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Estimate the Shape Parameter of Generalize Rayleigh Distribution Using Bayesian – Shrinkage Technique

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Abstract:

This paper is concerned with preliminary test single stage bayesian - shrinkage estimator for the unknown shape parameter (α) of two parameters generalized Rayleigh (GR) distribution when the scale parameter (λ) is known, addition a prior knowledge about the shape parameter (α) is available as initial value (α_0), via shrinkage weight factor Ψ(.) and pretest region R. This prior knowledge about the unknown parameter may be obtained from past experiences or from similar situations.

Expressions for the Bias, Mean Squared Error [MSE] and Relative Efficiency [R.Eff(-)] for the proposed estimator are derived. Numerical results and conclusions of mentioned expressions are carried out to assess the effects of the considered estimator and to illustrate these results. Table of these numerical results annexed. Comparisons between introduced estimator with respect to Bayes estimator and with some existing studies in the sense of Mean Squared Error or Relative Efficiency are performed.

Keywords: Generalized Rayleigh Distribution, Maximum Likelihood Estimator, Bayesian Estimator, Single Stage Shrinkage Estimator, Mean Squared Error and Relative Efficiency

1. Introduction

Burr [5] introduced different forms of cumulative distribution functions for modeling lifetime data. Among those distributions, Burr Type X and Burr Type XII are the most popular ones. Several authors consider different aspects of Burr Type X and Burr Type XII distribution see for example Surles and Padgett [10] and Raqab and Kundu [9].

Kundu and Raqab [8] , considered different estimators and studied how the estimator of different unknown parameter behave for different sample size and different parameter value, they showed that the two parameters Generalized Rayleigh distribution (Burr Type X) can be used quite effectively in modeling strength data and also modeling general lifetime data. It has application in the field of acoustics, spatial statistics and random walks; [7].

We prefer to call the two parameters Burr Type X distribution as the two parameter Generalized Rayleigh (GR) distribution.

The two parameters GR distribution has the following distribution function

$$F(x;\alpha,\lambda) = (1 - e^{-(\lambda x)^2})^{\alpha} \quad \text{For } x>0,\ \alpha > 0,\ \lambda > 0 \qquad \ldots (1)$$

Therefore, GR distribution has the density function

$$f(x;\alpha,\lambda)=\begin{cases}2\alpha\lambda^2 x\, e^{-(\lambda x)^2}(1-e^{-(\lambda x)^2})^{\alpha-1} & \text{for } x>0,\alpha>0,\lambda>0\\ 0 & \text{o.w.}\end{cases} \qquad \dots (2)$$

Here, α and λ are the shape and scale parameters respectively.

In conventional notation, we write x~GR (α,λ).

In this paper we introduce the problem of estimating the shape parameter (α) of two parameter GR distribution with known scale parameter (λ) when some prior information (α_0) regarding the actual value (α) available. More specifically we assume that the prior information regarding due the following reasons; [11].

1. We believe that (α_0) is closer to the true value of α, or
2. We fear that (α_0) may be near the true value of (α), i.e. ;something bad happens if (α) approximately equal to (α_0) and we do not know about it.

In such a situation it is natural to start with classical estimator $\hat{\alpha}$ (MLE for example) of α and modify it by moving it closure to (α_0) using shrinkage weight factor $\psi_1(\cdot)$; $0 \le \psi_1(\cdot) \le 1$, so that the resulting linear combination estimator $\tilde{\alpha}$ though perhaps biased has a smaller mean squared error [MSE] than that $\hat{\alpha}$ in some interval around (α_0),

i.e. $\tilde{\alpha}=\psi_1(\cdot)\hat{\alpha}+(1-\psi_1(\cdot))\alpha_0$ ... (3)

which is well-known as single stage shrinkage estimator (SSE) .

Preliminary test single stage shrinkage estimator (PTSSE) is introduce in this paper which is a testimator of level of significance (Δ) for test the hypothesis H_0: $\alpha = \alpha_0$ vs. H_A: $\alpha \neq \alpha_0$ using test statistics T ($\hat{\alpha}/\alpha_0$).

If H_0 accepted, the shrinkage estimator $\tilde{\alpha}$ which is defined in (3) will be utilizing to estimate α.

However if H_0 rejected, we consider shrinkage estimator via another shrinkage weight factor $\psi_2(\cdot)$; $0 \le \psi_2(\cdot) \le 1$, so the shrinkage estimator will be:

$\tilde{\alpha}=\psi_2(\cdot)\hat{\alpha}+(1-\psi_2(\cdot))\alpha_0$ … (4)

Thus, the general form of preliminary test single stage shrinkage estimator (PTSSE) for the shape parameter (α) will be:

$$\tilde{\alpha}=\begin{cases}\Psi_1(\hat{\alpha})\hat{\alpha}+\left(1-\Psi_1(\hat{\alpha})\right)\alpha_0 & ,if\ H_0\,accepted\\ \Psi_2(\hat{\alpha})\hat{\alpha}+\left(1-\Psi_2(\hat{\alpha})\right)\alpha_0 & ,if\ H_0\,rejected\end{cases} \qquad \dots (5)$$

Where $\psi_i(\hat{\alpha}), 0 \le \psi_i(\hat{\alpha}) \le 1$, i = 1, 2 is a shrinkage weight factor specifying the belief in ($\hat{\alpha}$) while (1 – ψ_i ($\cdot$)) specifying the belief in (α_0), and $\psi_i(\hat{\alpha})$ may be a function of $\hat{\alpha}$ or may be a constant (ad hoc basis).

The prior information may be incorporated in the estimation process using a preliminary test estimator; see for example [7], thus improving the estimation process.

Several authors have been studied (PTSSE) defined in (5) for various parameters of different distributions and for estimating the parameters of regression models, for instance, see [2], [3], [4] and [11].

The idea of this paper is concerned with the development of preliminary single stage shrinkage estimators (5) for estimate the shape parameter (α) of two parameters GR distribution with known scale parameter (λ) using Bayesian estimation technique under improper prior distribution and quadratic loss function($\hat{\alpha}_B$) instead of classical estimator ($\hat{\alpha}$) via study the Performance of Bias, Mean Squared Error and Efficiency expressions of the proposed estimator and make comparisons of the numerical results with ($\hat{\alpha}_B$) and existing studies.

2. Bayes Estimate

Consider the two parameters Generalized Rayleigh Distribution, which has the density function defined in equation (2).

Assume that α has Improper prior density defined as :

$$g(\alpha) \propto \frac{1}{\alpha} \quad ; \alpha > 0$$

And the posterior distribution function of α is defined as below

$$g(\alpha|x_1, x_2, \dots, x_n) = \frac{[\prod_{i=1}^{n} f(x;\alpha)]\, g(\alpha)}{\int_\alpha [\prod_{i=1}^{n} f(x;\alpha)]\, g(\alpha)\, d\alpha}$$

$$g(\alpha|x_1, x_2, \dots, x_n) = \frac{\left[2\lambda^{2n}\alpha^n \sum_{i=1}^{n} x_i e^{-\sum_{i=1}^{n}\lambda x_i}\left(1-e^{-\sum_{i=1}^{n}\lambda x_i}\right)^{\alpha-1}\right] 1/\alpha}{\int_0^\infty \left[2\lambda^{2n}\alpha^n \sum_{i=1}^{n} x_i e^{-\sum_{i=1}^{n}\lambda x_i}\left(1-e^{-\sum_{i=1}^{n}\lambda x_i}\right)^{\alpha-1}\right] 1/\alpha\, d\alpha}$$

$$\therefore g(\alpha|x_1, x_2, \dots x_n) = \frac{\alpha^{n-1}}{\Gamma(n)}\left(\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}\right)^n e^{-\alpha\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}} \; ; \alpha >, \lambda > 0, x > 0 \quad \dots (6)$$

Which implies that α is distributed as gamma distribution with parameters n and $\sum_{i=1}^{n}\left(1-e^{-\lambda x_i}\right)^{-1}$, i.e. ; α ~ G (n, $\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}$)

Therefore, one can obtain the Bayes estimator under quadratic loss function and risk function as follow:[6]

$$\hat{\alpha}_B = E(\alpha|x_1, x_2, \dots x_n) = \int_0^\infty \alpha\, g(\alpha|x_1, x_2, \dots x_n) d\alpha$$

$$= \int_0^\infty \alpha \left(\frac{\alpha^{n-1}}{\Gamma(n)}\left(\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}\right)^n e^{-\alpha\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}}\right) d\alpha$$

$$\therefore \hat{\alpha}_B = \frac{n-2}{\sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}} = \frac{n-2}{y} \quad \dots (7)$$

Where $y = \sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right)^{-1}$ is a complete sufficient statistics for α .

The risk function of the estimator $\hat{\alpha}_B$ is :

$$R(\alpha_B) = E[l(\alpha,\hat{\alpha}_B)] = \frac{1}{\alpha^2}E[(\alpha-\hat{\alpha}_B)^2]$$

$$= \frac{1}{\alpha^2}\left[\alpha^2 - 2\alpha(n-2)E\left(\frac{1}{y}\right) + (n-2)^2 E\left(\frac{1}{y^2}\right)\right]$$

$$\because y = \sum_{i=1}^{n} \ln\left(1-e^{-\lambda x_i}\right)^{-1} \quad then \;\; y\sim G(n,\alpha)\,; E\left(\frac{1}{y}\right) = \frac{\alpha}{n-1} \; and$$

$$E\left(\frac{1}{y^2}\right) = \frac{\alpha^2}{(n-1)(n-2)}$$

$\therefore R_B = \frac{1}{n-1}$ Which is constant. … (8)

As well-known, the maximum likelihood estimator (MLE) for the shape parameter of two parameter GR (α,λ) when $\lambda = 1$ (λ is konwn), is [1]

$$\hat{\alpha}_{MLE} = -\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-x_i^2})} \quad \ldots (9)$$

Note that, if $x_i \sim$ GR (α,1), then $-\alpha\sum_{i=1}^{n}\ln(1-e^{-x_i^2})$ follows Gamma distribution with shape parameter (n) and scale parameter 1; G(n,1), see [1]and [8] .

i.e.; $E(\hat{\alpha}_{MLE}) = \frac{n}{n-1}\alpha$ and $var(\hat{\alpha}_{MLE}) = \frac{n^2\alpha^2}{(n-1)^2(n-2)}$.

By using (9), $let\ \hat{\alpha}_B = \frac{n-2}{n}.\hat{\alpha}_{mle} = \frac{n-2}{y};\;\; y = -\alpha\sum\ln\left(1-e^{-x^2}\right)\sim G(n,1)$, then

$$E(\hat{\alpha}_B) = \frac{n-2}{n}E(\hat{\alpha}_{mle}) = \frac{(n-2)}{(n-1)}\alpha$$

$$\therefore Bias(\hat{\alpha}_B) = E(\hat{\alpha}) - \alpha = -\frac{\alpha}{(n-1)}$$

$$Var(\hat{\alpha}_B) = \left(\frac{n-2}{n}\right)^2.Var(\hat{\alpha}_{mle}) = \frac{(n-2)}{(n-1)^2}\alpha^2$$

$$\therefore MSE(\hat{\alpha}) = Var(\hat{\alpha}) + [Bias(\hat{\alpha})]^2 = \frac{\alpha^2}{n-1}$$

3. Preliminary Test Single Stage Bayesian-Shrinkage Estimator $\tilde{\alpha}_{BS}$

This section is concerned with pooling approach between shrinkage estimation that uses a prior information about the unknown parameter as initial values and Bayesian estimation that uses a prior information about the unknown parameter as a prior distribution for the scale parameter (α) of two parameters Generalized Rayleigh Distribution using specific shrinkage weight factors as well as a pretest Region (R) when a prior information about (α) is available as initial value (α_0).

General preliminary test single stage Bayesian-Shrinkage (PSSBS) estimator defined below [4]:-

$$\tilde{\alpha}_{BS} = \begin{cases} \Psi_1(\hat{\alpha}_B)\hat{\alpha}_B + \left(1-\Psi_1(\hat{\alpha}_B)\right)\alpha_0 & , if \;\; \hat{\alpha}_B \in R \\ \Psi_2(\hat{\alpha}_B)\hat{\alpha}_B + \left(1-\Psi_2(\hat{\alpha}_B)\right)\alpha_0 & , if \;\; \hat{\alpha}_B \notin R \end{cases} \quad \ldots (10)$$

In this section, we consider the (PSSBS) defined in (10) when $\psi_1(\hat{\alpha}) = 0$ and $\psi_2(\hat{\alpha}) = k$ (constant); $0 \le k \le 1$ for estimate the shape parameter α of two parameter GR distribution when $\lambda = 1$.

Thus PSSBS of α can be written as:

$$\tilde{\alpha}_{BS} = \begin{cases} \alpha_0 & , if \;\; \hat{\alpha}_B \in R \\ \mathrm{k}(\hat{\alpha}_B - \alpha_0) + \alpha_0 & , if \;\; \hat{\alpha}_B \notin R \end{cases} \quad \ldots (11)$$

Where R is a pretest region for testing the null hypothesis H_0: $\alpha = \alpha_0$ vs. the alternative hypothesis H_A: $\alpha \neq \alpha_0$ with level of significance (Δ) using test statistic $T = \frac{2(n-2)\alpha}{\hat{\alpha}}$

i.e.; $R = \left[a < \frac{2(n-2)\alpha_0}{\hat{\alpha}} < b\right]$ …(12)

Where $a = (X^2_{1-\Delta/2,2n})$ and $b = (X^2_{\Delta/2,2n})$, …(13)

are respectively the lower and upper 100(Δ/2) percentile point of Chi-Square distribution with degree of freedom (2n).

The expression for Bias of PSSBS ($\tilde{\alpha}_{BS}$) is defined as below

$$Bias(\tilde{\alpha}_{BS}|\alpha, R) = E(\tilde{\alpha}_{BS} - \alpha)$$

$$= \int_R (\alpha_0 - \alpha) f(\hat{\alpha}_B) d\hat{\alpha}_B + \int_{\bar{R}} (k\hat{\alpha}_B + (1-k)\alpha_0 \, f(\hat{\alpha}_B) d\hat{\alpha}_B$$

Where $\bar{R}$ is the complement region of R in real space and f($\hat{\alpha}$) is a p.d.f. defined in (14)below.

$$f(\hat{\alpha}_B, \alpha) = \begin{cases} \dfrac{\left(\frac{(n-2)\alpha}{\hat{\alpha}_B}\right)^{n+1} . e^{-\frac{(n-2)\alpha}{\hat{\alpha}_B}}}{\Gamma(n).(n-2)\alpha} & \hat{\alpha}_B > 0 \;, \alpha > 0 \\ 0 & o.w. \end{cases} \quad \ldots (14)$$

We conclude,

$$\mathrm{Bias}(\tilde{\alpha}_{BS}|\alpha; \mathrm{R}) = \alpha\Big\{(\zeta-1)\mathrm{J}_0(\mathrm{a}^*, \mathrm{b}^*) - \mathrm{k}\frac{1}{\mathrm{n}-1} + (\zeta-1)(1-\mathrm{k}) - \mathrm{k}(\mathrm{n}-2)\mathrm{J}_1(\mathrm{a}^*, \mathrm{b}^*) - (1-\mathrm{k})\zeta\mathrm{J}_0(\mathrm{a}^*, \mathrm{b}^*) + \mathrm{J}_0(\mathrm{a}^*, \mathrm{b}^*)\Big\} \quad \ldots (15)$$

Where: $J_\ell(a^*, b^*) = \int_{a^*}^{b^*} y^{-\ell} \frac{y^{n-1} e^{-y}}{\Gamma(n)} dy; \ell = 0,1,2$, …(16)

Also:

$\zeta = \frac{\alpha_0}{\alpha}$, $a^* = (2\zeta)^{-1} . a$, $b^* = (2\zeta)^{-1} . b$ and $y = \frac{(n-2)}{\hat{\alpha}_B}\alpha$ …(17)

i.e.; $R^* = [\zeta^{-1} a^*, \zeta^{-1} b^*]$

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The Bias ratio [B (·)] of PSSBS ($\tilde{\alpha}_{BS}$) is defined as follows

$$\text{Bias}(\tilde{\alpha}_{BS}) = \frac{Bias(\tilde{\alpha}_{BS}/\alpha,R)}{\alpha} \qquad \ldots(18)$$

The expression of Mean Squared Error (MSE) of ($\tilde{\alpha}_{BS}$) given as follows:-

$$\text{MSE}(\tilde{\alpha}_{BS}/\alpha,R) = E(\tilde{\alpha}_{BS}-\alpha)^2$$

$$= \int_R [\alpha_0-\alpha]^2 f(\hat{\alpha}_B)d\hat{\alpha}_B + \int_{\bar{R}}[k\hat{\alpha}+(1-k)\alpha_0-\alpha]^2 f(\hat{\alpha}_B)d\hat{\alpha}_B$$

and by simple computations, one can get:

$$MSE(\tilde{\alpha}_{BS}\,|\alpha;R) = \alpha^2\Big\{k^2\frac{1}{(n-1)} + 2k^2(\zeta-1)\frac{1}{(n-1)} + k^2(\zeta-1)^2 - 2k\frac{(\zeta-1)}{(n-1)} - 2k(\zeta-1)^2 + \alpha^2(\zeta-1)^2 - k^2(n-2)^2J_2(a^*,b^*) + 2k^2(n-2)\zeta J_1(a^*,b^*) - k^2\zeta^2J_0(a^*,b^*) - 2k(\zeta-1)(n-2)J_1(a^*,b^*) + 2k\zeta(\zeta-1)J_0(a^*,b^*)\Big\} \qquad \ldots(19)$$

Now, the Efficiency of $\tilde{\alpha}_{BS}$ relative to the $\tilde{\alpha}_B$ denoted by R.Eff($\tilde{\alpha}_{BS}$/α,R) is defined as

$$\text{R.Eff}(\tilde{\alpha}_{BS}|\alpha,R) = \frac{\text{MSE}(\hat{\alpha}_B)}{\text{MSE}(\tilde{\alpha}_{BS}|\alpha,R)} \text{ ; see [3], [11]} \qquad \ldots(20)$$

4. DISCUSSION AND NUMERICAL RESULTS

The computations of Relative Efficiency [R.Eff (·)] and Bias Ratio [B(·)] expression were used for the considered testimators $\tilde{\alpha}_{BS}$. These computations were performed for the constants Δ = 0.01, 0.05, 0.1, n = 4,6,8,10,12, $k = \text{EXP}(-X^2_{1-\Delta/2,2n})$ and ζ = 0.25(0.25)2. Some of these computations are displayed in attached table for some samples of these constants. The observation mentioned in the table leads to the following results:

i. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}_{BS}$ are adversely proportional with small value of Δ especially when $\zeta = 1$, i.e. Δ = 0.01 yield highest efficiency
ii. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}_{BS}$ has maximum value when $\alpha=\alpha_0(\zeta=1)$, for each n, Δ, and decreasing otherwise ($\zeta\neq1$). This feature shown the important usefulness of prior knowledge which given higher effects of proposed estimator as well as the important role of shrinkage technique and its philosophy.
iii. Bias ratio [B(·)] of $\tilde{\alpha}_{BS}$ increases when ζ increases.
iv. Bias ratio [B(·)] of $\tilde{\alpha}_{BS}$ are reasonably small when $\alpha=\alpha_0$ for each n, Δ, and increases otherwise. This property shown that the proposed estimator $\tilde{\alpha}_{BS}$ is very closely to unbiasedness especially when $\alpha=\alpha_0$.
v. The Relative efficiency [R.Eff(·)] of $\tilde{\alpha}_{BS}$ decreases function when the value of k increases for each n, Δ, ζ. This property employ the role of the prior information for proposed shrinkage estimator via takes high weight for prior information which leads to maximum efficiency.
vi. The Effective Interval [the value of ζ that makes R.Eff.(·) greater than one] using proposed estimator $\tilde{\alpha}$ is approximately [0.5,1.5]. Here the pretest criterion is very important for

guarantee that prior information is very closely to the actual value and prevent it far away from it, which get optimal effect of the considered estimator to obtain high efficiency.

vii. The considered estimator $\tilde{\alpha}_{BS}$ is better than the Bayes estimator $\tilde{\alpha}_{B}$, especially when $\alpha \approx \alpha_0$, which is given the effective of $\tilde{\alpha}_{BS}$ and important weight of prior knowledge as well as the increment of efficiency may be reach to tens of billions time .

viii. The proposed estimator $\tilde{\alpha}_{BS}$ has smaller MSE than some existing estimators introduced by the authors, see for examples [1] and [8].

5. CONCLUSIONS

From the above discussions it is obvious that by using guess point value one can improve the Bayes estimator by using shrinkage technique. It can be noted that if the guess point α_0 is very close to the true value of the parameter α (i.e.; ζ is approximate close to one), the proposed estimators perform better than the Bayes estimator. If one has no confidence in the guessed value then proposed preliminary test shrunken estimators can be suggested. We can safely use the proposed estimators for small sample size at usual level of significance Δ and moderate value of shrunken weight factor $\Psi(.)$.

References

1. Al-Joboori, A.N,(2011),One Significance Test Estimator for the Shape Parameter of Generalized Rayleigh Distribution , Journal of AL-Qadisyia /College of Computer and Math . Sciences /AL-Qadisyia Univ., Vol. (3), 2. PP.1-8.
2. Al-Joboori, A.N., et al (2014), Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases , International Journal of Statistics , Vol.(38),2,PP,1127 -1134.
3. Al-Joboori, A.N., Khalef, B.A. and Haydar S. K., (2014), Improved of Stein-Type Estimator of the Variance of Normal Distribution via Shrinkage Estimation Technique, International Journal of Inventive Engineering and Sciences (IJIES), Vol.(2),6,PP.14-17.
4. Al-Joboori, A.N. and Raeeda A.S., (2014), Bayesian Shrinkage Estimator for the Scale Parameter of Exponential Distribution Under Improper Prior Distribution, International Journal of Statistics and Applications, Vol. (4),3;pp.135-143 .
5. Burr, I.W., (1942), Cumulative Frequency Distribution, Annals of Mathematical Statistics, Vol. (13), pp.215-232.
6. Iman Makhdoom , (2011), Minimax Estimation Of Parameter Of The Generalized Exponential Distribution , International Journal Of Academic Research , vol.(3),2,PP.515-527 .
7. Johnson, N.L., Kotz, S. and Balakrishnan, (1994), Continuous Univariate Distributions, New York: Wiley.
8. Kundu, D. and Raqab, M.Z., (2005), Generalized Rayleigh Distribution: Deferent Methods of Estimation, Computational Statistics and Data Analysis, Vol.(49), pp.187-200.
9. Raqab, M.Z. and Kundu, D., (2006), Burr Type X Distribution: Revisited, Journal of Probability and Statistical Sciences, Vol. (4),2, pp.179-193.
10. Surles, J.G. and Padgett, W.J., (1998), Inference for P(Y<X) in the Burr Type X model, Journal of Applied Statistical Science, Vol. (7), pp.225-238.

11. Thompson, J.R., (1968), Some Shrinkage Techniques for Estimating the Mean, J. Amer. Statist. Assoc, Vol.(63), pp.113-122.

12. Toyoda, T. and Wallace, D., (1975), Estimation of Variance after a preliminary test of Homogeneity and Optimum Levels of Significance for the Pre-Test, J. Econometrics, Vol.(3), pp.395-404.

Table : Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff.(·)] of $\tilde{\alpha}_{BS}$

			ζ							
Δ	n	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	4	R.Eff(-)	.592592592855	1.3333333337	5.3333333308	6.0410528E(20)	5.33333334764	1.3333333358	.592592593505	.333333333785
		B(-)	-.74999999992	-.49999999997	-.2499999999	3.31971E(-12)	.249999999994	.49999999997	.749999999945	.999999999899
	6	R.Eff(-)	.355555555737	.80000000045	3.2000000005	7.4682238E(20)	3.2000000065	.80000000121	.355555556013	.200000000227
		B(-)	-.74999999987	-.49999999996	-.2499999999	2.21447E(-12)	.249999999992	.49999999997	.749999999928	.999999999869
	8	R.Eff(-)	.253968254098	.57142857180	2.2857142866	8.5430830E(20)	2.28571428975	.57142857220	.253968254262	.14285714300
		B(-)	-.74999999984	-.49999999995	-.24999999990	1.62132E(-12)	.249999999991	.49999999996	.749999999913	.99999999984
	10	R.Eff(-)	.197530864298	.44444444475	1.7777777787	9.3249450E(20)	1.7777777806	.44444444500	.197530864409	.11111111121
		B(-)	-.74999999816	-.49999999994	-.24999999998	1.27381E(-12)	.249999999990	.49999999995	.749999999899	.99999999981
	12	R.Eff(-)	.161616161699	.36363636388	1.4545454554	9.9156373E(20)	1.4545454567	.36363636407	.161616161778	9.0909090988
		B(-)	-.74999999981	-.49999999993	-.2499999999	1.04648E(-12)	.24999999998	.49999999995	.749999999886	.99999999979
0.05	4	R.Eff(-)	.592592592801	1.3333333336	5.3333333303	3.5881672E(20)	5.33333334713	1.3333333356	.592592593421	.333333333736
		B(-)	-.74999999989	-.49999999995	-.2499999999	5.85032E(-12)	.249999999983	.49999999994	.749999999892	.999999999827
	6	R.Eff(-)	.355555555713	.80000000035	3.2000000001	3.5836258E(20)	3.20000000619	.80000000110	.355555555960	.200000000198
		B(-)	-.74999999985	-.49999999993	-.2499999999	4.61686E(-12)	.249999999979	.49999999993	.749999999872	.999999999796
	8	R.Eff(-)	.253968254089	.57142857172	2.2857142864	3.6705548E(20)	2.28571428948	.57142857212	.253968254224	.142857142983
		B(-)	-.74999999982	-.49999999992	-.2499999999	3.60720E(-12)	.249999999977	49999999992	.749999999857	.999999999773
	10	R.Eff(-)	.197530864296	.44444444468	1.7777777785	3.7463155E(20)	1.77777778042	.44444444493	.197530864380	.111111111201
		B(-)	-.74999999980	-.49999999991	-.2499999999	2.93282E(-12)	.249999999975	.49999999992	.749999999844	.999999999756
	12	R.Eff(-)	.161616161699	.36363636383	1.4545454553	3.8059148E(20)	1.45454545655	.36363636401	.161616161755	9.09090909780
		B(-)	-.74999999981	-.49999999990	-.2499999999	2.46205E(-12)	.249999999973	.24999999997	.749999999833	.999999999742
0.1	4	R.Eff(-)	.592592592788	1.3333333335	5.3333333300	2.9350280E(20)	5.33333334663	1.3333333355	.592592593368	.333333333709
		B(-)	-.74999999988	-.49999999994	-.2499999999	4.88022E(-12)	.249999999971	.49999999992	.749999999858	.999999999785

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	6	R.Eff(-) B(-)	.355555555709 -.74999999984	.80000000031 -.49999999992	3.1999999999 -.2499999999	2.7128483E(20) 4.74666E(-12)	3.20000000590 .249999999968	.80000000102 .24999999996	.355555555930 .749999999841	.200000000184 .999999999760
	8	R.Eff(-) B(-)	.253968254088 -.74999999982	.57142857169 -.49999999990	2.2857142862 -.2499999999	2.66469415260 3.93985E(-12)	2.28571428927 .249999999965	.57142857206 .49999999990	.253968254204 .749999999828	.142857142974 .999999999742
	10	R.Eff(-) B(-)	.197530864296 -.74999999981	.44444444466 -.4999999998	1.7777777784 -.2499999999	2.6502179E(20) 3.29718E(-12)	1.7777777802 .249999999963	.44444444489 .49999999989	.19753086436 .749999999817	.111111111193 .999999999730
	12	R.Eff(-) B(-)	.161616161699 -.74999999980	.36363636382 -.4999999998	1.4545454551 -.2499999999	2.6458903E(20) 2.81714E(-12)	1.45454545641 .249999999961	.36363636398 .4999999998	.161616161745 .749999999809	9.09090909741 .999999999721