

Analytical Modeling of Droplet Burning Rate of Various Fuels

Saroj R. Chafle , Satish K. Bhele

Kavikulguru Institute of Technology and Science, Ramtek Dist:Nagpur

ABSTRACT:- This paper presents the analytical modeling of droplet burning characteristics at different conditions. In this paper we study the spray break up phenomenon. This will give the idea about how the droplet evaporates and then burns. Depending upon the physical and thermo physical properties of fuel, it takes the time for the burning of fuel. As the inlet temperature of fuel increases fuel droplet will burn within short time.

This paper satisfied D^2 – law of heat evaporation. As the time of burning increases droplet size will decrease. Result shows that when we compare the droplet burning time at 600K, 800K and 1200K at that time 1200K droplet burns faster as compared to other, also diesel evaporates faster than ethanol and gasoline. For the understanding of droplet evaporation we must know the droplet burning time so that it is easy to understand for the further study of combustion.

Keywords:- Droplet, diesel spray, burning rate, evaporation .

INTRODUCTION :- In the burning of a single fuel droplet in an oxidizing atmosphere, fuel is evaporated from the liquid surface and the fuel vapour diffuses to the flame front, while the oxygen moves from the surrounding to the flame front. The shape of the envelope flame can either be spherical or nonspherical. Nonspherical flames are generally caused by the convection effect due to the relative motion between the surrounding gases can entrain it easily and relative velocity between the droplet and nearby gases becomes small; then the diffusion flame surrounding the droplet becomes nearly spherical.

The rate at which the droplet evaporates and burns is generally considered to be determined by the rate of heat transfer from the flame front to fuel surface. In the pioneering development of a theoretical model describing the gaseous diffusion flames surrounding the burning droplet, Spalding proposed a double-film model for the temperature and chemical species distributions around the single droplet.

Evaporation of a Single Fuel Droplet :- A quasi-steady assumption is utilized in Spalding's analysis for determining the distributions of fuel vapour mass fraction and temperature around the evaporating spherical droplet. The main advantage of this assumption is to temporally avoid the consideration of time variation associated with the droplet regression process. The justification of this assumption is based on the relatively slow regression rate of the fuel droplet. Using this assumption, one can imagine that the droplet evaporates so slowly that the droplet can be replaced by a porous sphere of a fixed diameter; the surface of this sphere is wetted by liquid fuel, and the mass evaporation rate of fuel from its surface into the warm surrounding is equal to the feeding rate of the liquid fuel from a small feeding tube to the porous sphere. For spherical droplet of fixed size with radius r_s , the steady-state species and energy equations can then be written as

Fuel species continuity equation :

$$r^2 \rho v \frac{dY_F}{dr} = \frac{d}{dr} \left(r^2 D \rho \frac{dY_F}{dr} \right) \dots\dots\dots(1)$$

Energy equation:

$$\rho v \frac{dC_p T}{dr} = \frac{d}{dr} \left(\frac{k}{C_p} r^2 \frac{dC_p T}{dr} \right) \dots\dots\dots(2)$$

Let us assume that the ambient gases do not dissolve into the fuel droplet; just below the droplet surface , only fuel (F) exists. For the vaporizing droplet, the total fuel mass flow rate leaving the droplet surface must be equal to the sum of the fuel mass flow rate due to the bulk convective motion of the gaseous mixture at the surface and the mass diffusion rate of the fuel vapour in the radial direction caused by the existence of fuel concentration gradient above the droplet surface.

$$\rho_s v_s = \rho_s (Y_F)_s v_s + \rho_s (Y_F)_s (V_F)_s = \rho_s (Y_F)_s v_s - \rho_s D_s \left(\frac{dY_F}{dr} \right) \dots\dots\dots(3)$$

After rearranging equation (3), we have

$$v_s = \frac{D(dY_F/dr)_s}{(Y_F)_s - 1} \dots\dots\dots(4)$$

Splading defined parameter

$$b = \frac{Y_F}{((Y_F)_s - 1)} = \frac{\text{mass fraction of F at any location r}}{\text{mass fraction of species other than F at surface r = r_s}} \dots\dots\dots(5)$$

Then v_s becomes

$$v_s = D_s \left(\frac{db}{dr} \right)_s \dots\dots\dots(6)$$

Using parameter b , the fuel species species conservation equation becomes ,

$$r^2 \rho v \frac{db}{dr} = \frac{d}{dr} (r^2 D \rho \frac{db}{dr}) \dots\dots\dots(7)$$

The boundry condition at $r = \infty$ is $Y_F = Y_{F\infty}$ or

$$r = \infty ; b = b_\infty = \frac{Y_{F\infty}}{Y_{F_s} - 1} \dots\dots\dots(8)$$

From integrated continuity equation

$$r^2 \rho v = \text{constant} = r_s^2 \rho_s v_s \dots\dots\dots(9)$$

Equation (9) and (7) integrating w. r. to r we get

$$r^2 \rho v b = r^2 \rho D \frac{db}{dr} + \text{constant}$$

Applying boundry condition at surface

$$r_s^2 \rho_s v_s (b - b_s + 1) = r^2 \rho D \frac{db}{dr}$$

By separating variables, we have

$$\frac{r_s^2 \rho_s v_s}{r^2 \rho_s D_s} * dr = \frac{db}{(b - b_s + 1)}$$

Integrating we have,

$$-\frac{r_s^2 v_s}{D_s} \left(\frac{1}{r} \right) = \ln(b - b_s + 1) + \text{constant} \dots\dots\dots(10)$$

Using boundry condition (8) we have

$$\frac{r_s^2 v_s}{r D_s} = \ln \left(\frac{b_\infty - b_s + 1}{b - b_s + 1} \right) \dots\dots\dots(11)$$

At $r = r_s$

$$\frac{r_s v_s}{D_s} = \ln(b_\infty - b_s + 1) \dots\dots\dots(12)$$

$(b_\infty - b_s)$ is called the Splading transfer number B or B_M , where the subscript M stands for mass, i.e.,

$$B = B_M = (b_\infty - b_s) = \frac{Y_{Fs} - Y_{F\infty}}{1 - Y_{Fs}} \dots\dots\dots(13)$$

Now equation (12) becomes

$$r_s v_s = D_s \ln(1 + B_M) \dots\dots\dots(14)$$

Mass flow rate per unit area is usually called as G_F ,

$$G_F = \frac{\dot{m}_F}{4\pi r_s^2} = \rho_s v_s \dots\dots\dots(15)$$

The fuel evaporation rate expression can be given as

$$G_F = \frac{\rho_s D_s \ln(1 + B_M)}{r_s} \dots\dots\dots(16)$$

In order to calculate the mass evaporation rate from Eq. (16), the value of B_M must be evaluated; before that, Y_F must be determined. A reasonable assumption would be that the gas surrounding the droplet is saturated by the fuel vapour at the surface temperature. Thus, the problem now becomes to determine T_s , since the vapour- pressure data for most fuels are available.

In order to determine the surface temperature T_s , we must consider the energy equation. Using the integrated continuity equation of (9) and assuming C_p of the gas around the droplet is independent of temperature Eq. (2) becomes

$$r_s^2 \rho_s v_s C_p \frac{dT}{dr} = \frac{d}{dr} \left(r^2 k \frac{dT}{dr} \right) \dots\dots\dots(17)$$

Integrating the above equation with respect to r, we have

$$r_s^2 \rho_s v_s C_p T = r^2 k \frac{dT}{dr} + constant \dots\dots\dots(18)$$

For getting constant apply B. C. at the droplet surface

$$k \left(\frac{dT}{dr} \right) = \rho_s v_s \Delta h_v \quad T(r = r_s) = T_s \dots\dots\dots(19)$$

After applying B.C. at droplet surface, we have

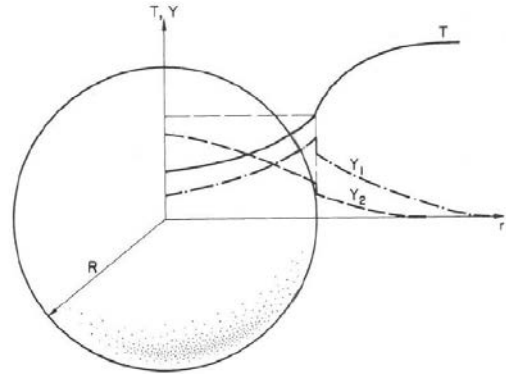
$$r_s^2 \rho_s v_s C_p \left(T - T_s + \frac{\Delta h_v}{C_p} \right) = r^2 k \frac{dT}{dr} \dots\dots\dots(20)$$

After separating the variables and integrating the the above equation we have

$$\frac{r_s^2 \rho_s v_s C_p}{rk} = \ln \left(T - T_s + \frac{\Delta h_v}{C_p} \right) + constant \dots\dots\dots(21)$$

Using the boundary condition that $T \rightarrow T_\infty$ as $r \rightarrow \infty$, we have

$$\frac{r_s^2 \rho_s v_s c_p}{rk} = \ln \left(\frac{T_\infty - T_s + \left(\frac{\Delta h_v}{c_p}\right)}{T - T_s + \frac{\Delta h_v}{c_p}} \right) \dots\dots\dots(22)$$



Temperature distribution of an evaporating liquid droplet

Setting r equal to r_s at the surface, we have

$$\frac{r_s \rho_s v_s c_p}{k} = \ln \left(1 + \frac{c_p(T_\infty - T_s)}{\Delta h_v} \right) \dots\dots\dots(23)$$

Since $\alpha = \frac{k}{\rho c_p}$,

$$r_s v_s = \alpha_s \ln \left[1 + \frac{c_p(T_\infty - T_s)}{\Delta h_v} \right] = \alpha_s \ln(1 + B_T) \dots\dots\dots(24)$$

Comparing equation (24) with Eq. (14)

$$r_s v_s = \alpha_s \ln \left(1 + \frac{c_p(T_\infty - T_s)}{\Delta h_v} \right) = D_s \ln \left(1 + \frac{Y_{F\infty} - Y_{FS}}{Y_{FS} - 1} \right) \dots\dots\dots(25)$$

$$r_s v_s = \alpha_s \ln(1 + B_T) = D_s \ln(1 + B_T) \dots\dots\dots(26)$$

If $\alpha_s = D_s$ (i. e., $Le = 1$), then

$$B_T = B_M, B_T = \frac{c_p(T_\infty - T_s)}{\Delta h_v} = \frac{Y_{F\infty} - Y_{FS}}{Y_{FS} - 1} = B_M = B \dots\dots\dots(27)$$

This equation relates the two unknowns T_s and Y_{FS} ; one additional equation is needed to solve for them.

The following equation relates Y_F to the partial pressure P_F

$$Y_F = \frac{\rho_F}{\rho} = \frac{n_F M_{WF}}{n M_W} = \frac{P_F M_{WF}}{P M_W} \quad \text{or} \quad Y_{FS} = \frac{P_{FS} M_{WF}}{P M_W} \dots\dots\dots(28)$$

This equation is helpful, since we can further relate the partial pressure of the fuel to the surface temperature T_s by the Clausius-Clapeyron vapour-pressure equation

$$\frac{d \ln P_F}{dT_s} = \frac{\Delta h_v}{RT_s^2} \dots\dots\dots(29)$$

$$\text{Or } \ln\left(\frac{P_{Fs}}{P_{Fs,ref}}\right) = \frac{\Delta h_v}{R} \left(\frac{1}{T_{s,ref}} - \frac{1}{T_s} \right) \dots\dots\dots(30)$$

Now we have three equations Eq.(27),(28),(30) for three unknowns Y_{Fs} , T_s , and P_{Fs} .

The lifetime or evaporation time, t_v , of a liquid droplet can be calculated using Eq.(15). The droplet evaporation time is an important parameter in combustion-chamber design, since the lifetime of the largest droplet in a spray determines the minimum time the droplet must be allowed to reside in the combustion chamber. Also, the residence time is related to other design parameters such as the air-stream velocity through the combustor, the velocity of the spray injection, the angle of injection, and the combustor geometry. Using mass continuity at the droplet surface $\left(-\frac{\rho_l dr_s}{dt} = \rho_s v_s\right)$ and rearranging Eq.(15) we have

$$\frac{dr_s}{dt} = -\frac{\rho_s D_s}{\rho_l r_s} \ln(1 + B) \dots\dots\dots(31)$$

After integrating and expressing the result in terms of droplet diameter, we have

$$d^2 = d_0^2 - \left[\frac{8\rho_s \alpha_s}{\rho_l} \ln(1 + B) \right] t \dots\dots\dots(32)$$

$$\text{Now, } K = \frac{8\rho_s \alpha_s}{\rho_l} \ln(1 + B) \dots\dots\dots(33)$$

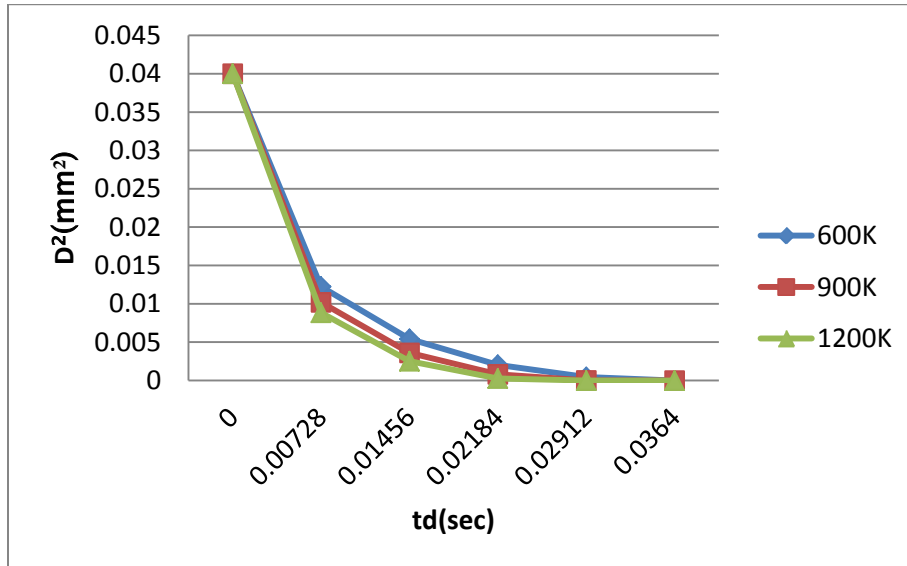
$$d^2 = d_0^2 - Kt \dots\dots\dots(33)$$

Where the evaporation coefficient K represents the magnitude of the negative slope of the straight line on the $d^2 - t$ plot. The lifetime of the droplet is therefore equal to

$$t_{life} = \frac{d_0^2}{K} \dots\dots\dots(34)$$

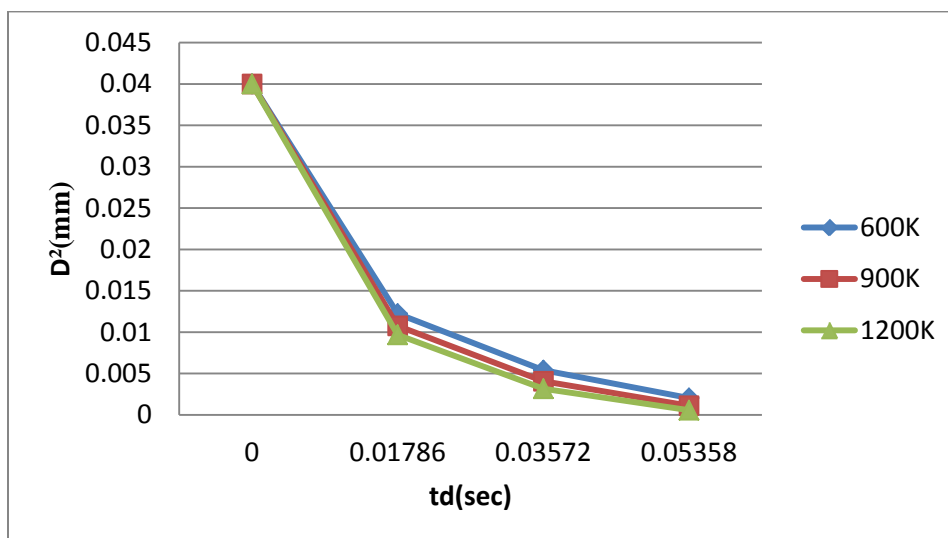
Results :- By using analytical modeling satisfied the $D^2 -$ law of heat evaporation. Plot the graph at various temperature for different fuels which is essential for the understanding of droplet evaporation rate. Various results of analytical modeling by using the equation (27) and equ (34) we got the results which are plotted below.

Vaporization of diesel fuel at 600K,900K,1200K:-



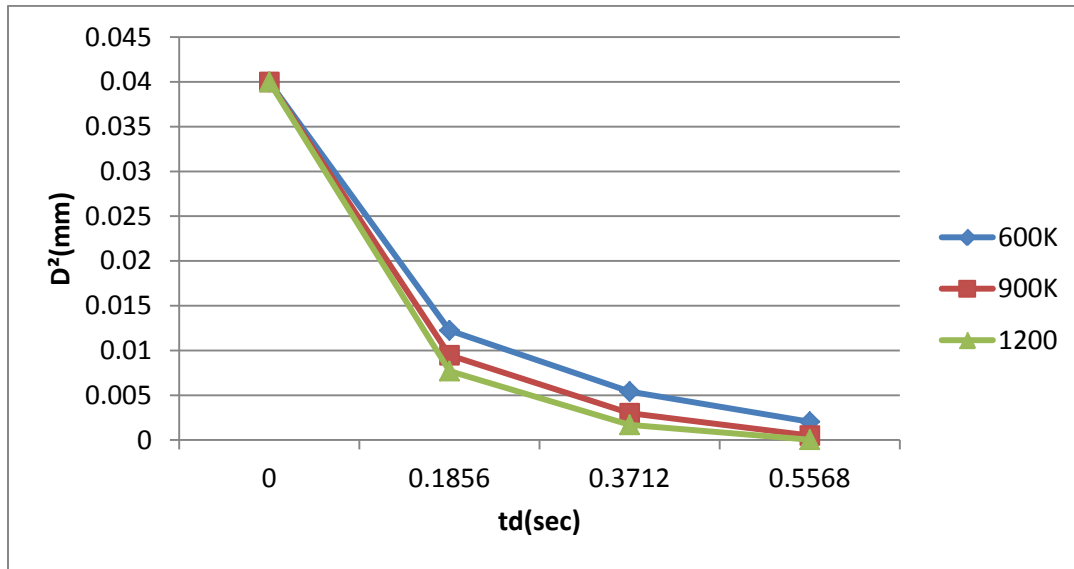
As the droplet lifetime increases diesel droplet get evaporates and burns. As the inlet atmospheric temperature is more it will evaporates and burns faster. From the above graph it shows that as compare to 600K and 900K , 1200K inlet temperature of air will evaporates faster. It satisfied the D^2 – law of heat evaporation.

Vaporization of gasoline at 600K,900K,1200K:-



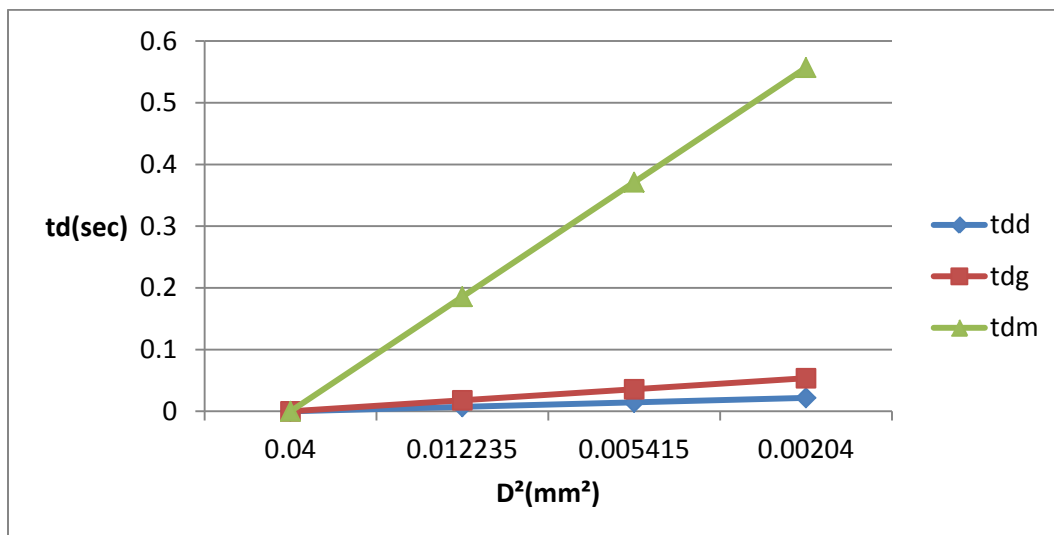
Above graph shows that gasoline evaporates faster at 1200K as compares to 600K and 900K as same as diesel but time required for evaporation is more s compares to diesel. Also it satisfied the D^2 – law of heat evaporation.

Vaporization of methanol at 600K,900K and 1200K



Above graph satisfied the D^2 – law nature of graph is same as that of petrol and diesel but time required for the evaporation of droplet is more as compare to petrol and diesel. This is because of thermo physical properties of methanol.

Droplet burning time of diesel, gasoline and methanol at 600K

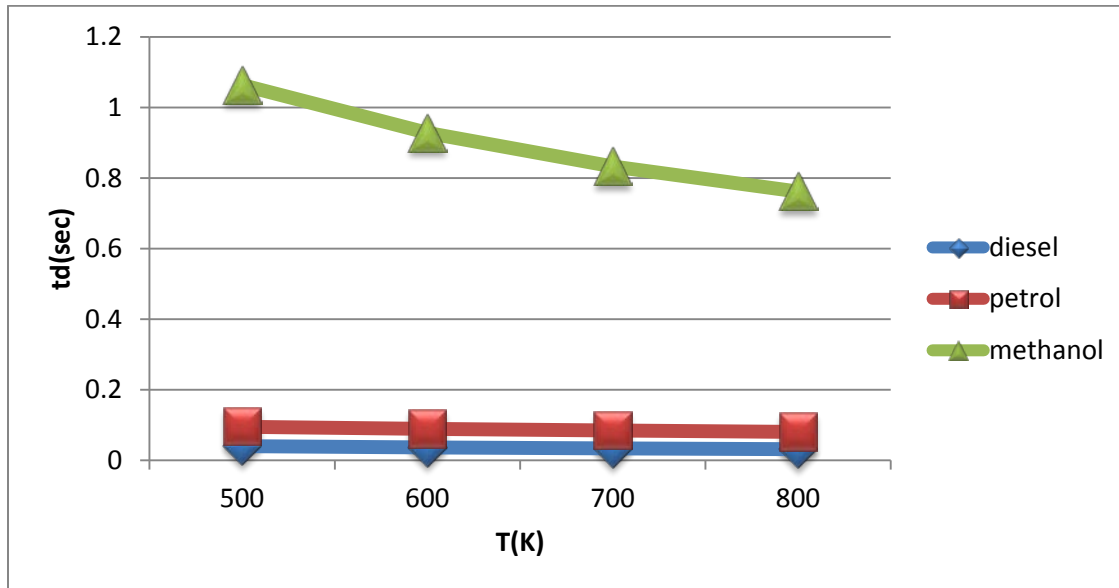


This graph shows diesel fuel evaporates faster than gasoline and ethanol. Because the boiling point and latent heat of vaporization or heat of combustion of diesel is more as compare to gasoline and methanol

that's why it burns within short time. Same graph will be obtain at 900K and 1200K only time required for the 600K is more as compare to 900K and 1200K.

Vaporization of different fuel at different temperature

As the inlet temperature of fuel is more that evaporates within short period of time below graph indicates diesel evaporates faster as compare to petrol and methanol.



Conclusion:-

This paper satisfied the D^2 – Law of evaporation. As by using analytical modeling calculated the droplet burning time which shows that by increase in ambient temperature lifetime of droplet is likely to get decreased further. From the above graph diesel fuel evaporates within short time as compare to petrol and ethanol because boiling point, heat of combustion or latent heat of vaporization is more of diesel fuel.

NOMENCLATURE

- \dot{m}_f droplet mass burning rate (kg/sec)
- ρ mass density (kg/m³)
- v_s bulk velocity of mixture at surface
- r radius of droplet (mm)
- k_g thermal conductivity of gases (W/mK)
- Δh_v heat of vaporization (kJ/kg)

- C_p specific heat (kJ/kg.K)
- B Splading transfer number
- K burning rate constant (m^2/sec)
- f fuel air ratio by mass
- T_∞ free stream temperature
- T_s surface temperature
- D diffusion coefficient (m^2/sec)
- t_{life} droplet burning time (sec)

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