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A New Perspective for solving Fully Fuzzy Multiple Objective Linear Bi level Programming Problems

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Abstract

In this paper a new method is proposed to solve a fully fuzzy objective linear bi level programming problem (FFMOLBLPP). Using the utility the MOLBLPP is transferred linear bi level programming problem and this problem is simply solved by one of the fuzzy approaches. A numerical example is then given to show applicability of the proposed approach.

Keywords: *Triangular fuzzy number, Multiple-objective linear bi level programming, fully fuzzy system.*

Introduction

Bi level programming problems is closely related to the economic problem of Stackelberg[1] in the filed of game theory. the concept of fuzzy linear programming was first proposed by Zimmermann[2]. Bellmam and Zadeh [3] proposed the concept of decision making in fuzzy environment Maleki [4] introduced a new method for solving linear programming with vagueness in constraints by using ranking function.

Pandian and Jayalashmi[5] proposed a new for solving fully fuzzy linear programming problem with fuzzy variables. Pandian and Jayalashmi[6] introduced a new method for finding an optimal fuzzy solution for fuzzy linear programming problems. Kumar [7] proposed a method for solving fully fuzzy linear programming problems. Safaei [8] proposed a new method for solving fully fuzzy linear bi level programming problems. In this paper a new method for solving a FFMOLBLPP is presented. This paper is organized as follows: In section 2 formulation of FFMOLBLP problems is Introduced. In section 3 method of steps for solving FFMOLBLP problems is proposed. In section 4 we solve an illustrative numerical example.

2. Fully fuzzy objective linear bi level programming problem

For $x \in X \subset R^n, y \in Y \subset R^m, F_{1,2} : X \times Y \to R$ $and \;\; f_{1,2} : X \times Y \to R$

$$\max_{\tilde{x}\in X} F_1 = \tilde{a} \otimes \tilde{x} \oplus \tilde{b} \otimes \tilde{y}$$
$$\max_{\tilde{x}\in X} F_2 = \tilde{c} \otimes \tilde{x} \oplus \tilde{d} \otimes \tilde{y}$$
$$s.t.: \quad \max_{\tilde{y}\in Y} f_1 = \tilde{m} \otimes \tilde{x} \oplus \tilde{n} \otimes \tilde{y}$$
$$\max_{\tilde{y}\in Y} f_2 = \tilde{p} \otimes \tilde{x} \oplus \tilde{q} \otimes \tilde{y}$$
$$s.t. \; \tilde{A} \otimes \tilde{x} \oplus \tilde{B} \otimes \tilde{y} = \tilde{t}$$
$$\tilde{x}, \tilde{y} \tilde{\geq} 0$$

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Where $a,c,m,p \in R^n, b,d,n,q \in R^m$

$, t \in R^p, A \in R^{p \times n}, B \in R^{p \times m}$.

3. The proposed method

In this section we describe the proposed method.
The steps are as follow.

Step1: The weighting problem of (FFMLBLLP) takes the from:

$$\begin{aligned}
\max_{\tilde{x}\in X} F &= w_1\left(\tilde{a}\otimes\tilde{x}\oplus\tilde{b}\otimes\tilde{y}\right)\oplus \\
&\quad w_2\left(\tilde{c}\otimes\tilde{x}\oplus\tilde{d}\otimes\tilde{y}\right) \\
s.t.:\quad \max_{\tilde{y}\in Y} f &= w_1\left(\tilde{m}\otimes\tilde{x}\oplus\tilde{n}\otimes\tilde{y}\right)\oplus \\
&\quad w_2\left(\tilde{p}\otimes\tilde{x}\oplus\tilde{q}\otimes\tilde{y}\right) \\
&s.t.\ \tilde{A}\otimes\tilde{x}\oplus\tilde{B}\otimes\tilde{y}=\tilde{t} \\
&\tilde{x},\tilde{y}\,\tilde{\geq}\,0\ ,\ w_1+w_2=1
\end{aligned}$$

Step2: By preference vector approach (FFMLBLLP) is transferred to a fuzzy linear bi level problem:

$$\begin{aligned}
\max_{\tilde{x}\in X} F &= \left(\tilde{a}\otimes\tilde{x}\oplus\tilde{b}\otimes\tilde{y}\right)\oplus\left(\tilde{c}\otimes\tilde{x}\oplus\tilde{d}\otimes\tilde{y}\right) \\
s.t.:\quad \max_{\tilde{y}\in Y} f &= \left(\tilde{m}\otimes\tilde{x}\oplus\tilde{n}\otimes\tilde{y}\right)\oplus\left(\tilde{p}\otimes\tilde{x}\oplus\tilde{q}\otimes\tilde{y}\right) \\
&s.t.\ \tilde{A}\otimes\tilde{x}\oplus\tilde{B}\otimes\tilde{y}=\tilde{t} \\
&\tilde{x},\tilde{y}\,\tilde{\geq}\,0
\end{aligned}$$

Step3: Substituting

$$\begin{aligned}
&\tilde{a}=[\tilde{a}_j]_{1\times n}, \tilde{c}=[\tilde{c}_j]_{1\times n}, \tilde{m}=[\tilde{m}_j]_{1\times n} \\
&, \tilde{p}=[\tilde{p}_j]_{1\times n}, \tilde{x}=[\tilde{x}_j]_{n\times 1}, \tilde{y}=[\tilde{y}_j]_{m\times 1} \\
&\tilde{b}=[\tilde{b}_j]_{1\times m}, \tilde{d}=[\tilde{d}_j]_{1\times m}, \tilde{n}=[\tilde{n}_j]_{1\times m}, \tilde{q}=[\tilde{q}_j]_{1\times m} \\
&, \tilde{A}=[\tilde{A}_{ij}]_{p\times n}, \tilde{B}=[\tilde{B}_{ij}]_{p\times m}, \tilde{t}=[\tilde{t}_j]_{m\times 1}
\end{aligned}$$

The above (FFLBL) problem may be written as:

$$\begin{aligned}
\max_{\tilde{x}\in X} F &= \left(\sum_{j=1}^{n}\left(\tilde{a}_j\otimes\tilde{x}_j\right)\oplus\sum_{j=1}^{m}\left(\tilde{b}_j\otimes\tilde{y}_j\right)\right)\oplus \\
&\quad\left(\sum_{j=1}^{n}\left(\tilde{c}_j\otimes\tilde{x}_j\right)\oplus\sum_{j=1}^{m}\left(\tilde{d}_j\otimes\tilde{y}_j\right)\right) \\
s.t.:\quad \max_{\tilde{y}\in Y} f &= \left(\sum_{j=1}^{n}\left(\tilde{m}_j\otimes\tilde{x}_j\right)\oplus\sum_{j=1}^{m}\left(\tilde{n}_j\otimes\tilde{y}_j\right)\right)\oplus \\
&\quad\left(\sum_{j=1}^{n}\left(\tilde{p}_j\otimes\tilde{x}_j\right)\oplus\sum_{j=1}^{m}\left(\tilde{q}_j\otimes\tilde{y}_j\right)\right) \\
&s.t.\ \sum_{j=1}^{n}\left(\tilde{a}_{ij}\otimes\tilde{x}_j\right)\oplus\sum_{j=1}^{m}\left(\tilde{b}_{ij}\otimes\tilde{y}_j\right)\leq\tilde{t}_i \\
&\tilde{x},\tilde{y}\,\tilde{\geq}\,0\ ,\ \forall i=1,2,\ldots,m.
\end{aligned}$$

Step4: If all the parameters

$$\begin{aligned}
&\tilde{a}_j=\left(a_{1j},a_{2j},a_{3j}\right),\tilde{b}_j=\left(b_{1j},b_{2j},b_{3j}\right),\tilde{c}_j=\left(c_{1j},c_{2j},c_{3j}\right) \\
&\tilde{d}_j=\left(d_{1j},d_{2j},d_{3j}\right),\tilde{m}_j=\left(m_{1j},m_{2j},m_{3j}\right),\tilde{n}_j=\left(n_{1j},n_{2j},n_{2j}\right), \\
&\tilde{p}_j=\left(p_{1j},p_{2j},p_{3j}\right),\tilde{q}_j=\left(q_{1j},q_{2j},q_{3j}\right) \\
&\tilde{A}_{ij}=\left(a_{1ij},a_{2ij},a_{3ij}\right),\tilde{B}_{ij}=\left(b_{1ij},b_{2ij},b_{3ij}\right),\tilde{t}_i=\left(t_{1i},t_{2i},t_{3i}\right), \\
&\tilde{x}_j=\left(x_{1j},x_{2j},x_{3j}\right),\tilde{y}_j=\left(y_{1j},y_{2j},y_{3j}\right)
\end{aligned}$$

By triangular fuzzy numbers are represented,
then the (FFLBL) problem, obtained in Step3, may be written as:

$$\begin{aligned}
\max_{\tilde{x}\in X} F &= \left(\sum_{j=1}^{n}\left(\left(a_{1j},a_{2j},a_{3j}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)\right)\oplus\sum_{j=1}^{m}\left(\left(b_{1j},b_{2j},b_{3j}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)\right)\right)\oplus \\
&\quad\left(\sum_{j=1}^{n}\left(\left(c_{1j},c_{2j},c_{3j}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)\right)\oplus\sum_{j=1}^{m}\left(\left(d_{1j},d_{2j},d_{3j}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)\right)\right) \\
s.t.:\quad \max_{\tilde{y}\in Y} f &= \left(\sum_{j=1}^{n}\left(\left(m_{1j},m_{2j},m_{3j}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)\right)\oplus\sum_{j=1}^{m}\left(\left(n_{1j},n_{2j},n_{2j}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)\right)\right)\oplus \\
&\quad\left(\sum_{j=1}^{n}\left(\left(p_{1j},p_{2j},p_{3j}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)\right)\oplus\sum_{j=1}^{m}\left(\left(q_{1j},q_{2j},q_{3j}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)\right)\right) \\
&s.t.\ \sum_{j=1}^{n}\left(\left(a_{1ij},a_{2ij},a_{2ij}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)\right)\oplus\sum_{j=1}^{m}\left(\left(b_{1ij},b_{2ij},b_{3ij}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)\right)=\left(t_{1i},t_{2i},t_{3i}\right) \\
&\left(x_{1j},x_{2j},x_{3j}\right),\left(y_{1j},y_{2j},y_{3j}\right)\,\tilde{\geq}\,0\ ,\ \forall i=1,2,\ldots,m
\end{aligned}$$

Step5: Assuming

$$\begin{aligned}
&\left(a_{1ij},a_{2ij},a_{3ij}\right)\otimes\left(x_{1j},x_{2j},x_{3j}\right)=\left(g_{1ij},g_{2ij},g_{3ij}\right), \\
&\left(b_{1ij},b_{2ij},b_{3ij}\right)\otimes\left(y_{1j},y_{2j},y_{3j}\right)=\left(u_{1ij},v_{2ij},s_{3ij}\right)
\end{aligned}$$

The (FFLBL) problem, obtained in Step4, may be written as:

$$\max_{\tilde{x}\in X} F = \Re\left(\begin{array}{l}\left(\sum_{j=1}^{n}\left((a_{1j},a_{2j},a_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((b_{1j},b_{2j},b_{3j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\oplus\\ \left(\sum_{j=1}^{n}\left((c_{1j},c_{2j},c_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((d_{1j},d_{2j},d_{3j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\end{array}\right)$$

$$s.t.:\quad \max_{\tilde{y}\in Y} f = \Re\left(\begin{array}{l}\left(\sum_{j=1}^{n}\left((m_{1j},m_{2j},m_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((n_{1j},n_{2j},n_{2j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\oplus\\ \left(\sum_{j=1}^{n}\left((p_{1j},p_{2j},p_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((q_{1j},q_{2j},q_{3j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\end{array}\right)$$

$$s.t.\ \sum_{j=1}^{n}(g_{1ij},g_{2ij},g_{3ij})\oplus\sum_{j=1}^{m}(u_{1ij},v_{2ij},s_{3ij})=(t_{1i},t_{2i},t_{3i})$$

$$(x_{1j},x_{2j},x_{3j}),(y_{1j},y_{2j},y_{3j})\tilde{\geq}0.,\ \forall i=1,2,\ldots,m$$

$$where: \Re(\tilde{A})=\frac{1}{4}(a+2b+c)\ for\ \tilde{A}=(a,b,c).$$

Step6: The fuzzy linear bi level programming problem , obtained in Step5, is converted into the following CLBLP problem:

$$\max_{\tilde{x}\in X} F = \Re\left(\begin{array}{l}\left(\sum_{j=1}^{n}\left((a_{1j},a_{2j},a_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((b_{1j},b_{2j},b_{3j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\oplus\\ \left(\sum_{j=1}^{n}\left((c_{1j},c_{2j},c_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((d_{1j},d_{2j},d_{3j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\end{array}\right)$$

$$s.t.:\quad \max_{\tilde{y}\in Y} f = \Re\left(\begin{array}{l}\left(\sum_{j=1}^{n}\left((m_{1j},m_{2j},m_{3j})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((n_{1j},n_{2j},n_{2j})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\oplus\\ \left(\sum_{j=1}^{n}\left((p_{ij},p_{ij},p_{ij})\otimes(x_{1j},x_{2j},x_{3j})\right)\oplus\sum_{j=1}^{m}\left((q_{ij},q_{ij},q_{ij})\otimes(y_{1j},y_{2j},y_{3j})\right)\right)\end{array}\right)$$

$$s.t.:\left(\sum_{j=1}^{n}g_{1ij}\right)\oplus\left(\sum_{j=1}^{n}u_{1ij}\right)=t_{1i},\ \left(\sum_{j=1}^{n}g_{2ij}\right)\oplus\left(\sum_{j=1}^{n}u_{2ij}\right)=t_{2i},\ \left(\sum_{j=1}^{n}g_{3ij}\right)\oplus\left(\sum_{j=1}^{n}u_{3ij}\right)=t_{3i}$$

$$x_{2j}-x_{1j}\geq 0,\ x_{3j}-x_{2j}\geq 0,\ y_{2j}-y_{1j}\geq 0,\ y_{3j}-y_{2j}\geq 0,\ \forall i=1,2,\ldots,m$$

Step7: Find the optimal solution $\left(x_{1j},x_{2j},x_{3j}\right)$ and $\left(y_{1j},y_{2j},y_{3j}\right)$ by solving CLBLP problem obtained in Step6.

Step8: Find the fuzzy optimal solution by putting the values of $\left(x_{1j},x_{2j},x_{3j}\right)$ in $\tilde{x}_j=\left(x_{1j},x_{2j},x_{3j}\right)$ and $\left(y_{1j},y_{2j},y_{3j}\right)$ in $\tilde{y}_j=\left(y_{1j},y_{2j},y_{3j}\right)$.

Step9: Find the fuzzy optimal value by putting $\tilde{x}_j$ and $\tilde{y}_j$ in F and f.

4. Numerical Example

In this a numerical example is given to show Applicability for the proposed method.
Let us consider the following FFMLBLLP.

$$\underset{\tilde{x}_1}{Maximise} F_1=\left((1,4,4)\otimes\tilde{x}_1\oplus(2,4,8)\otimes\tilde{x}_2\right)$$

$$\underset{\tilde{x}_1}{Maximise} F_2=\left((0,2,4)\otimes\tilde{x}_1\oplus(2,6,4)\otimes\tilde{x}_2\right)$$

$$s.t.:\ \underset{\tilde{x}_2}{Maximise} f_1=\left((2,4,6)\otimes\tilde{x}_1\oplus(1,6,8)\otimes\tilde{x}_2\right)$$

$$\underset{\tilde{x}_2}{Maximise} f_1=\left((0,4,6)\otimes\tilde{x}_1\oplus(2,6,8)\otimes\tilde{x}_2\right)$$

$$s.t.:\ (2,3,4)\otimes\tilde{x}_1\oplus(1,2,3)\otimes\tilde{x}_2=(6,16,30)$$

$$(1-,1,2)\otimes\tilde{x}_1\oplus(1,3,4)\otimes\tilde{x}_2=(1,17,30)\ \tilde{x}_1,\ \tilde{x}_2\geq 0.$$

Solution: Let $\left(w_1,w_2\right)=\left(0.5,0.5\right)$, then given FFMLBLLP may be written as:

$$\underset{\tilde{x}_1}{Maximise}\left((0.5,2,2)\otimes\tilde{x}_1\oplus(1,2,4)\otimes\tilde{x}_2\right)\oplus$$

$$\left((0,1,2)\otimes\tilde{x}_1\oplus(1,3,2)\otimes\tilde{x}_2\right)=$$

$$\underset{\tilde{x}_1}{Maximise}\left((1.5,4,6)\otimes\tilde{x}_1\oplus(1,3,6)\otimes\tilde{x}_2\right)$$

$$s.t.:\ \underset{\tilde{x}_2}{Maximise}\left((1,2,3)\otimes\tilde{x}_1\oplus(0.5,3,4)\otimes\tilde{x}_2\right)\oplus$$

$$\left((0,2,3)\otimes\tilde{x}_1\oplus(1,3,4)\otimes\tilde{x}_2\right)=$$

$$\underset{\tilde{x}_2}{Maximise}\left((1.5,5,7)\otimes\tilde{x}_1\oplus(1,5,7)\otimes\tilde{x}_2\right)$$

$$s.t.:\ (2,3,4)\otimes\tilde{x}_1\oplus(1,2,3)\otimes\tilde{x}_2=(6,16,30)$$

$$(1-,1,2)\otimes\tilde{x}_1\oplus(1,3,4)\otimes\tilde{x}_2=(1,17,30)$$

$$\tilde{x}_1,\ \tilde{x}_2\geq 0.$$

Let $\tilde{x}_1=\left(x_{11},x_{12},x_{13}\right)$ and $\tilde{x}_2=\left(x_{21},x_{22},x_{23}\right)$

the given FFMLBLLP may be written as:

$$\underset{(x_{11},x_{12},x_{13})}{Maximise}\begin{pmatrix}(1.5,4,6)\otimes(x_{11},x_{12},x_{13})\oplus\\(1,3,6)\otimes(x_{21},x_{22},x_{23})\end{pmatrix}$$

$s.t.$

$$\underset{(x_{21},x_{22},x_{23})}{Maximise}\begin{pmatrix}(1.5,5,7)\otimes(x_{11},x_{12},x_{13})\oplus\\(1,5,7)\otimes(x_{21},x_{22},x_{23})\end{pmatrix}$$

$$s.t.:\ (2,3,4)\otimes(x_{11},x_{12},x_{13})\oplus$$
$$(1,2,3)\otimes(x_{21},x_{22},x_{23})=(6,16,30)$$
$$(-1,1,2)\otimes(x_{11},x_{12},x_{13})\oplus$$
$$(1,3,4)\otimes(x_{21},x_{22},x_{23})=(1,17,30).$$
$$(x_{11},x_{12},x_{13})\geq 0\ ,\ (x_{21},x_{22},x_{23})\geq 0.$$

Using Step5, the above FFLBL problem may be written as:

$$\underset{(x_{11},x_{12},x_{13})}{Maximise}\,\Re(1.5x_{11}+x_{21},4x_{12}+3x_{22},6x_{13}+6x_{23})$$

$s.t.$

$$\underset{(x_{21},x_{22},x_{23})}{Maximise}\,\Re(1.5x_{11}+5x_{21},5x_{12}+5x_{22},7x_{13}+7x_{23})$$
$$s.t.:\ (2x_{11}+x_{21},3x_{12}+2x_{22},4x_{13}+3x_{23})=(6,16,30)$$
$$(-x_{11}+x_{21},x_{12}+3x_{22},2x_{13}+4x_{23})=(1,17,30)$$
$$(x_{11},x_{12},x_{13})\geq 0\ ,\ (x_{21},x_{22},x_{23})\geq 0.$$

Using Step6 of the proposed method the above FFLBL problem is converted into the following CLBLP problem:

$$\underset{(x_{11},x_{12},x_{13})}{Maximise}(0.25(1.5x_{11}+x_{21}+8x_{12}+6x_{22}+6x_{13}+6x_{23}))$$

$s.t.$

$$\underset{(x_{21},x_{22},x_{23})}{Maximise}(0.25(1.5x_{11}+5x_{21}+10x_{12}+10x_{22}+7x_{13}+7x_{23}))$$
$$s.t.:\ 2x_{11}+x_{21}=6\ ,\ 3x_{12}+2x_{22}=16\ ,4x_{13}+3x_{23}=30$$
$$-x_{11}+x_{21}=1\ ,\ x_{12}+3x_{22}=17\ ,\ 2x_{13}+4x_{23}=30$$
$$x_{12}-x_{11}\geq 0\ ,\ x_{13}-x_{12}\geq 0\ ,\ x_{22}-x_{21}\geq 0\ ,\ x_{23}-x_{22}\geq 0$$

The fuzzy optimal solution is given by

$$\tilde{x}_1=(1.6667,2.0000,3.0000)$$
$$\tilde{x}_2=(2.6667,5.0000,6.0000),$$

$$F_1=(7.0001,28,60)\ ,\ F_2=(5.3334,34,36)$$
$$f_1=(6.0001,38,66)\ ,\ f_2=(5.0001,38,66).$$

5. Conclusion

In this article a systematic approach for solving fully fuzzy Multi objective linear bi level programming problem is used. By utility vector approach multi objective linear bi level programming problem transferred to linear bi level programming problem and this problem is simply solved by one of the fuzzy approaches. Finally, an example is illustrated for the proposed algorithm.

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