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Fixed Point Theorem with EA Property in Intuitionistic Fuzzy Metric Space

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Abstract

In this paper, the concept of the common fixed point theorem in an Intuitionistic Fuzzy Metric space by extending the use of E A property introduces by Aamri & Moutawakil using implicit function for three self map satisfying weakly compatible property.

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Keywords: Intuitionistic fuzzy metric space, common fixed point, t-norm and weakly compatible maps.

Introduction

The foundation of fuzzy mathematics is laid by Lofti A Zadeh with the introduction of fuzzy sets in 1965, as a way to represent vagueness in everyday life. Subsequently several authors have applied various form from general topology of fuzzy sets and developed the concept of fuzzy space. A number of fixed point theorems have been obtained by various authors in fuzzy metric space by using the concept of compatible map, weakly compatible map, R weakly compatible map. In the study of fixed point of metric space, Popa proved theorem for Weakly Compatible, Noncontinuous mapping using implicit function. Recently Aamri & Moutawakil introduces the E.A property and had generalized the concept of non compatible map. The main objective of this paper is to obtain some common fixed point theorem in fuzzy metric space using implicit function with EA property. The result is different because it is the first time implicit function is used with EA property which has three self maps holding weakly compatible property. Our result generalizes several Comparable results in existing literature.

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Preliminaries

2.1. Definition: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a * b = a for all a ∈ [0, 1];

(d) a* b ≤ c * d whenever a ≤ c and b ≤ d and a, b, c, d ∈ [0, 1].

2.2. Definition: A 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions; for all x, y, z ∈ X, s, t > 0.

(1)H_M (x, y, t) > 0;

(2)H_M(x, y, t) =1 if and only if x = y;

(3)H_M(x, y, t) = H_M(y, x, t);

(4)H_M(x, y, t)* H_M(y, z, s) ≤ H_M(x, z, t + s);

(5)H_M(x, y, .): $(0, \infty) \to [0, 1]$ is continuous.

Then H_M is called a fuzzy metric on X. The function H_M(x, y, t) denote the degree of nearness between x and y with respect to t.

2.3. Example: Let (X, d) be a metric space. Denote $a * b = ab$ for a, b ∈ [0, 1] and let M_d be a fuzzy set on $X^2 \times (0, \infty)$ defined as follows:M_d (x, y, t) $= \frac{t}{t+d(x,\ y)}$. Then (X, M_d, *) is a fuzzy metric space, we call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

2.4. Definition: Let (X, M,*) be a fuzzy metric space, then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each ε > 0 and each t > 0, there exists n_0 ∈ N such that H_M (x_n, x, t) > 1 − ε for all n ≥ n_0.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy if for each ε > 0 and each t > 0, there exist n_0 ∈ N such that H_M (x_n,x_m, t) > 1 − ε for all n, m ≥ n_0 .

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

2.5. Proposition: In a fuzzy metric space (X, M, *), if a * a ≥ a for a ∈ [0, 1] then a * b = min {a, b} for all a, b ∈ [0, 1].

2.6. Definition: Two self-mappings A and S of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} H_M$ (ASx_n, SAx_n, t) = 1 whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = x for some x in X.

2.7. Definition: Two self-maps A and B of a fuzzy metric space (X, M, *) are called weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

2.8. Definition E A property Let A and B be self maps on a fuzzy metric space (X, M, *). They are said to satisfy (EA) property if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$ for some x ∈ X.

2.9. Remark: If self-maps A and B of a fuzzy metric space (X, M, *) are compatible then they are weakly compatible. Let (X, M,*) be a fuzzy metric space with the following condition: (6) $\lim_{t\to\infty} H_M$ (x, y, t) =1 for all x, y ∈ X.

2.10. Lemma: Let (X, M, *) be a fuzzy metric space. If there exists k ∈ [0,1] such that

$$H_M(x, y, kt) \geq H_M(x, y, t) \text{ then } x = y.$$

2.11. Lemma: Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *) with the condition (6). If there exists k ∈ [0, 1] such that H_M (y_n, y_{n+1}, kt) ≥ H_M (y_{n-1}, y_n, t) for all t > 0 and n ∈ N. Then { y_n } is a Cauchy sequence in X.

Main Results:

Theorem 3.1: Let A, B, P and Q be self mappings of a complete fuzzy metric space (X, M, *) satisfy the following:

(a) For all x, y in X and for all t > 0 there exists k ∈ (0, 1) such that

$$M_{M_1}(\mathrm{Px},\mathrm{Qy},\mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(Ax,By,t) * M_{M_1}(By,Qx,t)\,, \\ M_{M_1}(By,Px,t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Px,Ax,t) + \\ M_{M_1}(Qx,Bx,t) \end{pmatrix} \end{matrix}\right\} \text{and}$$

$$N_{N_1}(\mathrm{Px},\mathrm{Qy},\mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(Ax,By,t) * N_{N_1}(By,Qx,t)\,, \\ N_{N_1}(By,Px,t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Px,Ax,t) + \\ N_{N_1}(Qx,Bx,t) \end{pmatrix} \end{matrix}\right\}$$

(b) P(X) ⊂ B(X) and Q(X) ⊂ A(X)

(c) If one of P(X), B(X), Q(X), A(X) is complete subset of X then P and A have a coincidence point.

If the pair (P, A) and (Q, B) are weakly compatible then A, B, P and Q have a unique common fixed point in X.

Proof: As P(X) ⊂ B(X) and Q(X) ⊂ A(X) so we can define sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n+1} = \mathrm{P}x_{2n} = \mathrm{B}x_{2n+1},\ \ y_{2n+2} = \mathrm{Q}x_{2n+1} = \mathrm{A}x_{2n+2}$$

$$M_{M_1}(\mathrm{P}x_{2n},\mathrm{Q}x_{2n+1},\mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(Ax_{2n},Bx_{2n+1},t) * M_{M_1}(Bx_{2n+1},Qx_{2n},t), \\ M_{M_1}(Bx_{2n+1},Px_{2n},t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Px_{2n},Ax_{2n},t) + \\ M_{M_1}(Qx_{2n},Bx_{2n},t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(\mathrm{P}x_{2n},\mathrm{Q}x_{2n+1},\mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(Ax_{2n},Bx_{2n+1},t) * N_{N_1}(Bx_{2n+1},Qx_{2n},t)\,, \\ N_{N_1}(Bx_{2n+1},Px_{2n},t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Px_{2n},Ax_{2n},t) + \\ N_{N_1}(Qx_{2n},Bx_{2n},t) \end{pmatrix} \end{matrix}\right\}$$

$$M_{M_1}(y_{2n+1},y_{2n+2},\mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(y_{2n},y_{2n+1},t) * M_{M_1}(y_{2n+1},y_{2n+1},t)\,, \\ M_{M_1}(y_{2n+1},y_{2n+1},t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(y_{2n+1},y_{2n},t) + \\ M_{M_1}(y_{2n+1},y_{2n},t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(y_{2n+1},y_{2n+2},\mathrm{kt})\,) \leq \min\left\{\begin{matrix} N_{N_1}(y_{2n},y_{2n+1},t) * N_{N_1}(y_{2n+1},y_{2n+1},t)\,, \\ N_{N_1}(y_{2n+1},y_{2n+1},t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(y_{2n+1},y_{2n},t) + \\ N_{N_1}(y_{2n+1},y_{2n},t) \end{pmatrix} \end{matrix}\right\}$$

$M_{M_1}(y_{2n+1},y_{2n+2},\mathrm{kt}) \geq M_{M_1}(y_{2n},y_{2n+1},t)$

$N_{N_1}(y_{2n+1},y_{2n+2},\mathrm{kt}) \leq N_{N_1}(y_{2n},y_{2n+1},t)$

and Similarly $M_{M_1}(y_{2n+2},y_{2n+3},\mathrm{kt}) \geq M_{M_1}(y_{2n+1},y_{2n+2},t)$

$N_{N_1}(y_{2n+2},y_{2n+3},\mathrm{kt}) \leq N_{N_1}(y_{2n+1},y_{2n+2},t)$ and so on

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Therefore in general M_{M_1} (y_n,y_{n+1}, kt) ≥ $M_{M_1}(y_{n-1}, y_n, t)$ and

$$N_{N_1}(y_n, y_{n+1}, kt) \leq N_{N_1}(y_{n-1}, y_n, t)$$

Hence, by lemma 2.11$\{y_n\}$ is a Cauchy sequence in X. By completeness of X, $\{y_n\}$ is converges to some point z in X. Therefore, subsequence's $\{y_{2n}\}$, $\{y_{2n+1}\}$, $\{y_{2n+2}\}$ converges to point z i.e.

$$\lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Px_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} Ax_{2n+2} = z.$$

Now suppose A (X) is complete, therefore, let w Є A^{-1}z then Aw = z. Now consider,

$$M_{M_1}(Pw, Qx_{2n+1}, kt) \geq \max\left\{\begin{matrix} M_{M_1}(Aw, Bx_{2n+1}, t) * M_{M_1}(Bx_{2n+1}, Qw, t), \\ M_{M_1}(Bx_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Pw, Aw, t) + \\ M_{M_1}(Qw, Bw, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(Pw, Qx_{2n+1}, kt) \leq \min\left\{\begin{matrix} N_{N_1}(Aw, Bx_{2n+1}, t) * N_{N_1}(Bx_{2n+1}, Qw, t), \\ N_{N_1}(Bx_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Pw, Aw, t) + \\ N_{N_1}(Qw, Bw, t) \end{pmatrix} \end{matrix}\right\}$$

$$M_{M_1}(Pw, y_{2n+1}, kt) \geq \max\left\{\begin{matrix} M_{M_1}(Aw, y_{2n+1}, t) * M_{M_1}(y_{2n+1}, Qw, t), \\ M_{M_1}(y_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Pw, Aw, t) + \\ M_{M_1}(Qw, Bw, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(Pw, y_{2n+1}, kt) \leq \min\left\{\begin{matrix} N_{N_1}(Aw, y_{2n+1}, t) * N_{N_1}(y_{2n+1}, Qw, t), \\ N_{N_1}(y_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Pw, Aw, t) + \\ N_{N_1}(Qw, Bw, t) \end{pmatrix} \end{matrix}\right\}$$

$$M_{M_1}(Pw, z, kt) \geq \max\left\{\begin{matrix} M_{M_1}(z, y_{2n+1}, t) * M_{M_1}(y_{2n+1}, z, t), \\ M_{M_1}(y_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(z, z, t) + \\ M_{M_1}(z, z, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(Pw, z, kt) \leq \min\left\{\begin{matrix} N_{N_1}(z, y_{2n+1}, t) * N_{N_1}(y_{2n+1}, z, t), \\ N_{N_1}(y_{2n+1}, Pw, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(z, z, t) + \\ N_{N_1}(z, z, t) \end{pmatrix} \end{matrix}\right\}$$

$As\ n \to \infty$

$$M_{M_1}(Pw, z, kt) \geq \max\left\{\begin{matrix} M_{M_1}(z, z, t) * M_{M_1}(z, z, t), \\ M_{M_1}(z, Pw, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(z, z, t) + \\ M_{M_1}(z, z, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}\ (\mathrm{Pw}, z, \mathrm{kt}) \leq \min \left\{ \begin{array}{c} N_{N_1}(z,z,t) * N_{N_1}(z,z,t)\,, \\ N_{N_1}(z,Pw,t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(z,z,t) + \\ N_{N_1}(z,z,t) \end{pmatrix} \end{array} \right\}$$

M_{M_1} (Pw, z, kt) ≥ 1 and N_{N_1} (Pw, z, kt) ≤ 0

This gives Pw = Aw = z. Therefore w is a coincidence point of P and A. Since P(X) ⊂ B(X), therefore, z = Pw Є P(X) ⊂ B(X), this gives, z Є B(X), let v Є B^{-1}z i.e. Bv = z.

$$M_{M_1}\ (\mathrm{P}x_{2n}, \mathrm{Qv}, \mathrm{kt}) \geq \max \left\{ \begin{array}{c} M_{M_1}(Ax_{2n},Bv,t) * M_{M_1}(Bv,Qx_{2n},t)\,, \\ M_{M_1}(Bv,Px_{2n}\,,t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Px_{2n},Ax_{2n},t) + \\ M_{M_1}(Qx_{2n},Bx_{2n},t) \end{pmatrix} \end{array} \right\}$$

$$N_{N_1}\ (\mathrm{P}x_{2n}, \mathrm{Qv}, \mathrm{kt}) \leq \min \left\{ \begin{array}{c} N_{N_1}(Ax_{2n},Bv,t) * N_{N_1}(Bv,Qx_{2n},t), \\ N_{N_1}(Bv,Px_{2n},t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Px_{2n},Ax_{2n},t) + \\ N_{N_1}(Qx_{2n},Bx_{2n},t) \end{pmatrix} \end{array} \right\}$$

$$M_{M_1}\ (y_{2n+1}, \mathrm{Qv}, \mathrm{kt}) \geq \max \left\{ \begin{array}{c} M_{M_1}(y_{2n},Bv,t) * M_{M_1}(Bv,y_{2n+1},t)\,, \\ M_{M_1}(Bv,y_{2n+1}\,,t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(y_{2n+1},y_{2n},t) + \\ M_{M_1}(y_{2n+1},y_{2n},t) \end{pmatrix} \end{array} \right\}$$

$$N_{N_1}\ (y_{2n+1}, \mathrm{Qv}, \mathrm{kt}) \leq \min \left\{ \begin{array}{c} N_{N_1}(y_{2n},Bv,t) * N_{N_1}(Bv,y_{2n+1},t), \\ N_{N_1}(Bv,y_{2n+1},t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(y_{2n+1},y_{2n},t) + \\ N_{N_1}(y_{2n+1},y_{2n},t) \end{pmatrix} \end{array} \right\}$$

$As\ n \to \infty$

$$M_{M_1}\ (\mathrm{z}, \mathrm{Qv}, \mathrm{kt}) \geq \max \left\{ \begin{array}{c} M_{M_1}(z,z,t) * M_{M_1}(z,z,t) \\ * M_{M_1}(z,z,t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(z,z,t) + \\ M_{M_1}(z,z,t) \end{pmatrix} \end{array} \right\}$$

$$N_{N_1}(\mathrm{z}, \mathrm{Qv}, \mathrm{kt}) \leq \min \left\{ \begin{array}{c} N_{N_1}(z,z,t) * N_{N_1}(z,z,t) \\ * N_{N_1}(z,z,t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(z,z,t) + \\ N_{N_1}(z,z,t) \end{pmatrix} \end{array} \right\}$$

M_{M_1}(z, Qv, kt) ≥ 1 and N_{N_1} (z, Qv, kt) ≤ 0

This gives Qv = z = Bv. So v is coincidence point of Q and B. Since the Pair (P, A)) is weakly compatible, therefore P and Q commute at coincidence point i. e. PAw = APw, this gives, Pz = Az and as (Q, B) is weakly compatible, therefore QBv = BQv this gives, Qz = Bz. Now, we will show that Pz = z, by 1, we have

$$M_{M_1}(\mathrm{Pz}, \mathrm{Q}x_{2n+1}, \mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(Az, Bx_{2n+1}, t) * M_{M_1}(Bx_{2n+1}, Qz, t), \\ M_{M_1}(Bx_{2n+1}, Pz, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Pz, Az, t) + \\ M_{M_1}(Qz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(\mathrm{Pz}, \mathrm{Q}x_{2n+1}, \mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(Az, Bx_{2n+1}, t) * N_{N_1}(Bx_{2n+1}, Qz, t), \\ N_{N_1}(Bx_{2n+1}, Pz, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Pz, Az, t) + \\ N_{N_1}(Qz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$$M_{M_1}(\mathrm{Pz}, y_{2n+2}, \mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(Pz, y_{2n+1}, t) * M_{M_1}(y_{2n+1}, Bz, t), \\ M_{M_1}(y_{2n+1}, Pz, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Pz, Pz, t) + \\ M_{M_1}(Bz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(\mathrm{Pz}, \mathrm{Q}x_{2n+1}, \mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(Pz, y_{2n+1}, t) * N_{N_1}(y_{2n+1}, Bz, t), \\ N_{N_1}(y_{2n+1}, Pz, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Pz, Pz, t) + \\ N_{N_1}(Bz, Bz, t) \end{pmatrix} \end{matrix}\right\}$$

$As\ n \to \infty$

M_{M_1} (Pz, z, kt) ≥ max $\{M_{M_1}(Pz, z, t) * 1 * M_{M_1}(z, Pz, t) * 1\}$,

N_{N_1} (Pz, z, kt) ≤ min $\{N_{N_1}(Pz, z, t) * 0 * N_{N_1}(z, Pz, t) * 0\}$ and

M_{M_1} (Pz, z, kt) ≥ 1 and N_{N_1} (Pz, z, kt) ≤ 0

This gives Pz = Az = z, similarly, we prove that Qz = z.

$$M_{M_1}(\mathrm{P}x_{2n}, \mathrm{Qz}, \mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(Ax_{2n}, Bz, t) * M_{M_1}(Bz, Qx_{2n}, t), \\ M_{M_1}(Bz, Px_{2n}, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Px_{2n}, Ax_{2n}, t) + \\ M_{M_1}(Qx_{2n}, Bx_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(\mathrm{P}x_{2n}, \mathrm{Qz}, \mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(Ax_{2n}, Bz, t) * N_{N_1}(Bz, Qx_{2n}, t), \\ N_{N_1}(Bz, Px_{2n}, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Px_{2n}, Ax_{2n}, t) + \\ N_{N_1}(Qx_{2n}, Bx_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$$M_{M_1}(y_{2n+1}, \mathrm{Qz}, \mathrm{kt}) \geq \max\left\{\begin{matrix} M_{M_1}(y_{2n}, Qz, t) * M_{M_1}(Qz, y_{2n+1}, t), \\ M_{M_1}(Qz, y_{2n+1}, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(y_{2n+1}, y_{2n}, t) + \\ M_{M_1}(y_{2n+1}, y_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$$N_{N_1}(y_{2n+1}, \mathrm{Qz}, \mathrm{kt}) \leq \min\left\{\begin{matrix} N_{N_1}(y_{2n}, Qz, t) * N_{N_1}(Qz, y_{2n+1}, t), \\ N_{N_1}(Qz, y_{2n+1}, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(y_{2n+1}, y_{2n}, t) + \\ N_{N_1}(y_{2n+1}, y_{2n}, t) \end{pmatrix} \end{matrix}\right\}$$

$As\ n \to \infty$

$$M_{M_1}\text{ (z, Qz, kt)} \geq \max \left\{ \begin{matrix} M_{M_1}(z, Qz, t) * M_{M_1}(Qz, z, t), \\ M_{M_1}(Qz, z, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(z, z, t) + \\ M_{M_1}(z, z, t) \end{pmatrix} \end{matrix} \right\},$$

$$N_{N_1}\text{ (z, Qz, kt)} \leq \min \left\{ \begin{matrix} N_{N_1}(z, Qz, t) * N_{N_1}(Qz, z, t), \\ N_{N_1}(Qz, z, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(z, z, t) + \\ N_{N_1}(z, z, t) \end{pmatrix} \end{matrix} \right\} \text{ and}$$

M_{M_1} (z, Qz, kt) ≥ 1 and N_{N_1} (z, Qz, kt) ≤ 0

This gives Qz = z = Bz. Therefore z is a common fixed point P, A, Q and B.

Uniqueness: Let w be another fixed point of P, A, Q and B then by (1), we have

$$M_{M_1}\text{(Pz, Qw, kt)} \geq \max \left\{ \begin{matrix} M_{M_1}(Az, Bw, t) * M_{M_1}(Bw, Qz, t), \\ M_{M_1}(Bw, Pz, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(Pz, Az, t) + \\ M_{M_1}(Qz, Bz, t) \end{pmatrix} \end{matrix} \right\}$$

$$N_{N_1}\text{ (Pz, Qw, kt)} \leq \min \left\{ \begin{matrix} N_{N_1}(Az, Bw, t) * N_{N_1}(Bw, Qz, t), \\ N_{N_1}(Bw, Pz, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(Pz, Az, t) + \\ N_{N_1}(Qz, Bz, t) \end{pmatrix} \end{matrix} \right\}$$

$$M_{M_1}\text{ (z, w, kt)} \geq \max \left\{ \begin{matrix} M_{M_1}(z, w, t) * M_{M_1}(w, z, t), \\ M_{M_1}(w, z, t) * \frac{1}{2}\begin{pmatrix} M_{M_1}(z, z, t) + \\ M_{M_1}(z, z, t) \end{pmatrix} \end{matrix} \right\},$$

$$N_{N_1}\text{ (z, w, kt)} \leq \min \left\{ \begin{matrix} N_{N_1}(z, w, t) * N_{N_1}(w, z, t), \\ N_{N_1}(w, z, t) * \frac{1}{2}\begin{pmatrix} N_{N_1}(z, z, t) + \\ N_{N_1}(z, z, t) \end{pmatrix} \end{matrix} \right\}$$

M_{M_1}(z, w, kt) ≥ 1 and N_{N_1} (z, w, kt) ≤ 0.

This gives z = w. Hence z is a unique common fixed point P, A, Q and B.

Conclusions

In the present work we introduced a new concept of fuzzy mappings in the fuzzy metric space on compact sets, which is a partial generalization of fuzzy contractive mappings in the sense of M. Aamri and D. E. Moutawakil. Also, we derived the existence of fixed point theorem for weakly compatible maps in fuzzy metric space. Moreover, we reduced our result from fuzzy mappings in fuzzy metric spaces. Finally, we showed some relation of multivalued mappings and fuzzy mappings, which can be utilized to derive fixed point for multivalued mappings.

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References

[1] M. Aamri And D. E. Moutawakil, Some New Common Fixed Point Theorems Under Strict Contractive Conditions, J. Math. Anal. Appl., 270(2002), 181-188.

[2] Mujahid Abbas, Common Fixed Point Theorem For Non Compatible Maps, Bulletin of Mathematical Analysis &Application Volume 1 Issue 2(2009).

[3] V. H. Badsha, F. Sayyed and N. Gupta, Weak** Commuting Mapping and Common Fixed point, News Bull. Cal. Math. Soc. 24 (2001), 11-20.

[4] A. George and P. Veeramani, On Some Results in Fuzzy Metric Spaces, Fuzzy Sets And System 64 (1994), 395-399.

[5] M. Imdad, S. Kumar and M.S. Khan, Remarks On Some Fixed Point Theorems Satisfying An Implicit Relations, Radovi Mathematics 11(2002), 135-143.

[6] Suman Jain Bhawna Mundra & Sangita Aske Common Fixed Point Theorem Using Implcit Function International Mathematical Forum ,4 ,2009,No.3, 135-141.

[7]V. Popa, Some Fixed Point Theorems For Weakly Compatible Mappings, Radovi Mathematics 10 (2001), 245-252.

[8]Sushil Sharma Hanifi Tokgozand Servet Kutukeu, A Fixed Point Theorem In Fuzzy Metric Space Int. Journal Of Math. Analysis, Vol.1, 2007, No. 18861-872.

[9] B. Singh And S. Jain, Semi Compatibility And Fixed Point Theorems In Fuzzy Metric Space Using Implicit Relation, International Journal of Mathematics and Mathematical Sciences, 16 (2005), 2617-2629.

[10] R. Vasuki, Common Fixed Point Theorem in A Fuzzy Metric Space, Fuzzy Sets And Systems, 97 (1998), 395-397.