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Flow Of a Dusty Viscous Fluid Though the Circular Pipe with the Consideration of the Volume Fraction of the Dust Particles

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Abstract: *This study deals with the study of the flow of a dusty viscous fluid though a circular pipe. It is assumed that the fluid and the cloud of the dust particles were both static the motion begins as the magnetic field is induced the fluid is assumed to be electrically conducted. The analytical solutions are obtained by solving the partial differential equations using variable separable and Bessel function.*

Key words: Uniform pipe, dusty fluid, megnetohydrodymamics, Bessel function, volume fraction Hartmann number

Considering the motion of a dusty viscous fluid in a tube of various cross sections was first studied by Saffman in 1962, in 1963 F.E. Marble has carried out a detailed study on the dynamics of dusty gas then in the direction of z axis Rudinger(1965) and Neyfeh(1966) gave the equations of motion in Cartesian coordinates taking into consideration the volume fraction of the dust particles which are expressed as:

$$\rho(1-\phi)\frac{\partial q}{\partial t} = (1-\phi)\left[-\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2}\right)\right] + KN_0\left(q_p - q\right)$$

$$m\frac{\partial q_p}{\partial t} = \phi\left[-\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2}\right)\right] + KN_0\left(q - q_p\right)$$

Where u and v are the velocity of the fluid and dust particles respectively in the axial direction, ϕ is the volume occupied by the dust particle respectively in the axial direction. ϕ Is the volume occupied by the dust particle per unit volume of the axis. No is the no density of the particle .Nag and Jana have considered the flow through a rigid tube, Kishore and and pandey have discussed the same problem but they found the effect of particles size on velocity of sedimentation, Dutta and Dalal studied the motion of unsteady flow of a dusty fluid through circular pipe with impulsive pressure gradient , Gireesha and Bagewadi have also studied the flow of an unsteady dusty fluid through porous media in a uniform circular pipe.

Basic equations

Consider the flow of a dusty viscous fluid through a circular pipe with radius r let the particles and the fluid be electromagnetically charged and the motion is considered under the influence of the electromagnetic field of strength B_0 along the axis of the motion. Using Saffman model the equations governing the flow of the dusty fluid may be written as (Dutta and Dalal)

(1.1) $$(1-\phi)\frac{\partial q}{\partial \bar{t}} = \left[\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \nu\left(\frac{\partial^2 q}{\partial r^2} + \frac{1}{r}\frac{\partial q}{\partial r}\right) + \frac{KN_0}{\rho}\left(q_p - q\right)\right] - \frac{B_0^2\sigma}{\rho}q$$

(1.2) $$\frac{\partial q_p}{\partial t} = \frac{K}{N_0 m}\left(q - q_p\right)$$

Where q is the velocity of the fluid phase

q_p is the velocity of the phase particles

ϕ is volume fraction of the particle

t is the time

ρ is the density of the fluid

μ is the coefficient of viscosity

$\frac{\partial p}{\partial x}$ is pressure gradient of fluid

N_0 is coefficient of Momentum transfer

σ is electrical conductivity of fluid

B_0 is the magnetic induction

r is the radial distance from the axis of the tube

m is the Hall Parameter

K is the Stroke's resistance cofficient

Let us assume that

(1.3) $$-\frac{\partial p}{\partial z} = c + de^{iwt}$$

Let us introduce the following non dimensional quantities

$$\bar{q} = \frac{q}{U}, \quad \bar{q}_p = \frac{q_p}{U}, \ \bar{x} = \frac{x}{L}, \ \bar{y} = \frac{y}{L}, \ \bar{z} = \frac{z}{L}, \ \bar{t} = \frac{tU}{L}, \ \bar{\tau} = \frac{\tau U}{L}, \ f = \frac{mN_0}{\rho},$$

$$\frac{KN_0L}{\rho U} = \frac{KmN_0L}{\rho mU} = \frac{f}{\tau}, \ \bar{p} = \frac{p}{\rho U^2}, \ R = \frac{UL}{\upsilon}, \ \tau = \frac{m}{K}$$

Using these non dimensial quantities (1.4) equation (1.1) and (1.2) will become

$$(1-\phi)\frac{\partial(\bar{q}U)}{\partial(\bar{t}L/U)} = \left[\left(-\frac{1}{\rho}\frac{\partial(\bar{p}\rho U^2)}{\partial(\bar{z}L)}\right) + \nu\left(\frac{\partial^2(\bar{q}U)}{\partial(\bar{r}L)^2} + \frac{1}{\bar{r}R}\frac{\partial(\bar{u}U)}{\partial(\bar{r}R)}\right) + \frac{KNo}{\rho}\left(\bar{q}_pU - \bar{q}U\right)\right] - \frac{B_0^2}{\rho}\bar{q}U$$

$$\frac{\partial(\bar{q}U)}{\partial(\bar{t}L/U)} = \frac{1}{(1-\phi)}\left[\left(-\frac{1}{\rho}\frac{\partial(\bar{p}\rho U^2)}{\partial(\bar{z}L)}\right) + \nu\left(\frac{\partial^2(\bar{q}U)}{\partial(\bar{r}L)^2} + \frac{1}{\bar{r}R}\frac{\partial(\bar{u}U)}{\partial(\bar{r}L)}\right) + \frac{KNo}{\rho}\left(\bar{q}_pU - \bar{q}U\right)\right] - \frac{B_0^2M^2\mu}{\rho B_0^2L^2(1-\phi)}\bar{q}U$$

$$\frac{\partial\bar{q}}{\partial\bar{t}} = \frac{1}{(1-\phi)}\left[\left(-\frac{\partial\bar{p}}{\partial\bar{z}}\right) + \frac{\nu}{UL}\left(\frac{\partial^2\bar{q}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{q}}{\partial\bar{r}}\right) + \frac{KmNoL}{\rho mU}\left(\bar{q}_p - \bar{q}\right)\right] - \frac{M^2\nu}{LU(1-\phi)}\bar{q}$$

(1.4) $$\frac{\partial\bar{q}}{\partial\bar{t}} = \varepsilon_1^2\left[\left(-\frac{\partial\bar{p}}{\partial\bar{z}}\right) + \frac{1}{R}\left(\frac{\partial^2\bar{q}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{q}}{\partial\bar{r}}\right) + \frac{f}{\tau}\left(\bar{q}_p - \bar{q}\right)\right] - \frac{M^2\varepsilon_1^2}{R}\bar{q}$$

Where $M^2 = B_0L\sqrt{\sigma/\mu}$ =Hartman number and $\varepsilon_1^2 = \frac{1}{1-\phi}$

After dropping bars from the last equation it can be rewritten as

(1.5) $$\frac{\partial q}{\partial t}=\varepsilon_1^2\left[\left(-\frac{\partial p}{\partial z}\right)+\frac{1}{R}\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}\right)+\frac{f}{\tau}\left(q_p-q\right)\right]-\frac{M^2\varepsilon_1^2}{R}q$$

$$\frac{\partial(\bar{q}_p U)}{\partial(\bar{t}L/U)}=\frac{K}{N_0 m}\left(\bar{q}U-\bar{q}_p U\right)$$

$$\frac{\partial \bar{q}_p}{\partial \bar{t}}=\frac{KL}{UN_0 m}\left(\bar{q}-\bar{q}_p\right)$$

$$\frac{\partial \bar{q}_p}{\partial \bar{t}}=\frac{1}{\tau}\left(\bar{q}-\bar{q}_p\right)$$

After dropping bars in above equation will become

(1.6) $$\frac{\partial q_p}{\partial \bar{t}}=\frac{1}{\tau}\left(q-q_p\right)$$

Let

(1.7) $q=Q_0(r)+Q_1(r)e^{iwt}$ and $q_p=V_0(r)+V_1(r)e^{iwt}$

Using equation (1.6) in equation (1.5)

$$\frac{\partial}{\partial t}(Q_0+Q_1e^{iwt})=\varepsilon_1^2\left[\left(-\frac{\partial p}{\partial z}\right)+\frac{1}{R}\left(\frac{\partial^2}{\partial r^2}(Q_0+Q_1e^{iwt})+\frac{1}{r}\frac{\partial}{\partial r}(Q_0+Q_1e^{iwt})\right)+\frac{f}{\tau}\left\{\left(V_0+V_1e^{iwt}\right)-\left(Q_0+Q_1e^{iwt}\right)\right\}\right]$$
$$-\frac{M^2\varepsilon_1^2}{R}(Q_0+Q_1e^{iwt})$$

(1.8) $$iwQ_1e^{iwt}=\varepsilon_1^2\left\{c+de^{iwt}+\frac{1}{R}\left[\frac{d^2Q_0}{dr^2}+\frac{d^2Q_1}{dr^2}e^{iwt}+\frac{1}{r}\left\{\frac{dQ_0}{dr}+\frac{dQ_1}{dr}e^{iwt}\right\}\right]+\frac{f}{\tau}\left[V_0+V_1e^{iwt}-Q_0-Q_1e^{iwt}\right]\right\}$$
$$-\frac{M^2\varepsilon_1^2}{R}\left(Q_0+Q_1e^{iwt}\right)$$

By comparing the coefficients of e^{iwt} in equation (1.8) we get

(1.9) $$\frac{1}{R}\frac{d^2Q_0}{dr}+\frac{1}{Rr}\frac{dQ_0}{dr}+\frac{f}{\tau}\left[V_0-Q_0\right]-\frac{M^2}{R}Q_0=-c$$

(1.10) $$Q_1iw=\varepsilon_1^2\left[d+\frac{1}{R}\frac{d^2Q_1}{dr^2}+\frac{1}{Rr}\frac{dQ_1}{dr}+\frac{f}{\tau}\left(V_1-Q_1\right)\right]-\frac{M^2\varepsilon_1^2Q_1}{R}$$

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Using equation (1.6) in equation (1.7) will become

(1.11) $$V_1 iwe^{iwt} = \frac{1}{\tau}\left[(Q_0 - V_0) + (Q_1 - V_1)e^{iwt}\right]$$

By comparing the coefficients of e^{iwt} in equation (1.11) we get

(1.12) $$Q_0 = V_0$$

And

(1.13) $$V_1 = \frac{Q_1}{1 + i\tau w}$$

Using equation (1.12) in equation (1.9)

$$\frac{1}{R}\frac{d^2 Q_0}{dr^2} + \frac{1}{Rr}\frac{dQ_0}{dr} - \frac{M^2}{R}Q_0 = -c$$

$$\frac{d^2 Q_0}{dr^2} + \frac{1}{r}\frac{dQ_0}{dr} - M^2 Q_0 = -Rc$$

$$r^2\frac{d^2 Q_0}{dr^2} + r\frac{dQ_0}{dr} - r^2 M^2 Q_0 = -Rcr^2$$

$$r^2\frac{d^2 Q_0}{dr^2} + r\frac{dQ_0}{dr} - r^2 M^2\left(Q_0 + \frac{Rc}{M^2}\right) = 0$$

$$Q_0 + \frac{Rc}{M^2} = L$$

(1.14) $$r^2\frac{d^2 L}{dr^2} + r\frac{dL}{dr} - r^2 M^2 L = 0$$

The solution of equation (1.14) is given as

(1.15) $$L = \lambda J_0(Mr)$$

$$Q_0 = \lambda J_0(Mr) + \gamma K_0(Mr) - \frac{Rc}{M^2}$$

Where J_0 and K_0 are the zeroth order modified Bessel function of first and second kind respectively and λ and γ are constants. Here we note that since the velocity at

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the centre of the cylinder is zero, $\gamma K_0(Mr) \to \infty$ as $r \to 0$, makes the constant $\gamma = 0$. In this way the solution of the above equation will be reduced to and given as

(1.16) $$Q_0 = \lambda J_0(Mr) + \gamma K_0(Mr) - \frac{Rc}{M^2}$$

Using the boundary condition $Q_0(1) = 0$

$$\lambda = \frac{Rc}{M^2 J_0(M)}$$

Putting λ in equation (1.16)

$$Q_0 = \frac{Rc}{M^2 J_0(M)} J_0(Mr) - \frac{Rc}{M^2}$$

(1.17) $$Q_0 = -\frac{Rc}{M^2}\left(1 - \frac{J_0(Mr)}{J_0(M)}\right)$$

By using equation (1.12) we get

(1.18) $$V_0 = -\frac{Rc}{M^2}\left(1 - \frac{J_0(Mr)}{J_0(M)}\right)$$

Again by equation 1.10 we will have

$$iwQ_1 = \varepsilon_1^2\left[d + \frac{1}{R}\frac{d^2Q_1}{dr^2} + \frac{1}{Rr}\frac{dQ_1}{dr} + \frac{f}{\tau}(V_1 - Q_1)\right] - \frac{M^2\varepsilon_1^2 Q_1}{R}$$

$$\varepsilon_1^2\left[\frac{1}{R}\frac{d^2Q_1}{dr^2} + \frac{1}{Rr}\frac{dQ_1}{dr} + \frac{f}{\tau}\left(\frac{Q_1}{1+i\tau w} - Q_1\right)\right] - \frac{M^2\varepsilon_1^2 Q_1}{R} - Q_1 iw = -\varepsilon_1^2 d$$

$$\varepsilon_1^2\left[\frac{1}{R}\frac{d^2Q_1}{dr^2} + \frac{1}{Rr}\frac{dQ_1}{dr} - \frac{fQ_1}{\tau}\left(\frac{\tau^2 w^2 + i\tau w}{1+\tau^2 w^2}\right)\right] - \frac{M^2\varepsilon_1^2 Q_1}{R} - Q_1 iw = -\varepsilon_1^2 d$$

$$\frac{1}{R}\frac{d^2Q_1}{dr^2} + \frac{1}{Rr}\frac{dQ_1}{dr} - \frac{w}{R}\left[\left(\frac{Rf\tau w}{(1+\tau^2 w^2)} + \frac{M^2}{w}\right) + iR\left(\varepsilon_2^2 + \frac{f}{(1+\tau^2 w^2)}\right)\right]Q_1 = -d$$

$$\frac{1}{R}\frac{d^2Q_1}{dr^2} + \frac{1}{Rr}\frac{dQ_1}{dr} - \frac{w}{R}(\theta_1 + i\theta_2)Q_1 = -d$$

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Where $\theta_1 = \left(\frac{Rf\tau w}{(1+\tau^2 w^2)} + \frac{M^2}{w}\right)$, $\theta_2 = R\left(\varepsilon_2^2 + \frac{f}{(1+\tau^2 w^2)}\right)$ and $\varepsilon_2^2 = \frac{1}{\varepsilon_1^2}$

$$r^2 \frac{d^2 Q_1}{dr^2} + r\frac{dQ_1}{dr} - r^2\left[Rd - w(\theta_1 + i\theta_2)Q_1\right] = 0$$

(1.19) $$r^2 \frac{d^2 Q_1}{dr^2} + r\frac{dQ_1}{dr} - r^2 w(\theta_1 + i\theta_2)\left[Q_1 - \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}\right] = 0$$

Taking

(1.20) $$Q_1 - \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)} = H$$

Substituting in (1.20) equation (1.19)

(1.21) $$r^2 \frac{d^2 H}{dr^2} + r\frac{dH}{dr} - r^2 w(\theta_1 + i\theta_2)H = 0$$

Solution of equation (1.21) is given as

$$H = C_2 J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right)$$

Using value of H equation (1.20) will become

$$Q_1 = \alpha J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) + \beta K_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) + \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}$$

Where J_0 and K_0 are the zeroth order modified Bessel function of first and second kind respectively and α and β are constants. Here we note that since the velocity at the centre of the cylinder is zero, $\beta K_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) \to \infty$ as $r \to 0$, makes the constant $\beta = 0$. Becouse of the stated reason the solution of the above equation thus reduced to

(1.22) $$Q_1 = \alpha J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) + \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}$$

Using boundary condition

(1.23) $Q_1(1) = 0$

$$C_2 = \frac{\dfrac{-Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}}{J_0\left(\sqrt{w(\theta_1 + i\theta_2)}\right)}$$

$$Q_1 = \frac{\dfrac{-Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}}{J_0(\sqrt{w(\theta_1 + i\theta_2)})} J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) + \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}$$

$$Q_1 = \frac{\dfrac{-Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}}{J_0(\sqrt{w(\theta_1 + i\theta_2)})} J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right) + \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}$$

(1.24) $$Q_1 = \frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}\left[1 - \frac{J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right)}{J_0\left(\sqrt{w(\theta_1 + i\theta_2)}\right)}\right]$$

We know that $q = Q_0 + Q_1$

By equation (1.17) and (1.23) will become

(1.25) $$q = \frac{-Rc}{M^2\varepsilon_1^2}\left[1 - \frac{J_0(M\varepsilon_1 r)}{J_0(M\varepsilon_1)}\right] + e^{iwt}\frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2 + \theta_2^2)}\left[1 - \frac{J_0\left(\ r\sqrt{w(\theta_1 + i\theta_2)}\ \right)}{J_0\left(\sqrt{\theta_1 + i\theta_2}\right)}\right]$$

$$V_1 = \frac{Q_1}{1 + i\tau w}$$

Using equation (1.24) q_p will become

(1.26) $$V_1 = \frac{Rd(\theta_1 - i\theta_2)}{(1 + iw\tau)w(\theta_1^2 + \theta_2^2)}\left[1 - \frac{J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right)}{J_0\left(\sqrt{w(\theta_1 + i\theta_2)}\right)}\right]$$

Using equations (1.18) and (1.26)

$$q_p = V_0 + V_1 e^{iwt}$$

(1.27) $$q_p = \frac{-Rc}{M^2\varepsilon_1^2}\left[1 - \frac{J_0(M\varepsilon_1 r)}{J_0(M\varepsilon_1)}\right] + e^{iwt}\frac{Rd(\theta_1 - i\theta_2)}{(1 + iw\tau)w(\theta_1^2 + \theta_2^2)}\left[1 - \frac{J_0\left(r\sqrt{w(\theta_1 + i\theta_2)}\right)}{J_0\left(\sqrt{w(\theta_1 + i\theta_2)}\right)}\right]$$

Shearing Stress (Skin Friction)

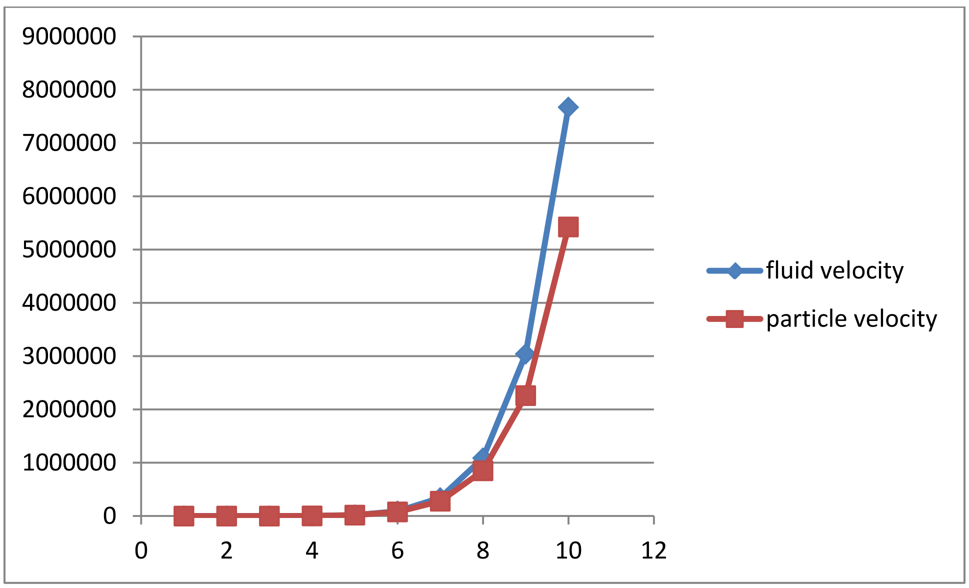
The shear stress at the boundaries $R=1$ and $R=b$ respectively, is given by

(1.28) $$D_1 = \frac{-Rc}{M^2\varepsilon_1^2}\left[-\frac{J_0^{'}(M\varepsilon_1)}{J_0(M\varepsilon_1)}\right] + e^{iwt}\frac{Rd(\theta_1 - i\theta_2)}{w(\theta_1^2+\theta_2^2)}\left[-\frac{J_0^{'}\left(\sqrt{w(\theta_1+i\theta_2)}\right)}{J_0\left(\sqrt{\theta_1+i\theta_2}\right)}\right]$$

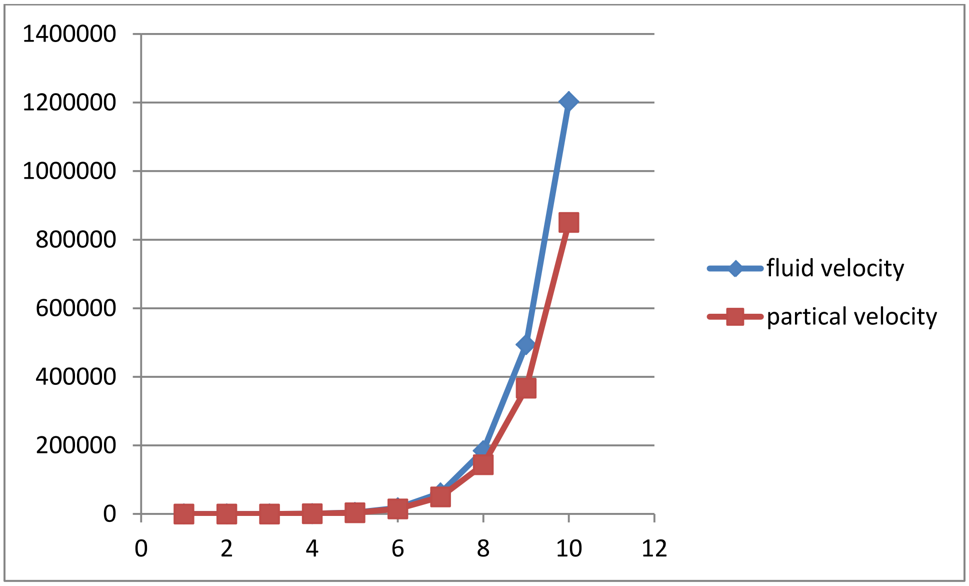
Mean velocity of the fluid can be given as

$$q_m = \frac{-1}{M^2\varepsilon_1^2}\left[1-\frac{J_0(M\varepsilon_1 r)}{J_0(M\varepsilon_1)}\right]$$

Graph between radius of the pipe and velocity of fluid phase and velocity of particle phase



Graph between magnetic flied (hartmann number) and velocity of fluid phase and velocity of particle phase .



By viewing both the graphs it is evident that the fluid velocity is more influenced by the varaing parameters and by increase in the parameters the velocities of both the fluid phase and particle phase are increasing.

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