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Significance Test Based On Rayleigh Distribution

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ABSTRACT

This paper deals with the Rayleigh distribution as a life time model. Moments of order statistics and an ordered sample are used to define a test statistic for the null hypothesis that the considered random variable has Rayleigh distribution. The percentiles of the test statistic are evaluated, Power's of the test with half-logistic distribution and gamma distribution with shape parameter 2 as alternatives are also evaluated.

Key Words: *Testing, Moments, Order statistics, Power of Test*

1. Introduction

The well known Weibull distribution is one of the most widely used probability distribution in Reliability engineering discipline and also is studied by many researchers with respect to various problems on statistical inference. Weibull distribution with shape parameter 2 is known as Rayleigh distribution. In the field of statistical research, life testing experiments and reliability studies Rayleigh is the one of the most commonly used increasing failure rate models, whose probability density function is given by

$$f(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}; x \geq 0 \qquad (1.1)$$
$$= 0, otherwise$$

where $\sigma > 0$, is the scale parameter of the distribution.
The cumulative distribution function is given by

$$F(x) = 1 - e^{-x^2/2\sigma^2};\ for\ x \in [0, \infty) \qquad (1.2)$$
$$= 0, otherwise$$

In this direction several authors contributed in this field with respect to references there in.

2. The Graphs Of Frequency Curve Of Rayleigh Distribution

The graph of frequency curve of Rayleigh distribution look similar to half logistic distribution, gamma distribution with shape parameter 2 models. Incidentally Rayleigh distribution is a combination of half logistic distribution and gamma distribution with shape parameter 2. We present below the frequency curve of Rayleigh distribution that look similar to half logistic distribution, gamma distribution with shape parameter 2.

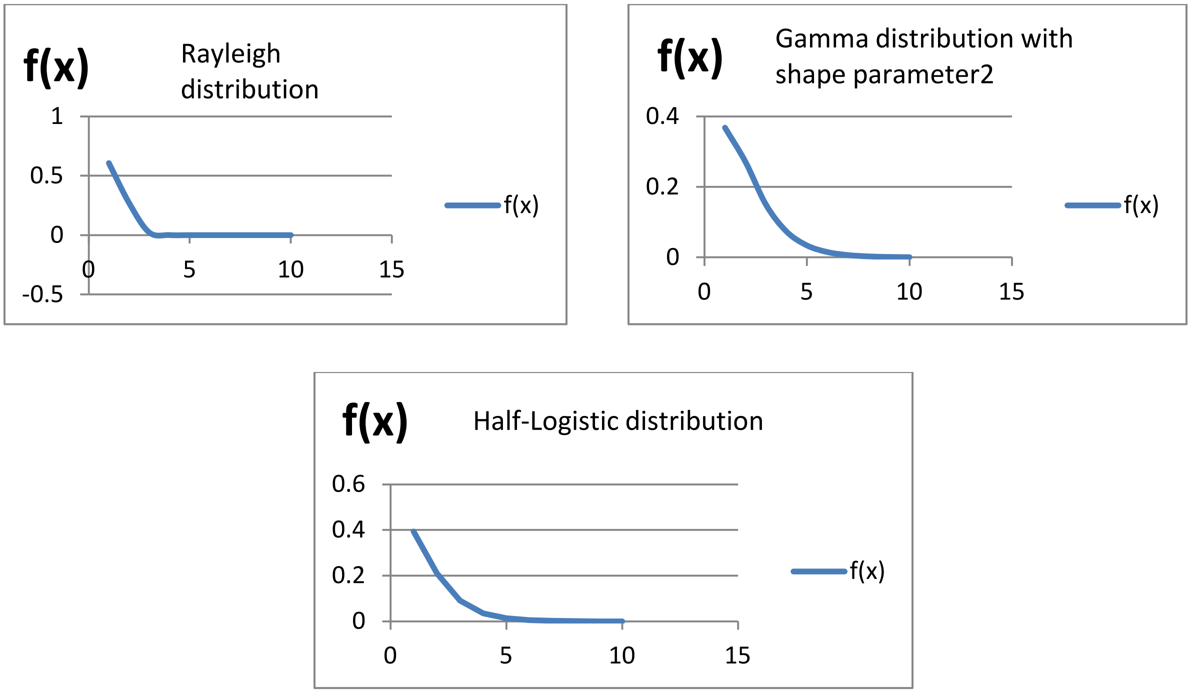

Fig (1)

In this paper we are interested to study the discrimination between Rayleigh distribution and half logistic distribution, gamma distribution with shape parameter 2. Another important of this study is to know whether half logistic distribution or gamma distribution with shape parameter 2 is a reasonable a better alternative to Rayleigh distribution in order to adopt the available simpler and admissible inferential procedures of half logistic distribution, gamma distribution with shape parameter 2 to Rayleigh distribution data. The similar studies of discrimination problems between probability models are made by Gupta *et al*(2002), Gupta and Kundu(2003A), Gupta and Kundu(2003B), Gupta and Kundu (2004),]Kundu and Manglick (2004), Kundu *et al* (2005), Kundu and Manglick (2006), Kundu (2005) and (2007), Kundu and Ragab(2005), Arabin and Kundu (2009,2010), Srinivasa Rao and Kantam (2011), Kantam et al (2015) studied the significance test for the log-logostic distribution. Arabin and Kundu (2012A), Arabin and Kundu (2012), and the references there in.

Sultan (2007) developed a test criterion to distinguish generalized exponential distribution from weibull, normal distributions, and moments of order statistics in samples drawn from generalized exponential distribution. In this paper we adopt the criterion suggested by Sultan (2007) to distinguish between Rayleigh and half- logistic, gamma distribution with shape parameter 2. A brief description of procedure developed by Sultan (2007) and its applications to our model is presented in Section 3. The methodology of arriving at the critical values and powers of the test procedures are given in Section 4 respectively. In all this cases, the percentiles of respective test statistic and powers of test procedures for selected sample sizes evaluated numerically and are tabulated in the respective situations. Rayleigh distribution is considered as null population (P_0) half-logistic distribution, gamma distribution with shape parameter 2 is considered as alternative populations (P_1).

3. Testing Of Hypothesis

Let us assume that the distribution of the life of the product is the scaled Rayleigh distribution where probability density function and cumulative distribution function give by

$$f_{x/\sigma}(x) = \frac{x}{\sigma^2} e^{-\frac{X^2}{2\sigma^2}}; \; x>0, \; \sigma>0 \qquad (3.1)$$

= 0 otherwise

$$F_{x/\sigma}(x) = 1- e^{-\frac{X^2}{2\sigma^2}} \qquad (3.2)$$

= 0 otherwise

and its probability density function of standard Rayleigh distribution as given by

$$f(z) = \frac{z}{\sigma} e^{-\frac{z^2}{2}}; \qquad z\geq 0 \qquad (3.3)$$

= 0 otherwise

$$F(z) = 1- e^{-\frac{z^2}{2}} \qquad \text{for } z€[0, \infty) \qquad (3.4)$$

= 0 otherwise

Probability density function of scaled half logistic distribution as given by

$$f_{x/\sigma}(x) = \frac{2e^{-\frac{X}{\sigma}}}{(1+e^{-\frac{X}{\sigma}})^2}; x\geq 0, \sigma>0 \qquad (3.5)$$

= 0 otherwise

$$F_{x/\sigma}(x) = \frac{1-e^{-\frac{X}{\sigma}}}{1+e^{-\frac{X}{\sigma}}}; \; x\geq 0, \; \sigma>0 \qquad (3.6)$$

= 0 otherwise

Probability density function of standard half- logistic distribution as given by

$$f(z) = \frac{2e^{-Z}}{(1+e^{-Z})^2}; \qquad z\geq 0 \qquad (3.7)$$

= 0 otherwise

and its cumulative distributive function

$$F(z) = \frac{1-e^{-Z}}{1+e^{-Z}} \qquad z\geq 0 \qquad (3.8)$$

= 0 otherwise

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Probability density function of scaled gamma distribution with shape parameter 2 is given by

$$f_{x/\sigma}(x) = \frac{x}{\sigma^2} e^{-\frac{X}{\sigma}}; \; x>0, \; \sigma>0 \qquad (3.9)$$

= 0 otherwise

and its cumulative distributive function

$$F_{x/\sigma}(x) = (1- e^{-\frac{X}{\sigma}})(1+\frac{x}{\sigma}) \qquad (3.10)$$

= 0 otherwise

Probability density function of standard gamma distribution with shape parameter 2 is given by

$$f(z)=\frac{x}{\sigma^2} e^{-\frac{X}{\sigma}}; \quad x>0 \, \sigma>0 \qquad (3.11)$$

= 0 otherwise

and its cumulative distributive function of standard gamma distribution with shape parameter 2 is given by

$$F(z)= (1- e^{-Z})(1+z) \qquad (3.12)$$

= 0 otherwise

Let $\mathbf{x_1, x_2, \ldots \ldots x_n}$ be a random sample of size n, here we test the null hypothesis P: the sample has come from Rayleigh distribution against each of the alternative hypothesis against the one of the alternative hypothesis.

(i) P_1: the sample has come from half logistic distribution.

(ii) P_1: the sample has come from gamma distribution with shape parameter 2.

As we mentioned in Section 1 that Rayleigh distribution is a combination of half- logistic distribution or gamma distribution with shape parameter 2. From this context we have considered Rayleigh distribution as null population and half-logistic distribution, gamma distribution with shape parameter 2 as alternative population. Sultan (2007) suggested a test statistic given by a formula

$$T=\frac{\sum_{i=0}^{n} X_{(i)}\alpha_{(i)}}{\sqrt{\sum_{i=0}^{n} X_{(i)}^{2} \sum_{i=0}^{n} \alpha_{(i)}^{2}}} \qquad (3.13)$$

where $x_{(i)}$ is the order observation in the sample, $\alpha_{(i)}$ is the expected value of i^{th} standard order statistic in a sample of size n from the null population. If $x_1, x_2, \ldots, x_n$ is a sample of size n from the null population the Equation (3.13) for T is used as a test statistic to discriminate a null population and the corresponding alternative population with the help of the critical values. Hence, the sampling distribution of T and its percentiles therefore are essential to make use of the test statistic T. Since, sample quantiles are consistent estimators of population quantiles. Hence we propose a statistic similar to T based on population quantiles rather than moments of order statistics and is given by

$$T^* = \frac{\sum_{i=1}^{n} \delta_{(i)} x_{(i)}}{\sqrt{\sum_{i=1}^{n} {x_{(i)}}^2 \sum_{i=1}^{n} {\delta_{(i)}}^2}} \tag{3.14}$$

where $\delta_{(i)} = F^{-1}(p) = \left(\frac{p_i}{1-p_i}\right)^{\frac{1}{\beta}}$ with $p_i = \frac{i}{n+1}$

We have tabulated the percentiles of empirical sampling distribution of T^* n=5(5)25 through 10,000 Monte-Carlo simulation runs and are given in Table 3.1. The percentiles of T^* would serve as critical values to test null hypothesis that a given sample comes from Rayleigh distribution and half logistic distribution, gamma distribution with shape parameter 2. Therefore a large value of T^* implies a strong linear relation between $x_{(i)}$ and $\delta_{(i)}$, which means that the sample is taken from Rayleigh distribution. On the other hand, a small value of T^* indicates that the sample is from a distribution different from Rayleigh distribution. Hence the statistic T is given by Equation (3.14) is approximate for testing the null hypothesis that X is distributed according to a Rayleigh distribution

The decision to reject the null hypothesis is based on the critical values of T with a pre-assigned level of significance. Accordingly, the percentiles of T^* are essential to carry out the test procedure regarding the above hypothesis. For example, let t_α denote the α – percentile of T^*, then if

$$P_T = \{\left({}^{t}/_{t} > t_\alpha\right)\} = 1 - \alpha \tag{3.15}$$

Holds, the null hypothesis cannot be rejected with level α. Thus, the percentiles of T^* are essential, but unfortunately it is not possible to obtain the distribution of T^* analytically. We determined the percentiles of T^* by means of simulation on the basis of 10,000 simulation runs for various sample sizes. The obtained percentiles are given in Table 3.2.

4. Power Of The Test

The power of a test refers to the probability of not rejecting a wrong null hypothesis and this probability depends heavily on the assumed alternative distribution. Here in this case, the null hypothesis given by Rayleigh distribution as the alternative distribution we consider half-logistic distribution. The test statistic T^* is calculated by generating a sample of a specified size from each of the alternative population with the $\delta_{(i)}$ in the formula for T^* being those of Rayleigh distribution. The corresponding calculated value of T^* is compared with the corresponding critical value given in Table 4.1. This process is repeated 10,000 times and the number of rejections of null hypothesis of the null hypothesis out of 10,000 times is taken as a measure of power of the test i.e.,

$$power\ of\ the\ test = \frac{number\ of\ rejections\ of\ H_0}{10{,}000} \tag{4.1}$$

The calculation of the power were performed for the above mentioned alternative with various sample sizes and various levels of significance for complete samples were presented in Tables

4.1 and 4.2 and the tables indicate the high power of the proposed test for all considered alternatives. The power indicates that the half logistic alternative is more powerful and gamma distribution with shape parameter 2 is less sensitive as alternative population. This shows that the test statistic clearly discriminates a Rayleigh distribution from half logistic distribution and gamma distribution with shape parameter 2 with decreasing power in that order.

Table 3.1 Percentiles of T^* for complete samples

p \ n	5	10	15	20	25
0.00135	0.867997	0.914191	0.99118	0.999216	0.99927
0.005	0.897317	0.938334	0.998805	0.998957	0.999082
0.01	0.919219	0.94823	0.98864	0.998782	0.998915
0.025	0.93836	0.961392	0.998315	0.998497	0.998691
0.05	0.952971	0.969693	0.99739	0.997915	0.998428
0.10	0.965421	0.977381	0.99739	0.997767	0.998058
0.90	0.99637	0.996767	0.983222	0.986755	0.988884
0.95	0.997511	0.997486	0.977828	0.982839	0.985089
0.975	0.998266	0.998019	0.971573	0.978253	0.9881579
0.99	0.998896	0.998433	0.962629	0.975645	0.975645
0.995	0.999311	0.998749	0.955778	0.9764860	0.970486
0.99865	0.999508	0.999105	0.941952	0.962572	0.962572

Table 3.2 Estimates of the power of test for two alternative distributions

n	Half-logistic distribution			Gamma(2)		
	0.99	0.95	0.90	0.99	0.95	0.90
5	0.9665	0.9823	0.9665	0.9955	0.9748	0.9428
10	0.9986	0.9947	0.9903	0.9962	0.983	0.9662
15	0.9997	0.998	0.9957	0.9975	0.9879	0.9746
20	0.9997	0.9991	0.9977	0.9982	0.9905	0.9825
25	1	0.9999	0.9995	0.9989	0.994	0.9861

Table 4.1 Power T values for Rayleigh vs HLD

S.No.	Sample no	99%	95%	90%
1	5	0.0335	0.0177	0.0020
2	10	0.0097	0.0053	0. 0014
3	15	0.0043	0.0020	0.0003
4	20	0.0003	0.0009	0.0003
5	25	0.0005	0.0001	0

Table4.2 Power T values for Rayleigh vs Gamma(2)

S.No.	Sample no	99%	95%	90%
1	5	0.0572	0.0252	0.0045
2	10	0.0338	0.0170	0.0038
3	15	0.0254	0.0121	0.0025
4	20	0.0175	0.0095	0.0018
5	25	0.0139	0.0060	0.0013

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