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A Study On Fuzzy Game Optimization Theory In Distributed System

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Abstract

In this paper, Game optimization theory is discussed based on Game theory and multi objective optimization problems. Optimization stability analysis is proposed as well. Game optimization theory could not only handle multi objective optimization problems effectively , but also could offset the disadvantages of traditional optimization theories, such as lack of framework and the insufficient consideration of relevant elements.

Keywords

Mixed strategy, Fuzzy optimization, Fuzzy Multi-weights Decision-making Method, Mutant species.

Introduction

Game Theory is the study of mathematical models of conflict and cooperation between intelligent individual decision makers. In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning and everything is a matter of degree. In fuzzy logic, knowledge is interpreted a collection of elastic or, equivalently, fuzzy constraint on a collection of variables. Inference is viewed as a process of propagation of elastic constraints. Any logical system can be fuzzified. There are two main characteristics of fuzzy systems that give them better performance for specific applications. Fuzzy systems are suitable for uncertain or approximate reasoning, especially for systems with mathematical models that are difficult to derive. Fuzzy logic allows decision making with estimated values under incomplete or uncertain information.

Fuzzy Optimization and decision making covers all aspects of the theory and practice of fuzzy optimization and decision making in the presence of uncertainty. It examines theoretical, empirical, and experimental work related to fuzzy modelling and associated mathematics, solution methods, and systems. The journal publishes papers in the following areas: modelling, theoretical developments, algorithmic developments, systems development and applications. This journal promotes research and the development of fuzzy technology and soft-computing methodologies to enhance our ability to address complicated optimization and decision making problems involving non-probabilistic uncertainty. It helps foster the understanding, development, and practice of fuzzy technologies for solving economic, engineering, management, and societal problems. The journal provides a forum for authors and readers in the fields of business, economics, engineering, mathematics, management science, operations research, and systems.

Basic Definitions

Definition 1. The normal-form representation of an n-player game specifies the players' strategy spaces ($S1,\cdots,Sn$) and their payoff functions ($u1,\cdots,un$).We denotes this game by $G = \{S1,\cdots,Sn;\ u1,\cdots,un\}$.

Definition 2. In the normal-form game $G = \{S1,\cdots,Sn;\ u1,\cdots,un\}$, let $s_i^{'}$ and $s_i^{''}$ be feasible strategies for player i, (*i.e.*, $s_i^{'}$ and $s_i^{''}$ are members of S_i).Strategy $s_i^{'}$ is strictly dominant by strategy $s_i^{''}$ if for each feasible combination of the other players' strategies, i's payoff from playing $s_i^{'}$ is strictly less than i's payoff from playing $s_i^{''}$: $u_i\,(s_1\ ,\ ,\ s_{i\ 1}\ ,\ s_i^{'}\ ,\ s_{i\ 1}\ ,\ ,\ s_n\ )\ u_i\ (s_1\ ,\ ,\ s_{i\ 1}\ ,\ s_i^{''}\ ,\ s_{i\ 1},\ ,\ s_n\ )$, for each strategies combination $(s_1\ ,\ ,\ s_{i\ 1}\ ,\ s_{i\ 1},\ ,\ s_n\ )$ that can be constructed from the other players' strategy spaces $S_1\ ,\ \ ,\ S_{i\ 1}\ ,\ S_{i\ 1},\ \ ,\ S_n$.

Each player's predicted strategy must be that player's best response to the predicted strategies of the other players. This prediction is called Nash equilibrium.

Definition 3. In the n-player normal-form game $G = \{S1,\cdots,Sn;\ u1,\cdots,un\}$, the strategies ($s_1^*\ ,\ ,\ s_n^*$) are a Nash equilibrium if, for each player i, s_i^* is (at least tied for) player i's best response to the strategies specified for the n-1 other players

$$(\ s_1^*\ ,\ \ ,\ s_{i\ 1}^*\ ,\ s_{i\ 1}^*,\ \ ,\ s_n^*\ ) : u_i\ (\ s_1^*\ ,\ ,\ s_{i\ 1}^*\ ,\ s_i^*\ ,\ s_{i\ 1}^*\ ,\ ,\ s_n^*\ )\ u_i\ (\ s_1^*\ ,\ ,\ s_{i\ 1}^*\ ,\ s_i\ ,\ s_{i\ 1}^*,\ ,\ s_n^*\ )\ .$$

These concepts are the foundations of Game Optimization Theory.

Optimization Stability Analysis

Analysis I

	A	B
A	a_{11} ; b_{11}	a_{12} ; b_{12}
B	a_{21} ; b_{21}	a_{22} ; b_{22}
	$1 - \delta$	δ

Analysis II

	A	B
A	a_{11} ; b_{11}	a_{12} ; b_{12}
B	a_{21} ; b_{21}	a_{22} ; b_{22}
	δ	$1 - \delta$

Analysis I and II represent two extreme situations, where one situation assumes the original species is the majority and another is when the mutant species is the majority. In the OSA process, δ is a small positive number, a and b are the payoffs to the original and mutant species when they choose their strategies, a represents the payoffs of the original species, b represents the payoffs of the mutant species. Strategies combination (A,A) denotes that the final state is only the scale expansion of the initial state, there is no improved approach is

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adopt to the optimal system, a_{11} denotes the payoffs of the initial state under the selected objective functions, b_{11} denotes the payoffs produced by the improved approach (b_{11} is 0 in this strategies combination); strategies combination (A,B) denotes that an improved approach is adopted to the optimal problem, a_{12} denotes the payoffs of the initial state when the improved approach is adopted, b_{12} denotes the payoffs produced by the improved approach; strategies combination (B,A) denotes the interchange of initial state and final state (*i.e.*, this strategies combination is the inverse process of optimization), a_{21} denotes the payoffs of the optimized initial state, b_{21} denotes the payoffs produced by the inversion process; strategies combination (B,B) denotes the initial state is the optimized state, and the final state is the expansion of the initial state, a_{22} is the payoff of the initial state, b_{22} denotes the payoffs produced by the improved approach.

When the original species is a majority in the population as shown in Analysis 1, the expectation payoffs of the original species and mutant species correspond to P and Q:

$$A: \; a_{11} \; X \, (1 - \delta) + a_{12} \, X \, \delta = P$$

$$B: \; a_{21} \; X \, (1 - \delta) + a_{22} \, X \, \delta = Q$$

If the optimization stability is stable, it should satisfy the condition Q > P; this result represents that the improved approach will produce better payoffs.

When the mutant species is the majority in the population as shown in Table 2, the expected payoffs of the original species and mutant species correspond to *R* and S:

$$A: b_{11} \; X \, \delta + a_{12} \, X \, (1 - \delta) = R$$

$$M: \; \alpha_{21} \; X \; \varepsilon + \alpha_{22} \, X(1 - \varepsilon) = S$$

If the optimization stability is stable, it should satisfy the condition S > R; this result represents that the initial state cannot produce better payoffs.

In addition to the conclusions made above, another two inequality constraints should be satisfied to guarantee the stability of optimization:

$$a_{12} + b_{12} \geq a_{11} + b_{11}$$

$$a_{22} + b_{22} \geq a_{21} + b_{21}$$

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The values of *a* and b are closely related to the object functions, and the OSA process needs a large number of historical data which relate to the optimal problem. From the analysis above, the OSA result could assess the improved approach from a comprehensive viewpoint. Then the general criteria of optimization stability could be summarized as two comparison and two criteria. This criterion formulates a mathematical model to evaluate the improved approach and can be used as guidance for the engineering practice.

FORMULATION OF MIXED STRATEGY GAME

Definition 1: In the normal-form game G = $\{S_1 , \quad , S_n ; u_1 , \quad ,u_n\}$, suppose S_i = $\{s_{i1},\cdots,s_{ik}\}$. Then a similar mixed strategy for player *i* is a fuzzy vector $P_i = \{s_{i1},\cdots, s_{ik}\}$, where k = 1,…,K, and $p_{iK} \leq 1$, P_{iK} is a degree of membership belongs to pure strategy s_{iK}.

		A1		
		Low	Middle	High
A2	Low	$X_1 ;y_1; z_1$	$x_1; y_2; z_1$	$x_1; y_3; z_1$
	Middle	$x_2; y_1; z_1$	$x_2; y_2; z_1$	$x_2; y_3; z_1$
	High	$x_3; y_1; z_1$	$x_3; y_2; z_1$	$x_3; y_3; z_1$

The set of strategies for each player is the non-dominant set generated by the optimization algorithm. The payoffs for each player relate to the decision maker's degree of attention . A 3-objective optimization problem works as an example to describe the structure of SMG and the process of generating a non-dominant set.

The objective functions are A_1, A_2 and A_3. We use fuzzy mathematics to determine the hierarchical organization for each object function, as the above table shows (the level of F_3 is low). (Low; Low; High), (Low, Middle, High), and other strategy pairs are the combinations of strategies. Distance Entropy Multi-Objective Particle Swarm Optimization (DEMPSO) is used to generate a non-dominant set and Fuzzy multi weights decision making method is used to quantify the trade offs in satisfying the different objectives. The trade offs in this game are called Similar Nash Equilibrium.

The SMG is suitable for the optimization programming problem in engineering practice, as it converts an optimization procedure into a process of seeking SNE in SMG. The SNE exists in the strategy space of the players in SMG, it is the optimal response and best trade-off for each player, for example, the SNE is the optimal scheme of the size and site of DG in the game of DG optimal planning. SMG, as a part of GOT, could offer integrity strategy space and supply a scientific method to find SNE in a specific game.

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Conclusion

Game Optimization theory is proposed in this paper. The Establishment of Game Optimization theory supplies a new method to explain multi objective optimization problems and it is used for solving the distribution system planning problem.

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