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Stability of the Quartic double centralizers and Quartic multipliers on non-Archimedean Banach algebras

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ABSTRACT

In this paper, we establish stability of quartic double centralizers and quartic multipliers on non-Archimedean Banach algebras.

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INTRODUCTION

The stability problem of functional equations originated from a question of Ulam [28] in 1940, concerning the stability of group homomorphisms. In 1941, D.H. Hyers [12] gave a first affirmative answer to the question of Ulam for Banach space. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \delta$$

for all $x, y \in E$, and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \delta$$

for all $x \in E$. Moreover, if $f(tx)$ is continuous in t for each fixed $x \in E$, then T is linear. In 1950, T. Aoki [1] was the second author to treat this problem for additive mapping. Finally in 1978, Th. M. Rassias [24] proved the following Theorem:
Theorem (Th. M. Rassias). Let $f : E \to E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \varepsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in E$, where ε and p are constants with $\varepsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \frac{2\varepsilon}{2-2^p} \|x\|^p$$

for all $x \in E$. Also, if the function $t \to f(tx)$ from R into E' is continuous for each fixed x in E, then T is linear.
This stability phenomenon of this kind is called the Hyers-Ulam-Rassias stability. In 1991, Z. Gajda [9] answered the question for the case $p < 1$, which was raised by Rassias. In 1994, a generalization of the Rassias' theorem was obtained by Gavruta as follows [10]. We refer the readers to [3]-[6], [7], [8], [13]-[15], [19], [22], [25], [26] and references therein for more detailed results on the stability problems of various functional equations.
The functional equation is called stable if any function satisfying that functional equation "approximately" is near to a true solution of functional equation. We say that a functional equation is superstable if every approximately solution is an exact solution of it.
Suppose that A is a Banach algebra. A linear mapping $L : A \to A$ is said to be left centralizer on A if $L(ab) = L(a)b$ for all $a, b \in A$. Similarly, a linear mapping $R : A \to A$ that $R(ab) = aR(b)$ for all $a, b \in A$ is called right centralized on A. A double centralizer on A is a pair (L, R), where L is a left centralizer, R is a right centralizer and $aL(b) = R(a)b$ for all

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$a, b \in A$. For example, (L_c, R_c) is a double centralizer, where $L_c(a) := ca$ and $R_c(a) := ac$. The set $D(A)$ of all double centralizers equipped with the multiplication $(L_1, R_1).(L_2, R_2) = (L_1L_2).(R_1R_2)$ is an algebra. The notion of double centralizer was introduced by Hochschild [11] and by Johnson [16]. Johnson [16] proved that if A is an algebra satisfying $A_l(A) = A_r(A) = \{0\}$, and L, R are mappings on A fulfilling $aL(b) = R(a)b$, ($a, b \in A$), then (L, R) is a double centralizer.

We can show that if $A^2 = A$ or $A_l(A) \cap A_r(A) = \{0\}$, then $L = R$ if and only if L and R are both left and right centralizer.

The functional equation

$$f(2x+y)+f(2x-y)=4f(x+y)+4f(x-y)+24f(x)-6f(y) \qquad (1)$$

is called the quartic functional equation, since the function $f(x) = x^4$ is a solution of this functional equation. Note that f is called quartic because of the identity

$$(2x+y)^4+(2x-y)^4=4(x+y)^4+4(x-y)^4+24x^4-6y^4 \qquad (2)$$

A Banach algebra A is said to be quartic commutative if $(ab)^4 = a^4b^4$ for all $a, b \in A$. We can show that there is a Banach algebra quartic commutative that is not commutative (see Example 1.4 of the present paper).

Every solution of the quartic functional equation is said to be a quartic mapping. It is In proved in [18] that a function $f : X \to Y$ between real normed spaces is quartic if and only if there exists a symmetric biquadratic function $f : X \times X \to Y$ such that $f(x) = F(x, x)$ for all $x \in X$. The first result on the stability of the quartic functional equation was obtained by J. M. Rassias [23]. Also H. -M. Kim [17], S. H. Lee, S. M. Im and I. S. Hwang [18], Najati [19] and C. Park [24] investigated the stability of quartic functional equation.

Let K be a field. A non- Archimedean absolute value on K is a function $|.| : K \to R$ such that for any $a, b \in K$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,

(ii) $|ab| = |a||b|$,

(iii) $|a+b| \leq \max\{|a|, |b|\}$.

The condition (iii) is called the strict triangle inequality. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integern. We always assume in addition that $|.|$ is non trivial, i.e., that there is an a_0 ∈k such that $|a_0| \notin \{0,1\}$.

Let X be a linear space over a scalar field K with a non- Archimedean non- trivial Valuation $|.|$. A function $\|.\| : X \to R$ is a non- Archimedean norm (valuation) if it satisfies the following conditions:

(NA1) $\|x\| = 0$ if and only if $x = 0$;

(NA2) $\|rx\| = |r|\|x\|$ for all $r \in K$ and $x \in X$;

(NA3) the strong triangle inequality (ultrametrie); namely,

$$\|x+y\| \leq \max\{\|x\|, \|y\|\} \qquad (x, y \in X).$$

Then $(X, \|.\|)$ is called a non-Archimedean spase.

It follows from (NA3) that

$$\|x_m + x_l\| \leq \max\{\|x_{j+1} - x_j\| : l \leq j \leq m-1\} \qquad (m > l),$$

Therefore a sequence $\{x_m\}$ is Cauchy in X if and if $\{x_{m+1}-x_m\}$ converges to zero in non-Archimedean space. By a complete non-Archimedean space we mean on in which every Cauchy sequence is convergent. A non-Archimedean Banach algebra is a complete non-Archimedean algebra A wich satisfies $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in A$. For more detailed definitions of non-Archimedean Banach algebra, we can refer to [27].

Recently, Baghban and Molaei [2] established the stability of double centralizers to quadratic functional equations in the framework of non-Archimedean Banach algebras. They also proved the superstability of double centralizers on non-Archimedean Banach algebras which are weakly without order as follows.

Theorem. Suppose that $s \in \{-1,1\}$ and that $f : A \to A$ is a mapping with $f(0) = 0$ for which there exist a mapping $g : A \to A$ with $g(0) = 0$ and functions $\phi_j : A \times A \times A \times A \to [0, \infty), \psi_i : A \times A \to [0, \infty)$ $(1 \leq j \leq 2, 1 \leq i \leq 3)$ such that

$$\tilde{\phi}_j(a,b,c,d) := \sum_{k=0}^{\infty} \frac{\phi_j(2^{sk}a, 2^{sk}b, 2^{sk}c, 2^{sk}d)}{|4|^{sk}} < \infty \qquad (1 \le j \le 2),$$

$$\lim_{n\to\infty} \frac{\psi_i(2^{sn}a, b)}{|4|^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi_i(a, 2^{sn}b)}{|4|^{sn}} \qquad (1 \le j \le 3),$$

$$\left\| f(\lambda a + \lambda b) + f(\lambda a - \lambda b) - 2\lambda^2 f(a) - 2\lambda^2 f(b) - 2d^2 f(c) \right\| \le \phi_1(a,b,c,d)$$

$$\left\| g(\lambda a + \lambda b) + g(\lambda a - \lambda b) - 2\lambda^2 g(a) - 2\lambda^2 g(b) - 2d^2 g(c) \right\| \le \phi_2(a,b,c,d)$$

$$\left\| f(ab) - f(a)b^2 \right\| \le \psi_1(a,b)$$

$$\left\| g(ab) - a^2 g(b) \right\| \le \psi_2(a,b)$$

$$\left\| a^2 f(b) - g(a)b^2 \right\| \le \psi_3(a,b)$$

for all $a, b \in A$ and all $\lambda \in T = \{\lambda \in T : |\lambda| = 1\}$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ and $t \to g(ta)$ from R to A are continuous, then there exists a unique quadratic double centralizer (L, R) on A satisfying

$$\left\| f(a) - L(a) \right\| \le \frac{1}{|4|} \tilde{\phi}_1(a, a, 0, 0)$$

$$\left\| g(a) - R(a) \right\| \le \frac{1}{|4|} \tilde{\phi}_2(a, a, 0, 0)$$

for all $a \in A$.
In this paper, we introduce the quartic double centralizers and quartic multipliers on non-Archimedean Banach algebras, and we establish the stability of both of them.

MAIN RESULTS

1. Stability of quartic double centralizer

In this section, let A be a non-Archimedean Banach algebra. We establish the stability of quartic double centralizers.

Definition 1.1. A mapping $L : A \to A$ is a quartic left centralizer if L satisfies the following properties:
1) L is a quartic mapping,
2) L is a quartic homogeneous, that is, $L(\lambda a) = |\lambda|^4 L(a)$ for all $a \in A$ and $\lambda \in C$,
3) $L(ab) = L(a)b^4$ for all $a, b \in A$.

Definition 1.2. A mapping $R : A \to A$ is a quartic right centralizer if R satisfies the following properties:
1) R is a quartic mapping,
2) R is quartic homogeneous, that is, $R(\lambda a) = |\lambda|^4 R(a)$ for all $a \in A$ and $\lambda \in C$,
3) $R(ab) = a^4 R(b)$ for all $a, b \in A$.

Definition 1.3. A quartic double centralizer of an algebra A is a pair (L, R), where L is a quartic left centralizer, R is a quartic right centralizer and $a^4 L(b) = R(a)b^4$ for all $a, b \in A$.
The following example introduces a quartic double centralizer.

Example 1.4. Let $(A, \|.\|)$ be a non-Archimedean Banach algebra. Let $B = A \times A \times A \times A \times A$. We define $|||a||| = \|a_1\| + \|a_2\| + \|a_3\| + \|a_4\| + \|a_5\|$ for all $a = (a_1, a_2, a_3, a_4, a_5)$ in B. It is not hard to see that $(B, |||.|||)$ is a banach space for arbitrarily elements $a = (a_1, a_2, a_3, a_4, a_5)$ and $b = (b_1, b_2, b_3, b_4, b_5)$ in B, we define $ab = (0, a_1 b_5, a_2 b_4, a_3 b_3, 0)$.

since A is a non-Archimedean Banach algebra, we conclude that B is a non-Archimedean Banach algebra. It is easy to see that $B^5 = \{abcde : a,b,c,d,e \in B\} = \{0\}$ But $B^4 = \{abcd : a,b,c,d \in B\}$ is not zero. Now we consider the mapping $T : B \to B$ defined by

$$T(a) = a^4 (a \in B).$$

Then T is a quartic mapping and quartic homogeneous. Since $B^5 = \{0\}$, we get

$$T(ab) = (ab)^4 = 0 = a^4 b^4 = T(a)b^4 = a^4 T(b)$$

and

$$a^4 T(b) = a^4 b^4 = 0 = T(a)b^4$$

For all $a,b \in B$. Hence (T,T) is a quartic double centralizer of B.

In the above example, B is a quartic commutative algebra, but it is not commutative.

Theorem 1.5. Suppose that $s \in \{-1,1\}$ and that $f : A \to A$ is a mapping with $f(0) = 0$ for which there exist a mapping $g : A \to A$ with $g(0) = 0$ and functions $\phi_j, \psi_i : A \times A \to [0,\infty)$ $(1 \le j \le 2, 1 \le i \le 3)$ such that

$$\tilde{\phi}_j(a,b) := \sum_{k=0}^{\infty} \frac{\phi_j(2^{sk}a, 2^{sk}b)}{|16|^{sk}} < \infty \qquad (1 \le j \le 2), \tag{1.1}$$

$$\lim_{n\to\infty} \frac{\psi_i(2^{sn}a, b)}{|16|^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi_i(a, 2^{sn}b)}{|16|^{sn}} \qquad (1 \le j \le 3),$$

$$\left\| f(2\lambda a + \lambda b) + f(2\lambda a - \lambda b) - 4\lambda^4 f(a+b) - 4\lambda^4 f(a-b) - 24\lambda^4 f(a) + 6\lambda^4 f(b) \right\| \le \varphi_1(a,b) \tag{1.2}$$

$$\left\| g(2\lambda a + \lambda b) + g(2\lambda a - \lambda b) - 4\lambda^4 g(a+b) - 4\lambda^4 g(a-b) - 24\lambda^4 g(a) + 6\lambda^4 g(b) \right\| \le \varphi_2(a,b)$$

$$\left\| f(ab) - f(a)b^4 \right\| \le \psi_1(a,b) \tag{1.3}$$

$$\left\| g(ab) - a^4 g(b) \right\| \le \psi_2(a,b)$$

$$\left\| a^4 f(b) - g(a)b^4 \right\| \le \psi_3(a,b) \tag{1.4}$$

for all $a,b \in A$ and all $\lambda \in T = \{\lambda \in T : |\lambda| = 1\}$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ and $t \to g(ta)$ from R to A are continuous, then there exists a unique quartic double centralizer (L,R) on A satisfying

$$\left\| f(a) - L(a) \right\| \le \frac{1}{|32|} \tilde{\varphi}_1(a,a), \tag{1.5}$$

$$\left\| g(a) - R(a) \right\| \le \frac{1}{|32|} \tilde{\varphi}_2(a,a), \tag{1.6}$$

for all $a \in A$.

Proof: Let $s = 1$. Putting $b = 0$ and $\lambda = 1$ in (1.2), we have

$$\left\| f(2a) - 16 f(a) \right\| \le \frac{1}{|2|} \phi_1(a,a)$$

for all $a \in A$. One can use induction to show that

$$\left\| \frac{f(2^n a)}{16^n} - \frac{f(2^m a)}{16^m} \right\| \le \frac{1}{|32|} \sum_{k=m}^{n-1} \frac{\phi_1(2^k a, 2^k a)}{|16|^k} \tag{1.7}$$

for all $n > m \ge 0$ and all $a \in A$. It follows from (1.7) and (1.1) that sequence $\left\{ \frac{f(2^n a)}{16^n} \right\}$ is Cauchy. Since A is a banach algebra, this sequence is convergent. Define

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$$L(a) := \lim_{n\to\infty} \frac{f(2^n a)}{16^n}. \qquad (1.8)$$

Replacing a and b by $2^n a$ and $2^n b$, respectively, in (1.2), we get

$$\| \frac{f(2^n(2\lambda a+\lambda b)}{16^n} + \frac{f(2^n(2\lambda a-\lambda b))}{16^n} - 4\lambda^4 \frac{f(2^n(a+b))}{16^n} - 4\lambda^4 \frac{f(2^n(a-b))}{16^n}$$

$$-24\lambda^4 \frac{f(2^n a)}{16^n} + 6\lambda^4 \frac{f(2^n a)}{16^n} \| \le \frac{\phi_1(2^n a, 2^n b)}{|16|^n}$$

Taking the limit as $n \to \infty$, we obtain

$$L(2\lambda a+\lambda b) + L(2\lambda a-\lambda b) = |4||\lambda|^4 L(a+b) + |4||\lambda|^4 L(a-b) + |24||\lambda|^4 L(a) - |6||\lambda|^4 L(b) \qquad (1.9)$$

for all $a, b \in A$ and all $\lambda \in T$. Putting $\lambda = 1$ in (1.9), we obtain that L is a quartic mapping. Setting $b := 0$ in (1.9), we get

$$L(2\lambda a) = |16||\lambda|^4 L(a)$$

for all $a \in A$, $\lambda \in T$. But L is a quartic mapping. So

$$L(\lambda a) = |\lambda|^4 L(a)$$

for all $a \in A$ and all $\lambda \in T$. Under the assumption that $f(ta)$ is continuous in $t \in R$ for each fixed $a \in A$, by the same reasoning as in the proof of [26], $L(\lambda a) = \lambda^4 L(a)$ for all $a \in A$ and all $\lambda \in R$. And [2], we obtain

$$L(\lambda a) = |\lambda|^4 L(a)$$

for all $a \in A$ and $\lambda \in C (\lambda \neq 0)$. This means that L is quartic homogeneous. It follows from (1.3) and (1.8) that

$$\left\| L(ab) - L(a)b^4 \right\| = \lim_{n\to\infty} \frac{1}{|16|^n} \left\| f(2^n ab) - f(2^n a)b^4 \right\| \le \lim_{n\to\infty} \frac{\psi_1(2^n a, b)}{|16|^n} = 0$$

for all $a, b \in A$. Hence L is a quartic left centralizer on A. Applying (1.7) with $m = 0$, we get $\|L(a) - f(a)\| \le \frac{1}{|32|} \tilde{\phi}_1(a, a)$

for all $a \in A$. It is well known that the quartic mapping L satisfying (1.5) is unique. A similar argument gives us a unique quartic right centralizer R defined by

$$R(a) := \lim_{n\to\infty} \frac{g(2^n a)}{16^n}$$

which satisfies (1.6). Now we let $a, b \in A$ arbitrarily. Since L is a quartic homogeneous, it follows from (1.4) and (1.5) that

$$\begin{aligned}
\left\| a^4 L(b) - R(a)b^4 \right\| &= \frac{1}{|16|^n} \left\| a^4 L(2^n b) - 16 R(a) b^4 \right\| \\
&\le \frac{1}{|16|^n} [\left\| a^4 L(2^n b) - a^4 f(2^n b) \right\| + \left\| a^4 f(2^n b) - g(a)(16^n b^4) \right\| \\
&+ \left\| 16^n g(a) b^4 - 16^n R(a) b^4 \right\|] \\
&\le \frac{1}{|16|^{n+1}} \tilde{\phi}_1(2^n b, 2^n a) \ \|a\|^4 + \frac{\psi_3(a, 2^n b)}{|16|^n} + \|g(a) - R(a)\| \ \|b\|^4.
\end{aligned}$$

The right hand side of the last inequality tends to $\|g(a) - R(a)\| \ \|b\|^4$ as $n \to \infty$.

By (1.6), we obtain

$$\left\| a^4 L(b) - R(a) b^4 \right\| = \frac{1}{|32|} \tilde{\phi}_2(a, a) \left\| b^4 \right\|.$$

Since R is a quartic mapping, we thus obtain

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$$\left\|a^4L(b)-R(a)b^4\right\| = \frac{1}{|16|^n}\left\|16^n a^4L(b)-R(2^n a)b^4\right\|$$

$$\leq \frac{1}{|32|}\tilde{\phi}_2(2^n a, 2^n a)\ \|a\|^4$$

$$= \frac{1}{|32|}\sum_{k=n}^{\infty}\frac{\phi_2(2^k a, 2^k a)}{|16|^k}\left\|b^4\right\|.$$

Passing to the limit as $n \to \infty$, we conclude $a^4L(b) = R(a)b^4$. Thus (L, R) is a quartic double centralizer.
The proof for $s = -1$ is similar to $s = 1$.

Corollary 1.6. Suppose that $f : A \to A$ is a mapping for which there exist a mapping $g : A \to A$ and constants $\varepsilon > 0$ and $0 < p < 4$ such that

$$\left\|f(2\lambda a+\lambda b)+f(2\lambda a-\lambda b)-4\lambda^4 f(a+b)-4\lambda^4 f(a-b)-24\lambda^4 f(a)+6\lambda^4 f(b)\right\| \leq \varepsilon(\|a\|^p+\|b\|^p),$$

$$\left\|g(2\lambda a+\lambda b)+g(2\lambda a-\lambda b)-4\lambda^4 g(a+b)-4\lambda^4 g(a-b)-24\lambda^4 g(a)+6\lambda^4 g(b)\right\| \leq \varepsilon(\|a\|^p+\|b\|^p),$$

$$\left\|f(ab)-f(a)b^3\right\| \leq \varepsilon\|a\|^p\|b\|^p,$$

$$\left\|g(ab)-a^3g(b)\right\| \leq \varepsilon\|a\|^p\|b\|^p,$$

$$\left\|a^3f(b)-g(a)b^3\right\| \leq \varepsilon\|a\|^p\|b\|^p$$

for all $a, b \in A$ and all $\lambda \in T$.Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ and $t \to g(ta)$ from R to A are continuous, then there exists a unique quartic double centralizer (L, R) on A satisfying

$$\left\|f(a)-L(a)\right\| \leq \frac{\varepsilon}{|16|-|2|^p}\|a\|^p,$$

$$\left\|g(a)-R(a)\right\| \leq \frac{\varepsilon}{|16|-|2|^p}\|a\|^p$$

for all $a \in A$.

Proof: For $j = 1,2$, putting $\phi_j(a,b) = \varepsilon(\|a\|^p + \|b\|^p)$ and for $i = 1,2$, putting $\psi_i(a,b) = \varepsilon\|a\|^p\|b\|^p$ in Theorem 1.5, we get the desired results.

2. Stability of quartic multipliers

Throughout this section, assume that A is a non-Archimedean Banach algebra.

Definition 2.1. We say that a mapping $T : A \to A$ is a quartic multiplier if T satisfies the following properties:
1) T is a quartic mapping,
2) T is quartic homogeneous, that is, $T(\lambda a) = |\lambda|^4 T(a)$ for all $a \in A$ and $\lambda \in C$,
3) $a^4T(b) = T(a)b^4$ for all $a, b \in A$.

Example 1.4 introduces a quartic multiplier. We investigate the stability of quartic. multipliers.

Theorem 2.2. Suppose that $s \in \{-1,1\}$ and that $f : A \to A$ is a mapping with $f(0) = 0$ for which there exist functions, $\psi : A \times A \to [0,\infty)$ such that

$$\tilde{\phi}(a,b) := \sum_{k=0}^{\infty}\frac{\phi(2^{sk}a, 2^{sk}b)}{|16|^{sk}} < \infty, \tag{2.1}$$

$$\lim_{n\to\infty} \frac{\psi(2^{sn}a,b)}{|16|^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi(a,2^{sn}b)}{|16|^{sn}},$$

$$\left\| f(2\lambda a+\lambda b)+f(2\lambda a-\lambda b)-4\lambda^4 f(a+b)-4\lambda^4 f(a-b)-24\lambda^4 f(a)+6\lambda^4 f(b)\right\| \le \phi(a,b),$$

$$\left\| a^4 f(b) - f(a)b^4 \right\| \le \psi(a,b) \tag{2.2}$$

for all $a,b \in A$ and all $\lambda \in T$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ from R to A are continuous, then there exists a unique quartic multiplier T on A satisfying

$$\| f(a) - T(a) \| \le \frac{1}{|32|}\tilde{\varphi}(a,a), \tag{2.3}$$

for all $a \in A$.

Proof. Let $s = 1$. By the same reasoning as in the proof of Theorem 1.5, there existsa unique quartic mapping $T : A \to A$ defined by

$$T(a) := \lim_{n\to\infty} \frac{f(2^n a)}{16^n}$$

with satisfying $T(\lambda a) = |\lambda|^4 T(a)$ for all $a \in A$ and all $\lambda \in C$. Also, $\| f(a) - T(a)\| \le \frac{1}{|32|}\tilde{\phi}(a,a)$ for all $a \in A$ .Let $a,b \in A$ be arbitrarily. Then T is quartic homogeneous.

By using (2.2) and (2.3), we have

$$\begin{aligned}
\left\| a^4 T(b) - T(a)b^4 \right\| &= \frac{1}{|16|^n}\left\| a^4 T(2^n b) - 16^n T(a) b^4 \right\| \\
&\le \frac{1}{|16|^n}[\left\| a^4 T(2^n b) - a^4 f(2^n b)\right\| + \left\| a^4 f(2^n b) - f(a)(16^n b^4)\right\| \\
&+ \left\| 16^n f(a)b^4 - 16^n T(a) b^4 \right\|] \\
&\le \frac{1}{|16|^{n+1}}\tilde{\phi}(2^n b, 2^n a)\ \|a\|^4 + \frac{\psi(a,2^n b)}{|16|^n} + \frac{1}{|32|}\tilde{\phi}(a,a)\ \|b\|^4 .
\end{aligned}$$

It follows from (2.1) that

$$\left\| a^4 T(b) - T(a)b^4 \right\| = \frac{1}{|32|}\tilde{\phi}(a,a)\ \|b\|^4 .$$

Finally, we obtain

$$\begin{aligned}
\left\| a^4 T(b) - T(a)b^4 \right\| &= \frac{1}{|16|^n}\left\| 16^n a^4 T(b) - T(2^n a) b^4 \right\| \\
&\le \frac{1}{|32|}\tilde{\phi}_2(2^n a, 2^n a)\ \|b\|^4 \\
&= \frac{1}{|32|}\sum_{k=n}^{\infty} \frac{\phi(2^k a, 2^k a)}{|16|^k}\|b\|^4 \\
&\to 0 \qquad as \quad n \to \infty .
\end{aligned}$$

So $a^4 T(b) = T(a) b^4$. Hence T is a quartic multiplier.

The proof for $s = -1$ is similar.

Corollary 2.3. Suppose that $f : A \to A$ is a mapping for which there exist nonnegative real numbers ε and p with $p \ne 4$ such that

$$\left\| f(2\lambda a+\lambda b)+f(2\lambda a-\lambda b)-4\lambda^4 f(a+b)-4\lambda^4 f(a-b)-24\lambda^4 f(a)+6\lambda^4 f(b)\right\| \le \varepsilon(\|a\|^p+\|b\|^p),$$

$$\left\| a^4 f(b)-f(a)b^4\right\| \le \varepsilon\|a\|^p\|b\|^p$$

for all $a,b\in A$ and all $\lambda\in T$. Also, if for each fixed $a\in A$ the mappings $t\to f(ta)$ from R to A are continuous, then there exists a unique quartic multiplier T on A satisfying

$$\| f(a)-T(a)\| \le \frac{\varepsilon}{|16|-|2|^p}\|a\|^p$$

for all $a\in A$.

Proof: Putting $\phi(a,b)=\varepsilon(\|a\|^p+\|b\|^p)$ and $\psi(a,b)=\varepsilon\|a\|^p\|b\|^p$ in Theorem 2.2, we get the desired results.

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