur at the point $\bar{x}^{2j} = (\bar{x}_1^{2j}, \bar{x}_2^{2j}, ..., \bar{x}_q^{2j})$ (j=1, 2,..., v_2) **.** Based on first order Taylor series linearize μ_{2j} about the point $\bar{x}^{2j} = (\bar{x}_1^{2j}, \bar{x}_2^{2j}, ..., \bar{x}_q^{2j})$, we obtain

$$\mu_{2j}(Z_{2j}(\bar{x})) \approx \mu_{2j}(Z_{2j}(\bar{x}^{2j})) + \sum_{k=1}^{q}(\bar{x}_k - \bar{x}_k^{2j})(\frac{\partial}{\partial x_k}\mu_{2j}(Z_{2j}(\bar{x})))_{\bar{x}=\bar{x}^{2j}}$$

$$= \mu_{2j}^{*}(Z_{2j}(\bar{x})) \tag{9}$$

The FGP model is presented below in order to solve compromise solution for the 2nd level.

$\text{Min}\,\lambda_2$

$\mu_{2j}^{*}(Z_{2j}(\bar{x})) + d_{2j}^{-} = 1 \;\; (j = 1, 2, ..., v_2)$,

$\lambda_2 \geq d_{2j}^{-}$,

$0 \leq d_{2j}^{-} \leq 1$

$\bar{x} \in S$

The above model gives the compromise solution for the first level as $\bar{x}^{2*} = (\bar{x}_1^2, \bar{x}_2^2, ..., \bar{x}_q^2)$

We proceed similarly for the other levels.

Let $\underset{x\in S}{\text{Max}}\, \mu_{qj}$ occur at the point $\bar{x}^{qj} = (\bar{x}_1^{qj}, \bar{x}_2^{qj}, ..., \bar{x}_q^{qj})$ (j=1, 2,..., v_q) **.** Based on first order Taylor series linearizing μ_{qj} about the point $\bar{x}^{qj} = (\bar{x}_1^{qj}, \bar{x}_2^{qj}, ..., \bar{x}_q^{qj})$, we obtain:

$$\mu_{qj}(Z_{qj}(\bar{x})) \approx \mu_{qj}(Z_{qj}(\bar{x}^{qj})) + \sum_{k=1}^{q}(\bar{x}_k - \bar{x}_k^{qj})(\frac{\partial}{\partial x_k}\mu_{qj}(Z_{qj}(\bar{x})))_{\bar{x}=\bar{x}^{qj}}$$

$$= \mu_{qj}^{*}(Z_{qj}(\bar{x})) \tag{10}$$

The FGP model is presented below in order to solve compromise solution for the q-th level.

$\text{Min}\,\lambda_q$

$\mu_{qj}^{*}(Z_{qj}(\bar{x})) + d_{qj}^{-} = 1 \;\; (j = 1, 2, ..., v_q)$,

$\lambda_q \geq d_{qj}^{-}$,

$0 \leq d_{qj}^{-} \leq 1$

$\bar{x} \in S$

The above model gives the compromise solution for the first level as $\bar{x}^{q*} = (\bar{x}_1^q, \bar{x}_2^q, ..., \bar{x}_q^q)$ (11)

Thus the compromise solution for i-th level can be obtained as:

$\bar{x}^{i*} = (\bar{x}_1^i, \bar{x}_2^i, ..., \bar{x}_i^i, ..., \bar{x}_q^i)$ (i = 1, 2, ... , q)

where $\bar{x}_i^i = (x_{i1}^i, x_{i2}^i, ..., x_{in_i}^i)$.

5. Selection of Upper and Lower Preference Bounds of Decision Vectors

In the multi-level decision making situation, it is observed that lower level DM cannot be satisfied with the decision of the upper level DM. As a result, decision deadlock arises. To overcome deadlock situation, each level DM provides some possible relaxation in terms of preference bounds of the decision vector under his / her control. The cooperation between DMs is useful to obtain the overall satisfactory solution.

Let $\bar{\tau}_i^L = (\tau_{i1}^L, \tau_{i2}^L, ..., \tau_{in_i}^L)$ and $\bar{\tau}_i^U = (\tau_{i1}^U, \tau_{i2}^U, ..., \tau_{in_i}^U)$, (i = 1, 2,..., q) be the preference lower and upper bounds vectors on the decision vector $\bar{x}_i = (x_{i1}, x_{i2}, ..., x_{in_i})$, controlled by the i- th level DM (i = 1, 2, ..., q). Thus, the i-th level DM controls n_i variables $(x_{i1}, x_{i2}, ..., x_{in_i})$ out of n variables. The compromise solution for the i-th level DM is $\bar{x}^{i*} = (\bar{x}_1^i, \bar{x}_2^i, ..., \bar{x}_i^i, ..., \bar{x}_q^i)$. The relaxation is given as follows:

$\bar{x}_i^i - \bar{\tau}_i^L \leq \bar{x}_i \leq \bar{x}_i^i - \bar{\tau}_i^U$ (i = 1, 2, ..., q) and $\bar{\tau}_i^L \neq \bar{\tau}_i^U$. More precisely, preference bounds of the decision variables can be presented as follows:

$x_{i1}^i - \tau_{i1}^L \leq x_{i1} \leq x_{i1}^i - \tau_{i1}^U$,

$x_{i2}^i - \tau_{i2}^L \leq x_{i2} \leq x_{i2}^i - \tau_{i2}^U$,

.

.

.

$$x_{in_i}^i - \tau_{in_i}^L \leq x_{in_i} \leq x_{in_i}^i - \tau_{in_i}^U \tag{12}$$

6. FGP Model Formulation

The FGP model for the MOMLLPLFPP can be written as:

$$\mu_{ij}^{*}(Z_{ij}(\bar{x})) + d_{ij}^{-} = 1 \;\; (i = 1, 2, ..., q;\; j = 1, 2, ..., v_i) \tag{13}$$

$\bar{x} \in S$

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$d_{ij}^{-}(\geq 0)$ is the negative deviational variable associated with the membership function $\mu_{ij}^{*}(Z_{ij}(x))$.

Three FGP models are presented below in order to solve MOMLLPLFPP.

6.1 FGP Model - 1

$$\text{Min}\,\lambda$$
$$\mu_{ij}^{*}(Z_{ij}(\bar{x})) + d_{ij}^{-} = 1\,,$$
$$x_{i1}^{i} - \tau_{i1}^{L} \leq x_{i1} \leq x_{i1}^{i} - \tau_{i1}^{U},$$
$$x_{i2}^{i} - \tau_{i2}^{L} \leq x_{i2} \leq x_{i2}^{i} - \tau_{i2}^{U},$$
$$\vdots$$
$$x_{in_i}^{i} - \tau_{in_i}^{L} \leq x_{in_i} \leq x_{in_i}^{i} - \tau_{in_i}^{U}$$
$$\lambda \geq d_{ij}^{-},$$
$$0 \leq d_{ij}^{-} \leq 1, \text{ and } \bar{x} \in S, (i = 1, 2, ..., q; j = 1, 2, ..., v_i) \quad (14)$$

6.2 FGP Model - 2

$$\text{Min}\,\nu = \sum_{i=1}^{q}\sum_{j=1}^{v_i} d_{ij}^{-}$$
$$\mu_{ij}^{*}(Z_{ij}(\bar{x})) + d_{ij}^{-} = 1\,,$$
$$x_{i1}^{i} - \tau_{i1}^{L} \leq x_{i1} \leq x_{i1}^{i} - \tau_{i1}^{U},$$
$$x_{i2}^{i} - \tau_{i2}^{L} \leq x_{i2} \leq x_{i2}^{i} - \tau_{i2}^{U},$$
$$\vdots$$
$$x_{in_i}^{i} - \tau_{in_i}^{L} \leq x_{in_i} \leq x_{in_i}^{i} - \tau_{in_i}^{U}$$
$$0 \leq d_{ij}^{-} \leq 1 \text{ and } \bar{x} \in S, (i = 1, 2, ..., q; j = 1, 2, ..., v_i) \quad (15)$$

6.3 FGP Model - 3

$$\text{Min}\,\eta = \sum_{i=1}^{q}\sum_{j=1}^{v_i} w_{ij} d_{ij}^{-}$$
$$\mu_{ij}^{*}(Z_{ij}(\bar{x})) + d_{ij}^{-} = 1 \; (i = 1, 2, ..., q; j = 1, 2, ..., v_i),$$
$$x_{i1}^{i} - \tau_{i1}^{L} \leq x_{i1} \leq x_{i1}^{i} - \tau_{i1}^{U},$$
$$x_{i2}^{i} - \tau_{i2}^{L} \leq x_{i2} \leq x_{i2}^{i} - \tau_{i2}^{U},$$
$$\vdots$$
$$x_{in_i}^{i} - \tau_{in_i}^{L} \leq x_{in_i} \leq x_{in_i}^{i} - \tau_{in_i}^{U}$$
$$0 \leq d_{ij}^{-} \leq 1 \text{ and } \bar{x} \in S$$

where, $w_{ij} = \dfrac{1}{Z_{ij}^{B} - Z_{ij}^{W}}$ $(i = 1, 2, ..., q; j = 1, 2, ..., v_i)$ (16)

7. Distance Function

Generally, three FGP models offer distinct solutions. It is essential to select a particular model for a particular problem which provides best solution. Euclidean distance function [17, 18, and 19] is used to identify the best FGP model. The Euclidean distance function is defined as:

$$D = \Big[\sum_{i=1}^{q}\sum_{j=1}^{v_i}\{1 - \mu_{ij}(Z_{ij}(\bar{x}))\}^2\Big]^{1/2} \quad (17)$$

The solution with minimum D is considered as the best compromise solution.

8. Summary

The present work is summarized in the following way:

1. 1. Consider the proposed MOMLLPLFPP with linear set of constraints.
2. 2. Find out the maximum and minimum values of each objective function subject to the system constraints.
3. 3. Using minimum and maximum values as lower and upper limits, non linear membership function for each objective function is constructed.
4. 4. Applying first order Taylor's series to approximate non linear membership function to linear membership function for each level.
5. 5. Calculate compromise solution for each level.
6. 6. Each DM provides relaxation on the upper and lower bounds of the decision variables under his / her control.
7. 7. Three FGP models are formulated for solution.
8. 8. Euclidean distance function is used to identify the best model.

9. Numerical Example

Consider the following MOMLLPLFPP with linear constraints

Example:

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$\underset{x_1}{\text{Max}}$ $(Z_{11}(\bar{x}) = (x_1+2) + (x_1+x_2+x_3)/(x_2+7),\ Z_{12}(\bar{x}) = x_1 + (x_1+2)/(x_2+2))$

$\underset{x_2}{\text{Max}}$ $(Z_{21}(\bar{x}) = (x_3+3) + (2x_1+2x_2+3x_3)/(x_3+6),\ Z_{22}(\bar{x}) = (x_3+x_1) + (x_2+2)/(x_1+2))$

$\underset{x_3}{\text{Max}}$ $(Z_{31}(\bar{x}) = (-x_1 - x_2 - x_3+6) + (x_1+6x_2+x_3+4)/(x_1+2),$

$Z_{32}(\bar{x}) = x_3 + (x_1+2)/(x_2+1))$ (18)

Subject to

$5x_1+x_2+x_3 \le 8,$
$x_1+x_3 \le 5,$
$-x_1+2x_2+x_3 \le 9,$
$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$ (19)

The individual best solution i.e. $Z^B_{ij} = \underset{x\in T}{\text{Max}}\ Z_{ij}(\bar{x})$, (i = 1, 2, 3; j= 1, 2, 3) of the objective function $Z_{ij}(\bar{x})$, (i = 1, 2, 3; j=1, 2, 3) subject to the constraints (19), is given in the Table 1.

Table 1: The individual best solutions of the objective functions

Maximum value of the objective functions	Z^B_{11} , Z^B_{12}	Z^B_{21} , Z^B_{22}	Z^B_{31} , Z^B_{32}
$Z^B_{ij} = \underset{x\in T}{\text{Max}}\ Z_{ij}(\bar{x})$, (i = 1, 2, 3; j= 1, 2, 3)	3.83 at (1.6, 0, 0) 3.4at (1.6, 0, 0)	9.72 at (0, 2, 5) 7 at (0, 2, 5)	17 at (0, 4.5, 0) 7 at (0, 0, 5)

The individual worst solution i.e. $Z^W_{ij} = \underset{x\in T}{\text{Min}}\ Z_{ij}(\bar{x})$, (i = 1, 2, 3; j= 1, 2, 3) of the objective function $Z_{ij}(\bar{x})$, (i = 1, 2, 3; j=1, 2, 3) subject to the constraints (19), is given in the Table 2.

Table 2: The individual worst solutions of the objective functions

Minimum value of the objective functions	Z^W_{11} , Z^W_{12}	Z^W_{21} , Z^W_{22}	Z^W_{31} , Z^W_{32}
$Z^W_{ij} = \underset{x\in T}{\text{Min}}\ Z_{ij}(\bar{x})$, (i = 1, 2, 3; j= 1, 2, 3)	2 at (0, 0, 0) 0.31 at (0, 4.5, 0)	3 at (0, 0, 0) 1 at (0, 0, 0)	4.27 at (0.75, 0, 4.25) 0.36 at (0, 4.5, 0)

Using (7), the non-linear membership function $\mu_{ij}(Z_{ij}(\bar{x}))$ (i = 1, 2, 3; j= 1, 2, 3) can be formulated as follows:

$$\mu_{11}(Z_{11}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{11}(\bar{x}) \ge 3.83 \\ \dfrac{Z_{11}(\bar{x}) - 2}{3.83 - 2}, & \text{if } 2 \le Z_{11}(\bar{x}) \le 3.83 \\ 0, & \text{if } Z_{11}(\bar{x}) \le 2 \end{cases},$$

$$\mu_{12}(Z_{12}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{12}(\bar{x}) \ge 3.4 \\ \dfrac{Z_{12}(\bar{x}) - 0.31}{3.4 - 0.31}, & \text{if } 0.31 \le Z_{12}(\bar{x}) \le 3.4 \\ 0, & \text{if } Z_{12}(\bar{x}) \le 0.31 \end{cases},$$

$$\mu_{21}(Z_{21}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{21}(\bar{x}) \ge 9.72 \\ \dfrac{Z_{21}(\bar{x}) - 3}{9.72 - 3}, & \text{if } 3 \le Z_{21}(\bar{x}) \le 9.72 \\ 0, & \text{if } Z_{21}(\bar{x}) \le 3 \end{cases},$$

$$\mu_{22}(Z_{22}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{22}(\bar{x}) \ge 7 \\ \dfrac{Z_{22}(\bar{x}) - 1}{7 - 1}, & \text{if } 1 \le Z_{22}(\bar{x}) \le 7 \\ 0, & \text{if } Z_{22}(\bar{x}) \le 1 \end{cases},$$

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$$\mu_{31}(Z_{31}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{31}(\bar{x}) \ge 17 \\ \dfrac{Z_{31}(\bar{x}) - 4.27}{17 - 4.27}, & \text{if } 4.27 \le Z_{31}(\bar{x}) \le 17 \\ 0, & \text{if } Z_{31}(\bar{x}) \le 4.27 \end{cases},$$

$$\mu_{32}(Z_{32}(\bar{x})) = \begin{cases} 1, & \text{if } Z_{32}(\bar{x}) \ge 7 \\ \dfrac{Z_{32}(\bar{x}) - 0.36}{7 - 0.36}, & \text{if } 0.36 \le Z_{32}(\bar{x}) \le 7 \\ 0, & \text{if } Z_{32}(\bar{x}) \le 0.36 \end{cases},$$

The linear membership functions are then constructed as follows:

$$\mu_{11}(Z_{11}(\bar{x})) \approx 1 + ((x_1 - 1.6) * (1 + 1/7) + x_2 * (7 - 1.6)/49 + x_3/7)/(3.83 - 2) = \mu_{11}^*(Z_{11}((\bar{x}))$$

$$\mu_{12}(Z_{12}(\bar{x})) \approx 1 + ((x_1 - 1.6) * 1.5 - x_2 * (3.6/4))/(3.4 - 0.31) = \mu_{12}^*(Z_{12}((\bar{x}))$$

$$\mu_{21}(Z_{21}(\bar{x})) \approx 1 + (x_1 * (2/11) + (x_2 - 2) * (2/11) + (x_3 - 5) * (1 + 14/121))/(9.72 - 3) = \mu_{21}^*(Z_{21}((\bar{x}))$$

$$\mu_{22}(Z_{22}(\bar{x})) \approx 1 + ((x_2 - 2) * 0.5 + x_3 - 5)/(7 - 1) = \mu_{22}^*(Z_{22}((\bar{x}))$$

$$\mu_{31}(Z_{31}(\bar{x})) \approx 1 + (x_1 * (1 + (2 - 6 * 4.5 - 4)/4) + (x_{2\backslash} - 4.5) * (-1 + 3) + x_3(-1/2))/(17 - 4.27) = \mu_{31}^*(Z_{31}((\bar{x}))$$

$$\mu_{32}(Z_{32}(\bar{x})) \approx 1 + (x_1 + 2 * x_2 + x_3 - 5)/(7 - 0.36) = \mu_{32}^*(Z_{32}((\bar{x})) \quad (20)$$

We are to calculate compromise solution for each level. The FGP model for the first level is given below:

$\text{Min}\,\lambda_1$

$1 + ((x_1 - 1.6) * (1 + 1/7) + x_2 * (7 - 1.6)/49 + x_3/7)/(3.83 - 2) + d_{11}^- = 1$

$1 + ((x_1 - 1.6) * 1.5 - x_2 * (3.6/4))/(3.4 - 0.31) + d_{12}^- = 1$

$\lambda_1 \ge d_{11}^-,$

$\lambda_1 \ge d_{12}^-$

$0 \le d_{11}^- \le 1$

$0 \le d_{12}^- \le 1$

and the system constraints given in (19).

The FGP model for the second level is given below:

$\text{Min}\,\lambda_2$

$1 + (x_1 * (2/11) + (x_2 - 2) * (2/11) + (x_3 - 5) * (1 + 14/121))/(9.72 - 3) + d_{21}^- = 1$

$1 + ((x_2 - 2) * 0.5 + x_3 - 5)/(7 - 1) + d_{22}^- = 1$

$\lambda_2 \ge d_{21}^-,$

$\lambda_2 \ge d_{22}^-$

$0 \le d_{21}^- \le 1$

$0 \le d_{22}^- \le 1$

and the system constraints given in (19).

The FGP model for the third level is given below:

$\text{Min}\,\lambda_3$

$1 + (x_1 * (1 + (2 - 6 * 4.5 - 4)/4) + (x_{2\backslash} - 4.5) * (-1 + 3) + x_3(-1/2))/(17 - 4.27) + d_{31}^- = 1$

$1 + (x_1 + 2 * x_2 + x_3 - 5)/(7 - 0.36) + d_{32}^- = 1$

$\lambda_3 \ge d_{31}^-,$

$\lambda_3 \ge d_{32}^-$

$0 \le d_{31}^- \le 1$

$0 \le d_{32}^- \le 1$

and the system constraints given in (19).

Solving the above model, the compromise solution for first, second and third levels are obtained respectively as follows:

$x_1 = 1.6,\ x_2 = 0,\ x_3 = 0.$

$x_1 = 0,\ x_2 = 2,\ x_3 = 5.$

$x_1 = 0,\ x_2 = 2.5,\ x_3 = 0.$

The preference bounds of the decision variables given by three levels DM are:

$1 \le x_1 \le 2,\ 0 \le x_2 \le 2.5,\ 0 \le x_3 \le 1.$ (21)

Using FGP models (14), (15), (16) the solutions are presented in the following table 3.

Table 3: The optimal solutions

FGP Model - 1	*FGP Model - 2*	*FGP Model - 3*
$x_1 = 1$	$x_1 = 1$	$x_1 = 1$
$x_2 = 2$	$x_2 = 1.51$	$x_2 = 1.51$
$x_3 = 0$	$x_3 = 0.99$	$x_3 = 0.99$
$\mu_{11} = 0.7286$	$\mu_{11} = 0.77$	$\mu_{11} = 0.77$
$\mu_{12} = 0.466$	$\mu_{12} = 0.5$	$\mu_{12} = 0.5$
$\mu_{21} = 0.149$	$\mu_{21} = 0.32$	$\mu_{21} = 0.32$
$\mu_{22} = 0.222$	$\mu_{22} = 0.36$	$\mu_{22} = 0.36$

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$\mu_{31} = 0.345$	$\mu_{31} = 0.255$	$\mu_{31} = 0.255$
$\mu_{32} = 0.096$	$\mu_{32} = 0.275$	$\mu_{32} = 0.275$
$Z_{11} = 3.33$	$Z_{11} = 3.41$	$Z_{11} = 3.41$
$Z_{12} = 1.75$	$Z_{12} = 1.855$	$Z_{12} = 1.855$
$Z_{21} = 4$	$Z_{21} = 5.13$	$Z_{21} = 5.13$
$Z_{22} = 2.33$	$Z_{22} = 3.155$	$Z_{22} = 3.155$
$Z_{31} = 8.67$	$Z_{31} = 7.515$	$Z_{31} = 7.515$
$Z_{32} = 1$	$Z_{32} = 2.18$	$Z_{32} = 2.18$
D = 2.94	D = 2.255	D = 2.255

The FGP model – 2 and FGP model – 3 offer the solution with minimum D. Thus the compromise optimal solution is shown in the table 4.

Table 4: The compromise optimal solution

	x_1, x_2, x_3	$Z_{11}, Z_{12}, Z_{21}, Z_{22}, Z_{31}, Z_{32}$	$\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \mu_{31}, \mu_{32}$
FGP Model - 2 and FGP Model - 3	(1, 1.51, 0.99)	3.41, 1.855, 5.13, 3.155, 7.515, 2.18	0.77, 0.5, 0.32, 0.36, 0.255, 0.275

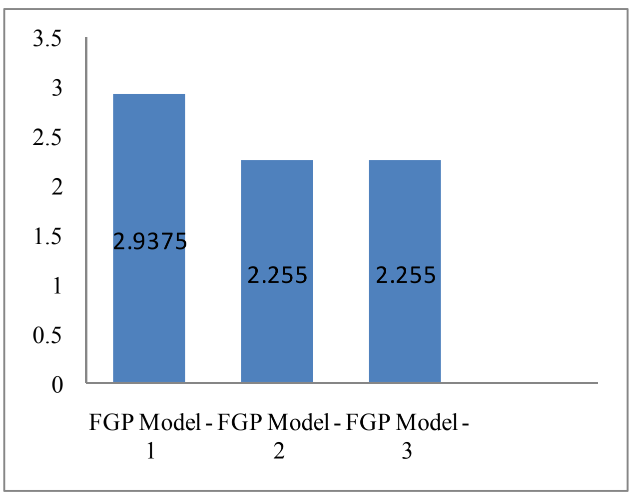

Fig. 1 Comparison of Distances obtained from three FGP models

10. Conclusion

Three FGP models are developed for dealing with MOMLLPLFPP. Distance function is used to select the best compromise solution. Numerical example is provided to illustrate the proposed FGP models.

The proposed approach can be further extended to solve decentralized MOMLLPLFPP, chance constrained MOMLLPLFPP, MOMLLPLFPP with fuzzy coefficients and to solve many real decision making problems involving MOMLLPLFPPs.

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