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IDEAL BITOPOLOGICAL b-R_0 (-R_1) SPACES

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ABSTRACT. In this paper we introduce and study (i,j)-b-$\mathcal{I}$-R_0 and (i,j)-b-$\mathcal{I}$-R_1 spaces.

1. Introduction and Preliminaries

The concept of ideal bitopological spaces was first introduced by Kelly [4]. After the introduction of the definition of a ideal bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. The concept of ideals in topological spaces has been introduced and studied by Kuratowski [5] and Vaidyanathasamy [9]. An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)^*$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [9] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^*(\tau, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator $\mathrm{Cl}^*(.)$ for a topology $\tau^*(\tau, \mathcal{I})$ called the $*$-topology, finer than τ is defined by $\mathrm{Cl}^*(A) = A \cup A^*(\tau, \mathcal{I})$ when there is no chance of confusion, $A^*(\mathcal{I})$ is denoted by A^* and τ_i-$\mathrm{Int}^*(A)$ denotes the interior of A in $\tau_i^*(\mathcal{I})$. If $\mathcal{I}$ is an ideal on X, then ideal bitopological spaces is called an ideal bitopological space. Let A be a subset of a ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$. We denote the closure of A and the interior of A with respect to τ_i by τ_i-$\mathrm{Cl}(A)$ and τ_i-$\mathrm{Int}(A)$, respectively. The notion of R_0 topological spaces introduced by Shanin [8] in 1943. Davis [1] introduced the notion of R_1 topological spaces which are independent of both T_0 and T_1 but strictly weaker than T_2. Some basic properties of the class of R_1 in topological spaces were discussed by Murdeshwar and Naimpally [7]. Bitopological forms of these concepts have appeared in the definitions of pairwise R_0 and pairwise R_1 spaces given by Mr$\widehat{s}$evi$\widehat{c}$ [6]. A subset A is called (i,j)-b-$\mathcal{I}$-$\mathcal{I}$-open [2] if $A \subset j\,\mathrm{Cl}^*(i\,\mathrm{Int}(A)) \cup i\,\mathrm{Int}(j\,\mathrm{Cl}^*(A))$. The complement of an

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(i,j)-b-$\mathcal{I}$-open set is called an (i,j)-b-$\mathcal{I}$-closed set. The the intersection of all (i,j)-b-$\mathcal{I}$-closed sets of X containing A is called the (i,j)-b-$\mathcal{I}$-closure of A and is denoted by (i,j)-$b\mathcal{I}\,\mathrm{Cl}(A)$. The the intersection of all (i,j)-b-$\mathcal{I}$-open sets of X containing A is called the (i,j)-b-$\mathcal{I}$-kernal of A and is denoted by (i,j)-$b\mathcal{I}\,\mathrm{Ker}(A)$. An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_0 [3] if for every pair of distinct points in X, there exists an (i,j)-b-$\mathcal{I}$-open set of X containing one of the points but not the other. An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_1 [3] if for every pair of distinct points x, y of X, there exists a pair of (i,j)-b-$\mathcal{I}$-open sets one containing x but not y and the other containing y but not x. An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_2 [3] if for every pair of distinct points x, y of X, there exists a pair of disjoint (i,j)-b-$\mathcal{I}$-open sets, one containing x and the other containing y. In this paper we introduce and study (i,j)-b-$\mathcal{I}$-R_0 and (i,j)-b-$\mathcal{I}$-R_1 spaces.

2. Ideal Bitopological b-R_0 (-R_1) spaces

In this section, we introduce and study (i,j)-b-$\mathcal{I}$-R_0 and (i,j)-b-$\mathcal{I}$-R_1 spaces. These axioms can be characterized in terms of the (i,j)-b-$\mathcal{I}$-closure and the (i,j)-b-$\mathcal{I}$-kernal of singletons.

Definition 1. *A bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-R_0 if for every (i,j)-b-$\mathcal{I}$-open set of X contains the (i,j)-b-$\mathcal{I}$-closure of each of its singletons.*

Remark 2.1. *Since a bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_1 if, and only if the singletons are (i,j)-b-$\mathcal{I}$-closed, it is clear that every (i,j)-b-$\mathcal{I}$-T_1 space (i,j)-b-$\mathcal{I}$-R_0. But the converse is not true in general as it can be seen from the following example:*

Example 2.2. *Let $X=\{a,b,c\}$, $\tau_1=\{\emptyset,\{a\},\{b,c\},X\}$, $\tau_2=\mathcal{P}(X)$ and $\mathcal{I}=\{\emptyset\}$. It is clear that, $(1,2)$-$B\mathcal{I}O(X)=\{\emptyset,\{a\},\{b,c\},X\}$. Then the bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is $(1,2)$-b-$\mathcal{I}$-R_0 but not $(1,2)$-b-$\mathcal{I}$-T_1.*

Remark 2.3. *Th following example shows that the notions (i,j)-b-$\mathcal{I}$-T_0-ness (i,j)-b-$\mathcal{I}$-R_0-ness are independent.*

Example 2.4. *Let $X=\{a,b,c\}$, $\tau_1=\{\emptyset,\{a\},\{a,b\},\{a,c\},X\}$, $\tau_2=\mathcal{P}(X)$ and and $\mathcal{I}=\{\emptyset\}$. It is clear that, $(1,2)$-$B\mathcal{I}O(X)=\{\emptyset,\{a\},\{a,b\},\{a,c\},X\}$. Then the bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is $(1,2)$-b-$\mathcal{I}$-T_0 but not $(1,2)$-b-$\mathcal{I}$-R_0. Also the bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ as in Example 2.2 is $(1,2)$-b-$\mathcal{I}$-R_0 but not $(1,2)$-b-$\mathcal{I}$-T_0.*

Definition 2. *An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-symmetric if for each $x,y\in X$, $x\in(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ implies $y\in(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$.*

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Theorem 2.5. *An ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-R_0 *if, and only if it is* (i, j)-b-$\mathcal{I}$-*symmetric.*

Proof. Assume that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0. Let $x \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and U be any (i, j)-b-$\mathcal{I}$-open set such that $y \in U$. Then by hypothesis, $x \in U$. Therefore, every (i, j)-b-$\mathcal{I}$-open set which contains y contains x. Hence, $y \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Conversely, let U be an (i, j)-b-$\mathcal{I}$-open set and $x \in U$. If $y \notin U$, then $x \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, and thus by assumption, $y \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Therefore, (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$, and hence, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0. □

Theorem 2.6. *An ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_1 *if, and only if* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_0 *and* (i, j)-b-$\mathcal{I}$-R_0

Proof. Let $x, y \in X$ and $x \neq y$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_0, we may assume without loss of generality that $x \in G \subset X \backslash \{y\}$ for some (i, j)-b-$\mathcal{I}$-open set G. Thus, $x \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, and so by Theorem 2.5, $y \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Therefore, $X \backslash (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ is an (i, j)-b-$\mathcal{I}$-open set containing y but not x. Hence, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_1. The converse is clear. □

Remark 2.7. *It is clear from Theorem 2.6 that if* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_0 *but not* (i, j)-b-$\mathcal{I}$-T_1*, then* $(X, \tau_1, \tau_2, \mathcal{I})$ *is not* (i, j)-b-$\mathcal{I}$-R_0.

Proposition 2.8. *For a bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$*, the following statements are equivalent:*

(1) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-R_0 *space;*
(2) *If for any* $F \in (i, j)$-$B\mathcal{I}C(X)$, $x \notin F$*, then* $F \subset U$ *and* $x \notin U$ *for some* $U \in (i, j)$-$B\mathcal{I}O(X)$*;*
(3) *If for any* $F \in (i, j)$-$B\mathcal{I}C(X)$ *such that* $x \notin F$*, then* $F \cap (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = \emptyset$*;*
(4) *If for any two distinct points* x *and* y *of* $(X, \tau_1, \tau_2, \mathcal{I})$*, then either* (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ *or* (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$.

Proof. (1)⇒(2): Let $F \in (i, j)$-$B\mathcal{I}C(X)$ and $x \notin F$. Then by (1) (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset X \backslash F$. Set $U = X \backslash (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$, then $U \in (i, j)$-$B\mathcal{I}O(X)$ with $F \subset U$ and $x \notin U$.
(2)⇒(3): Let $F \in (i, j)$-$B\mathcal{I}C(X)$ such that $x \notin F$. Then by (2), there exists $U \in (i, j)$-$B\mathcal{I}O(X)$ such that $F \subset U$ and $x \notin U$. Since $U \in (i, j)$-$B\mathcal{I}O(X)$, $U \cap (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = \emptyset$ and $F \cap (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = \emptyset$.
(3)⇒(4): Suppose that (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ for the distinct points $x, y \in X$. Then there exists $z \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ such that $z \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ (or $z \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ such that $z \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$). Then there exists $V \in (i, j)$-$B\mathcal{I}C(X, z)$ such that $y \notin$

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V, hence $x \in V$. Therefore, $x \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. By (3), we obtain $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$. The proof for the other case is similar.

(4)⇒(1): Let $V \in (i,j)\text{-}B\mathcal{I}O(X,x)$. For each $y \notin V$, we have $x \neq y$ and $x \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. This shows that $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Hence by (4), $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$ for each $y \in X \setminus V$ and hence $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (\bigcup_{y \in X \setminus V} (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})) = \emptyset$. On the other hand, since $V \in (i,j)\text{-}B\mathcal{I}O(X)$ and $y \in X \setminus V$, we have $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset X \setminus V$ and hence $X \setminus V = \bigcup_{y \in X \setminus V} (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Therefore, we obtain $(X \setminus V) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) = \emptyset$ and hence $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset V$. This shows that $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$. □

Theorem 2.9. *A bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$ if, and only if for any $x,y \in X$, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$ implies $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$.*

Proof. Suppose that $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$ and x, $y \in X$ such that $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exists $z \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ such that $z \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$ (or $z \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$ such that $z \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$). Since $z \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$, there exists $V \in (i,j)\text{-}B\mathcal{I}O(X,z)$ such that $y \notin V$. But $z \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ so $x \in V$. Therefore, $x \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Hence $x \in X \setminus (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) \in (i,j)\text{-}B\mathcal{I}O(X)$. Since $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset X \setminus (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Hence $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$. The proof for otherwise is similar. Conversely, let $V \in (i,j)\text{-}B\mathcal{I}O(X,x)$. We will show that $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset V$. Let $y \notin V$, that is, $y \in X \setminus V$. Then $x \neq y$ and $x \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. This shows that $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. By assumption, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$. Hence $y \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ and therefore $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset V$. Hence $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$. □

Theorem 2.10. *An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$ if, and only if for any points x and y in X, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{y\})$ implies $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{y\}) = \emptyset$.*

Proof. Suppose that $(X,\tau_1,\tau_2,\mathcal{I})$ is an $(i,j)\text{-}b\text{-}\mathcal{I}\text{-}R_0$ space. Then for any points x and y in X, if $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{y\})$, then $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Assume that $z \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{y\})$. By $z \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\})$, it follows that $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{z\})$. Thus by Theorem 2.9, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{z\})$. Similarly, we have $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\}) = (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{z\}) = ((i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$, a contradiction. Hence, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{y\}) = \emptyset$. Conversely, let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological

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space such that for any points x and y of X, (i,j)-$b\,\mathrm{Ker}(\{x\}) \neq (i,j)$-$b\,\mathrm{Ker}(\{y\})$ implies (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{y\}) = \emptyset$. Assume that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{y\})$, and therefore by assumption, (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{y\}) = \emptyset$. Now if $z \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$, then $x \in (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{z\})$, and therefore, (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \cap (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{z\}) \neq \emptyset$. By hypothesis, (i,j)-$b\,\mathrm{Ker}(\{x\}) = (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{z\})$. Thus $z \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ implies that (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) = (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{z\}) = (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{y\})$, a contradiction. Therefore (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ implies that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) = \emptyset$, and thus by Theorem 2.9, $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0. □

Theorem 2.11. *For a bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, the following statements are equivalent:*

(1) *$(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0.*
(2) *For any nonempty subsets A of X and $G \in (i,j)$-$B\mathcal{I}O(X)$ such that $A \cap G \neq \emptyset$, there exists $F \in (i,j)$-$B\mathcal{I}C(X)$ such that $A \cap F \neq \emptyset$ and $F \subset G$.*
(3) *For any $G \in (i,j)$-$B\mathcal{I}O(X)$, $G = \cup\{F : F \in (i,j)$-$B\mathcal{I}C(X)$, $F \subset G\}$.*
(4) *For any $F \in (i,j)$-$B\mathcal{I}C(X)$, $F = \cap\{G : G \in (i,j)$-$B\mathcal{I}O(X)$, $F \subset G\}$.*
(5) *For any $x \in X$, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i,j)$-$b\mathcal{I}\,\mathrm{Ker}(\{x\})$.*

Proof. (1)⇒(2): Let A be a nonempty set of X and $G \in (i,j)$-$B\mathcal{I}O(X)$ such that $A \cap G \neq \emptyset$. Then there exists $x \in A \cap G$. Since $x \in G \in (i,j)$-$B\mathcal{I}O(X)$, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset G$. Set $F = (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Then $F \in (i,j)$-$B\mathcal{I}C(X)$, $F \subset G$ and $A \cap F \neq \emptyset$.
(2)⇒(3): Let $G \in (i,j)$-$B\mathcal{I}O(X)$, then $G \supset \cup\{F : F \in (i,j)$-$B\mathcal{I}C(X)$, $F \subset G\}$. Let x be any point of G. Then there exists $F \in (i,j)$-$B\mathcal{I}C(X)$ such that $x \in F$ and $F \subset G$. Therefore, $x \in F \subset \cup\{F : F \in (i,j)$-$B\mathcal{I}C(X)$, $F \subset G\}$, and hence $G = \cup\{F : F \in (i,j)$-$B\mathcal{I}C(X)$, $F \subset G\}$.
(3)⇒(4): This is obvious.
(4)⇒(5): Let x be any point of X and $y \notin (i,j)$-$b\,\mathrm{Ker}(\{x\})$. Then there exists $V \in (i,j)$-$B\mathcal{I}O(X,x)$ any $y \notin$ V; hence (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \cap V = \emptyset$. By (4), $\cap\{G : G \in (i,j)$-$B\mathcal{I}O(X)$, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset G\}$, and hence there exists $G \in (i,j)$-$B\mathcal{I}O(X)$ such that $x \notin G$ and (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset G$. Therefore, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap G = \emptyset$ and hence $y \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})) = (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Consequently, we obtain (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i,j)$-$b\,\mathrm{Ker}(\{x\})$.
(5)⇒(1): Let $G \in (i,j)$-$BO(X,x)$. If $y \in (i,j)$-$b\,\mathrm{Ker}(\{x\})$, then $x \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and so $y \in G$. This implies that (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset$

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G. Therefore, $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset G$. This shows that $(X, \tau_1, \tau_2, \mathcal{I})$ is an (i,j)-b-$\mathcal{I}$-R_0 space. □

Corollary 2.12. *For a bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0 if, and only if $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ =$(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\})$ for each $x \in X$.*

Proof. Suppose that $(X, \tau_1, \tau_2, \mathcal{I})$ is an (i,j)-b-$\mathcal{I}$-R_0 space. By Theorem 2.11, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\})$ for each $x \in X$. Let $y \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\})$. Then we have $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and by Theorem 2.9 $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Therefore, $y \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ and hence $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$. This shows that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ =$(i,j)\text{-}b\,\mathrm{Ker}(\{x\})$. The converse follows from Theorem 2.11. □

Theorem 2.13. *For an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$, the following statements are equivalent:*

1. *$(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0.*
2. *$x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$ if, and only if $y \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ for any points x and y in X.*

Proof. (1)⇒(2): Assume that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0 and $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Hence $y \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$. The other part is similar.
(2)⇒(1): Let $x \in U \in (i,j)\text{-}B\mathcal{I}O(X,x)$. If $y \notin U$, then $x \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and hence $y \notin (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ (by (2)). Thus $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0. □

Theorem 2.14. *For an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$, the following statements are equivalent:*

1. *$(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0.*
2. *If F is an (i,j)-b-$\mathcal{I}$-closed subset of X, then $F = (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(F)$.*
3. *If F is an (i,j)-b-$\mathcal{I}$-closed subset of X and $x \in F$, then (i,j)-$b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset F$.*
4. *If $x \in X$, then $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$.*

Proof. (1)⇒(2): Let F be an (i,j)-b-$\mathcal{I}$-closed subset of X and $x \notin F$. Thus $X \backslash F \in (i,j)\text{-}B\mathcal{I}O(X,x)$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset X \setminus F$. Thus $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap F = \emptyset$ and $x \notin (i,j)\text{-}b\,\mathrm{Ker}(F)$. Therefore, $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(F) = F$.
(2)⇒(3): In general, $A \subset B$ implies $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(A) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(B)$. Therefore, it follows from (2) that $(i,j)\text{-}b\,\mathrm{Ker}(\{x\}) \subset (i,j)\text{-}b\,\mathrm{Ker}(F) = F$.
(3)⇒(4): Since $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ and $(i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$ is (i,j)-b-$\mathcal{I}$-closed, by (3) $(i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\}) \subset (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{x\})$.
(4)⇒(1): Let $x \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then $y \in (i,j)\text{-}b\mathcal{I}\,\mathrm{Ker}(\{x\})$. Since

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$x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$ and (i,j)-$b\mathcal{I}\operatorname{Cl}(\{x\})$ is (i,j)-b-$\mathcal{I}$-closed, by (4) we obtain $y \in (i,j)$-$b\mathcal{I}\operatorname{Ker}(\{x\}) \subset (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$. Therefore, $x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$ implies that $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$. Hence by Theorem 2.13, $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0. □

Definition 3. *A net $\{x_\alpha\}_{\alpha\in\Lambda}$ in an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is called (i,j)-b-$\mathcal{I}$-convergent to a point x in X if for every $U \in (i,j)$-$B\mathcal{I}O(X,x)$, there exists $\alpha_0 \in \Lambda$ such that $x_\alpha \in U$ for each $\alpha \geq \alpha_0$.*

Lemma 2.15. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space and let x and y any two points in X such that every net in X (i,j)-b-$\mathcal{I}$-converging to y (i,j)-b-$\mathcal{I}$-converges to x. Then $x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$.*

Proof. Suppose that $x_n = y$ for each $n \in \mathbb{N}$. Then $\{x_n\}_{n\in\mathbb{N}}$ is a net in X that (i,j)-b-$\mathcal{I}$-convergence to y. Thus by assumption, (i,j)-b-$\mathcal{I}$-converges to x. Hence $x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. □

Theorem 2.16. *For an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, the following statements are equivalent:*

(1) *$(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0.*
(2) *If x, $y \in X$, then $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$ if, and only if every net in X (i,j)-b-$\mathcal{I}$-converging to y also (i,j)-b-$\mathcal{I}$-converges to x.*

Proof. (1)⇒(2): Let x, $y \in X$ such that $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$. Suppose that $\{x_\alpha\}_{\alpha\in\Lambda}$ be a net in X such that $\{x_\alpha\}_{\alpha\in\Lambda}$ (i,j)-b-$\mathcal{I}$-converges to y. Since $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$, by Theorem 2.5, $x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. Conversely, let x, $y \in X$ such that every net in X (i,j)-b-$\mathcal{I}$-converging to y (i,j)-b-$\mathcal{I}$-converges to x. Then $x \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. By Theorem 2.13, $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$.
(2)⇒(1): Assume that x and y are any two points of X such that (i,j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) \cap (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\}) \neq \emptyset$. Let $z \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{x\}) \cap (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. So there exists a net $\{x_\alpha\}_{\alpha\in\Lambda}$ in (i,j)-$b\mathcal{I}\operatorname{Cl}(\{x\})$ such that $\{x_\alpha\}_{\alpha\in\Lambda}$ (i,j)-b-$\mathcal{I}$-converges to z. Since $z \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$, $\{x_\alpha\}_{\alpha\in\Lambda}$ also (i,j)-b-$\mathcal{I}$-converges to y. Hence by (2) $z \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. Therefore (i,j)-$b\mathcal{I}\operatorname{Cl}(\{z\}) \subset (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$ ($\star$). Hence $y \in (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{z\})$ gives (i,j)-$b\mathcal{I}\operatorname{Cl}(\{y\}) \subset (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{z\})$ ($\star\star$). Hence from ($\star$) and ($\star\star$), (i,j)-$b\mathcal{I}\operatorname{Cl}(\{y\}) = (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{z\})$. Similarly it can be shown that (i,j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) = (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{z\})$ by taking the net in (i,j)-$b\mathcal{I}\operatorname{Cl}(\{y\})$. So (i,j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) = (i,j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. By Theorem 2.9 $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0. □

Now we introduce and study (i,j)-b-$\mathcal{I}$-R_1 space. Also we obtain the relationship between this axiom and other separation axioms introduced in this chapter.

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Definition 4. *An ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be (i, j)-b-$\mathcal{I}$-R_1 if for each points x and y of X such that (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\})$ $\neq (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$, there exist disjoint (i, j)-b-$\mathcal{I}$-open subsets of X, say, U and V such that (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) \subset U$ and (i, j)-$b\mathcal{I}\operatorname{Cl}(\{y\}) \subset V$.*

Proposition 2.17. *If an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1, then it is (i, j)-b-$\mathcal{I}$-R_0.*

Proof. Let $U \in (i, j)$-$B\mathcal{I}O(X, x)$. If $y \notin U$, then $x \notin (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. So (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1, there exists an (i, j)-b-$\mathcal{I}$-open set V_y such that (i, j)-$b\mathcal{I}\operatorname{Cl}(\{y\})$ $\subset V_y$ and $x \notin V_y$, which implies that $y \notin (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{x\})$. Hence (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) \subset U$. Therefore, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0. □

The following example shows that the converse of Proposition 2.17 is not true in general.

Example 2.18. *Let X be an infinite set and p a fixed point of X and let $\tau_1 = \tau_2 = \{\emptyset, X, U\}$ with $U \subset (X \backslash \{p\})$ and $X \backslash U$ finite and $\mathcal{I} = \{\emptyset\}$. Then $(1, 2)$-$B\mathcal{I}O(X) = \{\emptyset, U, A, X\}$ such that for each $A \subset U$. Since $(1, 2)$-$b\mathcal{I}\operatorname{Cl}(\{x\}) = \{x\}$ for every $x \in X$, $(X, \tau_1, \tau_2, \mathcal{I})$ is $(1, 2)$-b-$\mathcal{I}$-R_0. But it is not $(1, 2)$-b-$\mathcal{I}$-R_1.*

Theorem 2.19. *For a bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$, the following properties are equivalent:*

(1) *$(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_2,*
(2) *$(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1 and (i, j)-b-$\mathcal{I}$-T_1,*
(3) *$(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1 and (i, j)-b-$\mathcal{I}$-T_0.*

Proof. (1)⇒(2): Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_2, then it is (i, j)-b-$\mathcal{I}$-T_1. Now if $x, y \in X$ such that (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$, then $x \neq y$ and there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that x $\in U$ and $y \in V$. Hence by Theorem 2.6, (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) = \{x\} \subset U$ and (i, j)-$b\mathcal{I}\operatorname{Cl}(\{y\}) = \{y\} \subset V$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1.
(2)⇒(3): Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_1, then it is (i, j)-b-$\mathcal{I}$-T_0.
(3)⇒(1): Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1 and (i, j)-b-$\mathcal{I}$-T_0, then by Proposition 2.17, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0 and (i, j)-b-$\mathcal{I}$-T_0. Hence by Theorem 2.6, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_1. Let x, $y \in X$ such that $x \neq y$. Then (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) = \{x\} \neq \{y\} = (i, j)$-$b\mathcal{I}\operatorname{Cl}(\{y\})$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1, there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that (i, j)-$b\mathcal{I}\operatorname{Cl}(\{x\}) = \{x\} \subset U$ and (i, j)-$b\mathcal{I}\operatorname{Cl}(\{y\}) =$ $\{y\} \subset V$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_2. and thus by Theorem 2.5 (X, τ) is an (i, j)-b-$\mathcal{I}$-R_0 space. □

Corollary 2.20. *For an (i, j)-b-$\mathcal{I}$-R_1 space $(X, \tau_1, \tau_2, \mathcal{I})$, the following statements are equivalent:*

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(1) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_2.
(2) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_1.
(3) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_0.

Theorem 2.21. *For an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-R_1 *if, and only if* $x \in X \backslash (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ *implies that* x *and* y *have disjoint* (i, j)-b-$\mathcal{I}$-*neighbourhoods.*

Proof. Let $x \in X \backslash (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and x and y have disjoint (i, j)-b-$\mathcal{I}$-neighbourhoods. Conversely, first we show that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0. Let U be an (i, j)-b-$\mathcal{I}$-open set and $x \in U$. Suppose that $y \notin U$. Then, (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \cap U = \emptyset$ and $x \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U_x and U_y such that $x \in U_x$ and $y \in U_y$ and $U_x \cap U_y = \emptyset$. Hence, (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(U_x)$ and (i, j)-$b\mathcal{I}\,\mathrm{Cl}(x) \cap U_y \subset (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{U_x\}) \cap U_y = \emptyset$. Therefore, $y \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Consequently, (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$ and $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0. Next, we show that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1. Suppose that (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then, we can assume that there exists $z \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ such that $z \notin (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exist disjoint (i, j)-b-$\mathcal{I}$-open sets V_z and V_y such that $z \in V_z$, $y \in V_y$. Since $z \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$, $x \in V_z$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_0, we obtain (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset V_z$, (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset V_y$ and $V_z \cap V_y = \emptyset$. This shows that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-R_1. and thus by Theorem 2.5 (X, τ) is an (i, j)-b-$\mathcal{I}$-R_0 space. □

Theorem 2.22. *For a bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$*, the following statements are equivalent:*

(1) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-R_1.
(2) *For each* x, $y \in X$ *one of the following holds:*
  (a) *If* U *is* (i, j)-b-$\mathcal{I}$-*open, then* $x \in U$ *if, and only if* $y \in U$.
  (b) *there exist disjoint* (i, j)-b-$\mathcal{I}$-*open sets* U *and* V *such that* $x \in U$ *and* $y \in V$.
(3) *If* x, $y \in X$ *such that* (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$*, then there exist* (i, j)-b-$\mathcal{I}$-*closed sets* F_1 *and* F_2 *such that* $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$*, and* $X = F_1 \cup F_2$.

Proof. (1)⇒(2): Let $x, y \in X$. Then (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ or (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. If (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ and U is (i, j)-b-$\mathcal{I}$-open, then $x \in U$ implies $y \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$ and $y \in U$ implies $x \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset U$. Thus consider the case that (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that $x \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$ and $y \in (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset V$.
(2)⇒(3): Let x, $y \in X$ such that (i, j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i, j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$.

Then $x \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ or $y \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$, say $x \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exists an (i,j)-b-$\mathcal{I}$-open set A such that $x \in$ A and $y \notin A$. Then by (2) there exist disjoint (i,j)-b-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$. Then $F_1 = X \setminus V$ and $F_2 = X \setminus U$ are (i,j)-b-$\mathcal{I}$-closed sets such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.
(3)$\Rightarrow$ (1): We shall first show that $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0 space. Let U be an (i,j)-b-$\mathcal{I}$-open set such that $x \in U$. We claim that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset U$. For suppose $y \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \cap (X \setminus U)$. Then (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ (for if (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) = (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, then $y \in U$) and hence by (3), there exist (i,j)-b-$\mathcal{I}$-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$. Then $y \in F_2 \setminus F_1 = X \setminus F_1 \in (i,j)$-$B\mathcal{IO}(X)$ and $x \notin X \setminus F_1$, a contradicts the fact that $y \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_0 space. Let $p, q \in X$ be such that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{p\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{q\})$. Then by the given condition there exist (i,j)-b-$\mathcal{I}$-closed sets H_1 and H_2 such that $p \in H_1$, $q \notin H_1$, $q \in H_2$, $p \notin H_2$ and $X = H_1 \cup H_2$. Thus $p \in H_1 \setminus H_2$ and $q \in H_2 \setminus H_1$, where $H_1 \setminus H_2$ and $H_2 \setminus H_1$ are disjoint (i,j)-b-$\mathcal{I}$-open sets. Hence (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{p\}) \subset H_1 \setminus H_2$ and (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{q\}) \subset H_2 \setminus H_1$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-R_1 space. and thus by Theorem 2.5 (X, τ) is an (i,j)-b-$\mathcal{I}$-R_0 space. □

In view of Theorems 2.19 and 2.22, it now follows that

Theorem 2.23. *An ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_2 if, and only if for each x, $y \in X$ such that $x \neq y$, there exist (i,j)-b-$\mathcal{I}$-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.*

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