Effects of Temperature Dependent Viscosity and Viscous Dissipation on Fluid Flow past a Moving Isothermal Flat Plate

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Abstract

In this study, the effects of temperature dependent viscosity and viscous dissipation on fluid flow over a moving flat isothermal plate have been investigated. A steady two dimensional laminar boundary layer flow of incompressible, Newtonian fluid past a flat plate with uniform surface velocity and temperature has been considered. The partial differential equations governing the flow have been transformed into non- dimensional form and solved using finite difference numerical method implemented in Java programing language. The numerical results obtained are presented graphically and discussed. It has been observed that increasing variable viscosity parameter increases velocity whereas it decreases fluid temperature. Increasing velocity ratio has been noted to increase fluid velocity but decreases fluid temperature. The effect of increasing viscous dissipation parameter (Eckert number) is observed to increase fluid temperature.

Keywords: *Boundary layer, Finite difference method, Isothermal plate, Temperature dependent viscosity, Viscous dissipation*

1. Introduction

Fluid flowing over a surface tends to adhere to it and so the flow is divided into two regions; a region that is adjacent to the surface referred to as boundary layer and a region away from the surface in which fluid properties are not affected by bounding surface referred to as free stream. This study considers boundary layer region.

Boundary layer flow over moving surfaces has a variety of applications in manufacturing processes such as in paper production, glass –fiber production, production of sheeting materials and cooling of infinite metallic plates in a cooling path along a fluid film. In these processes, heat transfer occurs between the continuous moving materials and the adjacent fluid in motion. Due to these immense applications, studies on fluid flow over moving surfaces have been carried out for a long period of time. Sakiadis [1] was the first to study laminar boundary layer flow of a quiescent fluid over a moving surface. Ishak.A *et al* [2] extended sakiadis equations by considering uniform free stream parallel to a moving plate with more practical significance. The plate was subjected to a constant heat flux and moves in the same direction or opposite direction to the free stream. Numerical results for velocity and temperature profiles indicated that dual solutions exist when the plate and the free stream move in the opposite directions. Habib *et al* [3] developed a mathematical model for fluid flow over a moving surface in a moving fluid. The governing partial differential equations were transformed into ordinary differential equations and solved by combined shooting and perturbation techniques. The results showed that heat transfer depends on relative velocity between the moving surface and the moving fluid. G.K Ramesh *et al* [4] carried out mathematical modeling for boundary layer flow of quiescent fluid and heat transfer past an inclined stationery and moving flat plates with convective boundary conditions. The equations governing the flow were solved numerically using shooting method. The numerical results showed that the temperature of the stationery flat plate was higher than the temperature of the moving flat plate.

The studies considered above have neglected effects of viscous dissipation. However, as fluid flow, mechanical energy is dissipated into thermal energy by the action of viscous forces. Viscous dissipation changes the temperature distribution by playing the role of energy source and therefore viscous and highly shear flows influences the fluid flow structure. Fang.T

[5] studied thermal boundary layer flow of a semi-infinite flat plate moving in a constant velocity free stream fluid considering viscous dissipation. The numerical results showed that wall heat fluxes increased with the increase in velocity ratio. With increasing Eckert number, the viscous dissipation heating became dominant. Further results showed that when the Eckert number was small, the wall heat fluxes increased with increasing Prandtl number and when the Prandtl number was large than the critical value, wall heat fluxes decreased with increasing Prandtl number. Palanisamy Geetha *et al* [6] carried a study on steady two dimensional boundary layer flow past a continuously moving semi -infinite flat plate taking into consideration viscous dissipation effects. Numerical results showed that there was significant changes in heat transfer due to viscous dissipation in the medium. The thermal boundary layer thickness was found to increase with increase in viscous dissipation. Waheed Mufutau Adekojo *et al* [7] presented a numerical study of the effects of varying dynamic viscosity coupled with viscous dissipation function on the convective heat transfer over a continuously moving horizontal plate. The study was carried out for different fluids such as oil with Prandtl number, Pr = 10, air with Prandtl number, Pr = 0.7 and liquid metal with Prandtl number, Pr = 0.01 for various dynamic viscosity parameters and heat capacity in order to characterize the nature of flow patterns and energy distribution. The results revealed that the dynamic viscosity has significant influence on the velocity and temperature profiles for particular specific heat capacity and Prandtl number higher than unity at fixed viscous dissipation. Further results showed that an increase in dynamic viscosity for Prandtl number greater than unity leads to a significance decrease in the velocity.

The studies discussed above have considered constant viscosity fluid flows. However, fluid viscosity changes noticeably if temperature differences exist between the surface of a plate and the free stream. To accurately describe flow regime, it is therefore important to consider the temperature dependent viscosity in boundary layer flow, Modather *et al* [8]. Cheng – Ren Lin *et al* [9] investigated the effect of temperature dependent viscosity on a boundary layer flow over a linearly stretching surface. The numerical results obtained indicated that the velocity and temperature distribution depends on viscosity parameter. Increase in viscosity parameter increases velocity whereas it decreases fluid temperature. Elbashbeshy *et al* [10] studied fluid flow over a continuous moving surface in a stationery fluid to investigate the effects of temperature dependent viscosity on the heat transfer neglecting the effects of viscous dissipation. The results indicated that increase in variable viscosity parameter increases temperature profiles whereas it decreased velocity profiles. Skin friction was found to decrease with increase in the variable viscosity parameter. Pantokratoras [11] presented a study on the effects of variable fluid properties for the classical Blassius and Sakiadis flow. The investigation considered engine oil, water and air taking into account the variation of their viscosity with temperature. From the results it was found that the variation of fluid viscosity has a strong influence on the velocity and temperature flow profiles. The results for oil and water were in general similar and were generalized to liquids whereas air results were different and were generalized to gases.

Motivated by the above mentioned researchers, this paper aims at studying the combined influence of moving flat surface in a moving fluid, variable viscosity and viscous dissipation effects in one study. The motivation behind this study is to contribute towards this gap in numerical study of boundary layer flow over a moving flat surface in a moving fluid with temperature dependent viscosity taking into consideration viscous dissipation effects. Viscosity is considered to vary as inverse function of temperature. The governing equations have been solved by employing finite difference numerical method. The effects of various parameters on velocity and temperature profiles are presented graphically and discussed.

2. Mathematical Formulation

A steady laminar two dimensional boundary layer flow of an incompressible, Newtonian fluid over a moving flat surface with temperature dependent viscosity is considered. The surface is considered to move with velocity U_w in the same direction to the fluid with velocity U_∞ in the free stream region where $U_\infty > U_w$. Moreover, the surface is kept at constant temperature T_w while the free stream temperature is T_∞ where $T_w > T_\infty$. The flow is considered to be caused by constant pressure gradient and movement of the surface. The x- axis is taken along direction of the surface and y- axis perpendicular to it as shown in figure 1.

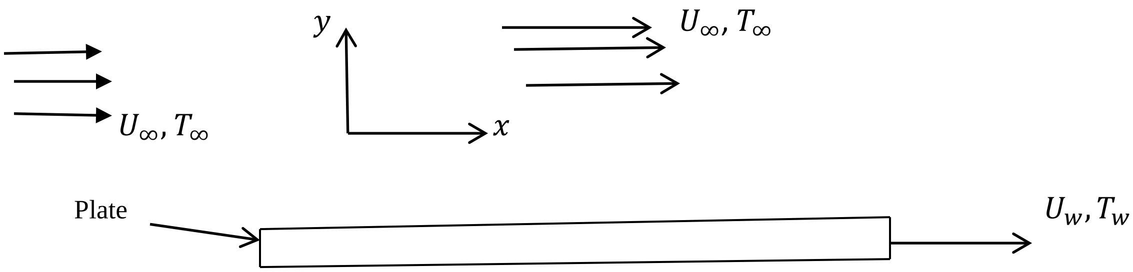

Figure 1: Flow configuration

The boundary layer equations governing the flow are:

Continuity;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum;

$$u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \frac{\mu(\theta)}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Energy;

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_P}K\frac{\partial^2 T}{\partial y^2} + \frac{\mu(\theta)}{\rho C_P}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Subject to the boundary conditions;

$$\text{At } y = 0,\; u(x,y) = U_{w,} \quad v(x,y) = 0, \quad T(x,y) = T_{w,} \tag{4}$$

$$\text{At } y \to \infty,\; u(x,y) = U_{\infty,} \quad v(x,y) = 0, \quad T(x,y) = T_{\infty,} \tag{5}$$

In this study, the fluid viscosity is considered to vary as inverse function of temperature given by Bercovici and Lin [12] model in the form;

$$\mu(\theta) = \frac{\mu_\infty}{1+\left(\frac{\mu_\infty - \mu_w}{\mu_w}\right)\theta} \tag{6}$$

Where μ_∞ is the viscosity at free stream temperature while μ_w is viscosity at surface temperature.

The non- dimensional variables used in transforming equations (1), (2) and (3) into non-dimensional form are

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{U_\infty}, \quad v^* = \frac{v}{U_\infty}, \quad \theta = \frac{T-T_\infty}{T_w - T_\infty}, \quad P^* = \frac{P}{\rho U_\infty^2} \tag{7}$$

Where $(*)$ denotes dimensionless variables.

2.1 Non- dimensional Numbers

Non- dimensional numbers are applicable in this study in order to transform the results obtained in the model into any other dynamically similar case. In this paper, the following numbers have been used;

2.1.1 Eckert Number

It is a dimensionless number that provides a measure of the kinetic energy of flow relative to the enthalpy difference across the thermal boundary layers. It represents the conversion of kinetic energy into internal energy by work that is done against the viscous fluid stresses. It is the ratio of kinetic energy to enthalpy. It is given as;

$$Ec = \frac{U^2}{C_p(T_w - T_\infty)}$$

2.1.2 Reynolds Number

The Reynolds number indicates the relative significance of the viscous effects compared to the inertia effect. It is the ratio of inertial forces to viscous forces. It is important in analyzing any flow with substantial velocity gradients. It is given as;

$$Re = \frac{\rho u l}{\mu}$$

2.1.3 The Prandtl Number

The Prandtl number is the ratio of fluid properties controlling the velocity and the temperature distributions. It is the ratio of the momentum diffusivity to the thermal diffusivity. It provides a measure of relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers respectively. Prandtl number is given by;

$$\text{Pr} = \frac{\mu C_P}{K}$$

The non – dimensional form of temperature dependent equation (6) takes the form;

$$\frac{\mu(\theta)}{\mu_\infty} = \frac{1}{1+\varepsilon\theta} \tag{8}$$

Where $\frac{\mu(\theta)}{\mu_\infty}$ is the dimensionless viscosity and ε is the variable viscosity parameter.

Equations (7) and (8) are used together with non –dimensional numbers to obtain governing equations in non- dimensional form.

Continuity, momentum and energy equations respectively in non-dimensional form are given as;

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{9}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dP^*}{dx^*} + \frac{1}{Re(1+\varepsilon\theta)}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{10}$$

$$u^*\frac{\partial \theta}{\partial x^*} + v^*\frac{\partial \theta}{\partial y^*} = \frac{1}{Pr.Re}\frac{\partial^2 \theta}{\partial y^{*2}} + \frac{Ec}{Re(1+\varepsilon\theta)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{11}$$

The boundary conditions in non- dimensional form are as follows;

$$\text{At } y = 0, \quad u^*(x^*, y^*) = \frac{U_w}{U_\infty} = \lambda, \quad v^*(x^*, y^*) = 0, \quad \theta(x^*, y^*) = 1 \tag{12}$$

$$\text{At } y \to \infty, \quad u^*(x^*, y^*) = 1, \quad v^*(x^*, y^*) = 0, \quad \theta(x^*, y^*) = 0 \tag{13}$$

Where $\frac{U_w}{U_\infty} = \lambda$ is the velocity ratio parameter. In this study we consider $0 < \lambda < 1$, when the fluid and plate move in the same direction, also the speed of the plate is less than those of fluid.

3. Method of Solution

Finite difference numerical approximation method is applied in solving equations (9), (10) and (11) subject to boundary conditions (12) and (13).

The forward finite difference form of non- dimensional equations (9), (10) and (11) respectively are in the form;

$$\frac{u(i+1,j)-u(i,j)}{\Delta x}+\frac{v(i,j+1)-v(i,j)}{\Delta y}=0 \tag{14}$$

$$u(i,j)\left[\frac{u(i+1,j)-u(i,j)}{\Delta x}\right]+v(i,j)\left[\frac{u(i,j+1)-u(i,j)}{\Delta y}\right]=-\frac{dP}{dx}+\frac{1}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j-1)-2u(i,j)+u(i,j+1)}{(\Delta y)^2}\right] \tag{15}$$

$$u(i,j)\left[\frac{\theta(i+1,j)-\theta(i,j)}{\Delta x}\right]+v(i,j)\left[\frac{\theta(i,j+1)-\theta(i,j)}{\Delta y}\right]=\frac{1}{\Pr.Re}\left[\frac{\theta(i,j-1)-2\theta(i,j)+\theta(i,j+1)}{(\Delta y)^2}\right]+\frac{Ec}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j+1)-u(i,j)}{\Delta y}\right]^2 \tag{16}$$

Making $v(i,j), u(i+1, j)$ and $\theta(i+1, j)$ subject of the formula in equations (14), (15) and (16) respectively yields;

$$v(i,j)=v(i,j-1)+\frac{\Delta y}{\Delta x}\{u(i,j-1)-u(i+1,j-1)\} \tag{17}$$

$$u(i+1,j)=u(i,j)+\frac{\Delta x\left\{-\frac{dP}{dx}+\frac{1}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j-1)-2u(i,j)+u(i,j+1)}{(\Delta y)^2}\right]-v(i,j)\left[\frac{u(i,j+1,)-u(i,j)}{\Delta y}\right]\right\}}{u(i,j)} \tag{18}$$

$$\theta(i+1,j)=\theta(i,j)+\frac{\Delta x\left\{\frac{1}{Pr.Re}\left[\frac{\theta(i,j-1)-2\theta(i,j)+\theta(i,j+1)}{(\Delta y)^2}\right]+\frac{Ec}{Re\{1+\varepsilon\theta(i,j)\}}\left[\frac{u(i,j+1)-u(i,j)}{\Delta y}\right]^2-v(i,j)\left[\frac{\theta(i,j+1)-\theta(i,j)}{\Delta y}\right]\right\}}{u(i,j)} \tag{19}$$

Equations (17), (18) and (19) are the final finite difference equations which are solved using Java programing language in order to obtain the results discussed in the next section.

4. Results and Discussion

In this study, the effects of temperature dependent viscosity and viscous dissipation on fluid flow past a moving isothermal flat plate have been investigated. Viscosity of the fluid has been considered as an inverse function of temperature. The results presented are for the case when Prandtl number, Pr = 0.71, Reynolds number, Re = 5.0 and pressure gradient, $\frac{dP}{dx}$ = -5.0. The values for the viscosity parameter (ε), viscous dissipation parameter (Ec) and velocity ratio (λ) are varied over a range listed in the figures. The computations have been performed using $\Delta x = 0.01$ and $\Delta y = 0.01$ to ensure convergence and stability of the finite difference method.

Figure 2 shows the effects of varying viscosity parameter (ε) on velocity profile. It is observed that an increase in viscosity parameter causes an increase in velocity profile. Viscosity parameter is inversely proportional to dimensionless viscosity as given in equation (8). Therefore an increase in viscosity parameter results to a decrease in dimensionless viscosity and in turn a decrease in viscous forces which oppose fluid motion. This translates to inertia forces dominating viscous forces and hence fluid accelerates.

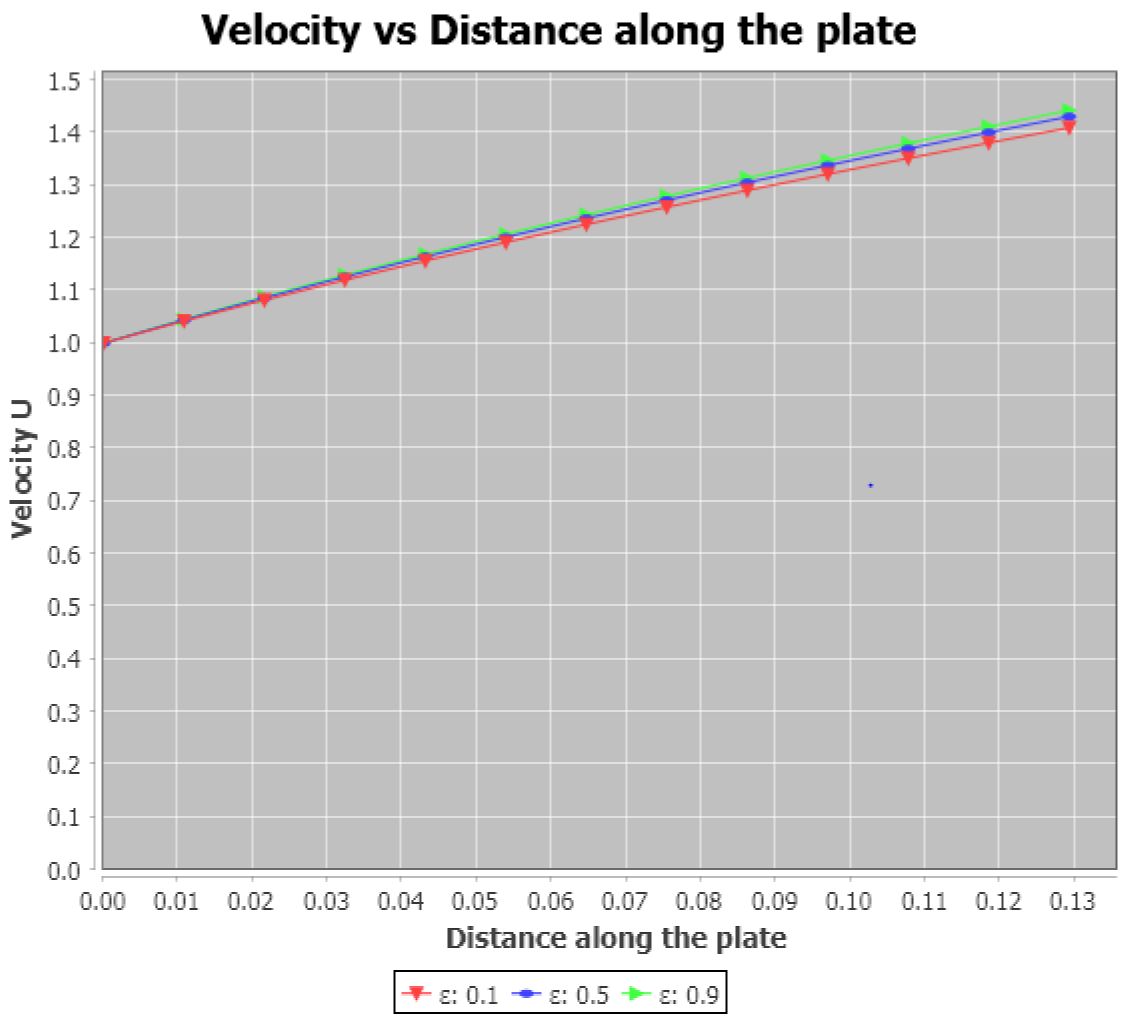

Figure 2: Velocity profiles varying viscosity parameter for ***Pr = 0.71, Re = 5, Ec = 0.4, dP/dx = -5,*** $\lambda = 0.1$

Figure 3 illustrates the effects of varying viscosity parameter (ε) on the temperature distribution. An increase in viscosity parameter (ε) causes a decrease in temperature profile. This is attributed to decrease in dimensionless viscosity translating into decrease in viscous forces. These smaller viscous forces results to reduced friction between fluid and the surface thus decrease in dissipation of heat within the boundary layer.

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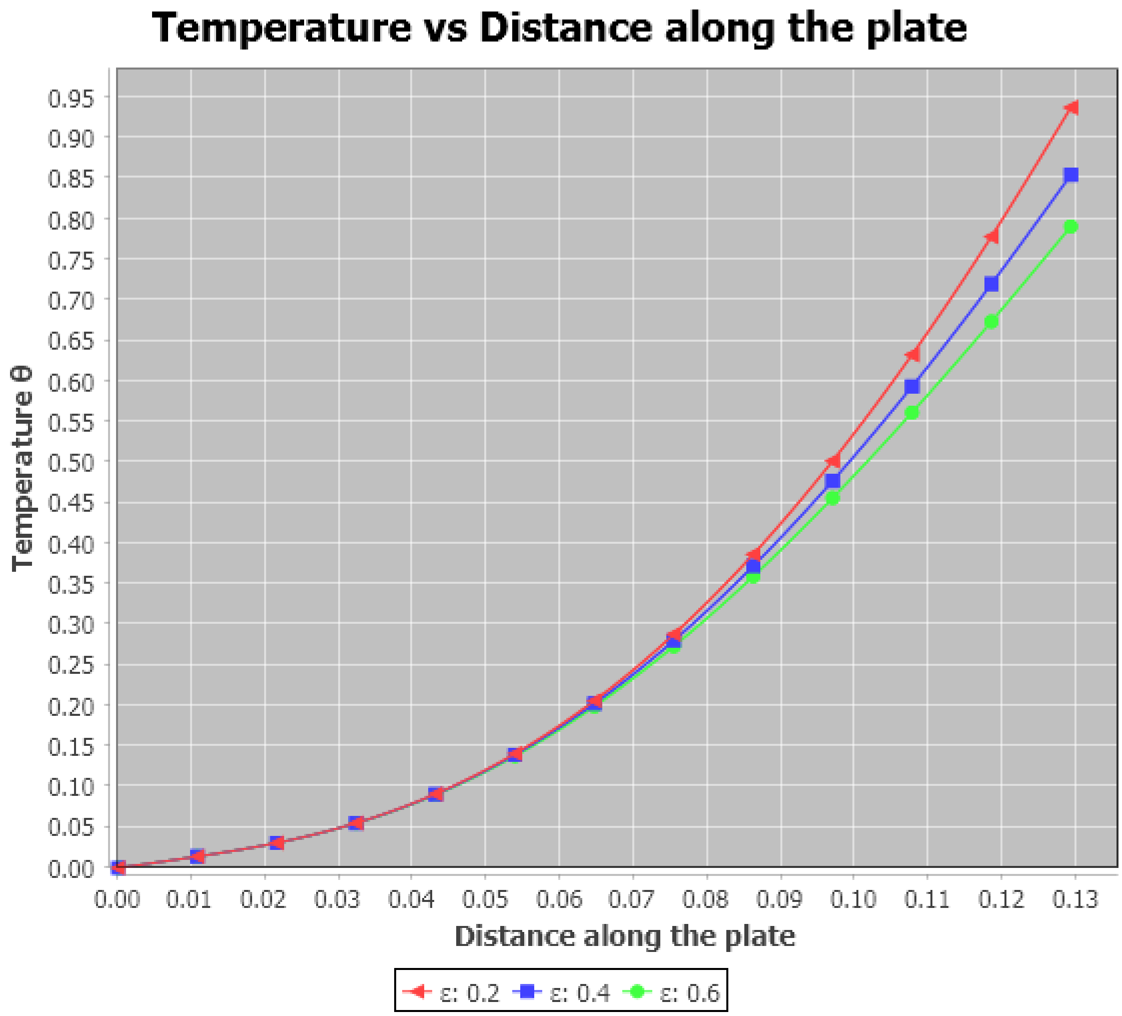

Figure 3: Temperature profiles varying viscosity parameter for Pr = *0.71, Re=5 , dP/dx =-5, Ec = 0.4,* $\lambda = 0.1$

Figure 4 shows the effects of varying velocity ratio (λ)on velocity profile. It is observed that an increase in velocity ratio causes an increase in velocity of the fluid. This is due to non-slip conditions whereby the layer of fluid in the immediate vicinity of a bounding surface attains the velocity of the boundary. An increase in velocity ratio therefore implies an increase in fluid velocity.

Figure 5 illustrates effects of varying velocity ratio (λ) on temperature profile. An increase in velocity ratio decreases temperature profile. An increase in velocity ratio results to an increase in fluid velocity implying a decrease in viscous forces. Decrease in viscous forces decreases friction between the surface and the fluid and hence reduced dissipation of heat within the thermal boundary layer thus decrease in temperature of the fluid.

Figure 6 shows the effects of varying viscous dissipation parameter (Ec) on temperature profile. It is observed that an increase in Eckert number causes an increase in temperature profile. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. An increase in Eckert number implies that the kinetic energy is large resulting to increased vibration of fluid leading to increased collision of fluid molecules. The increased collision of molecules increases dissipation of heat in the boundary layer region hence an increase in temperature profile.

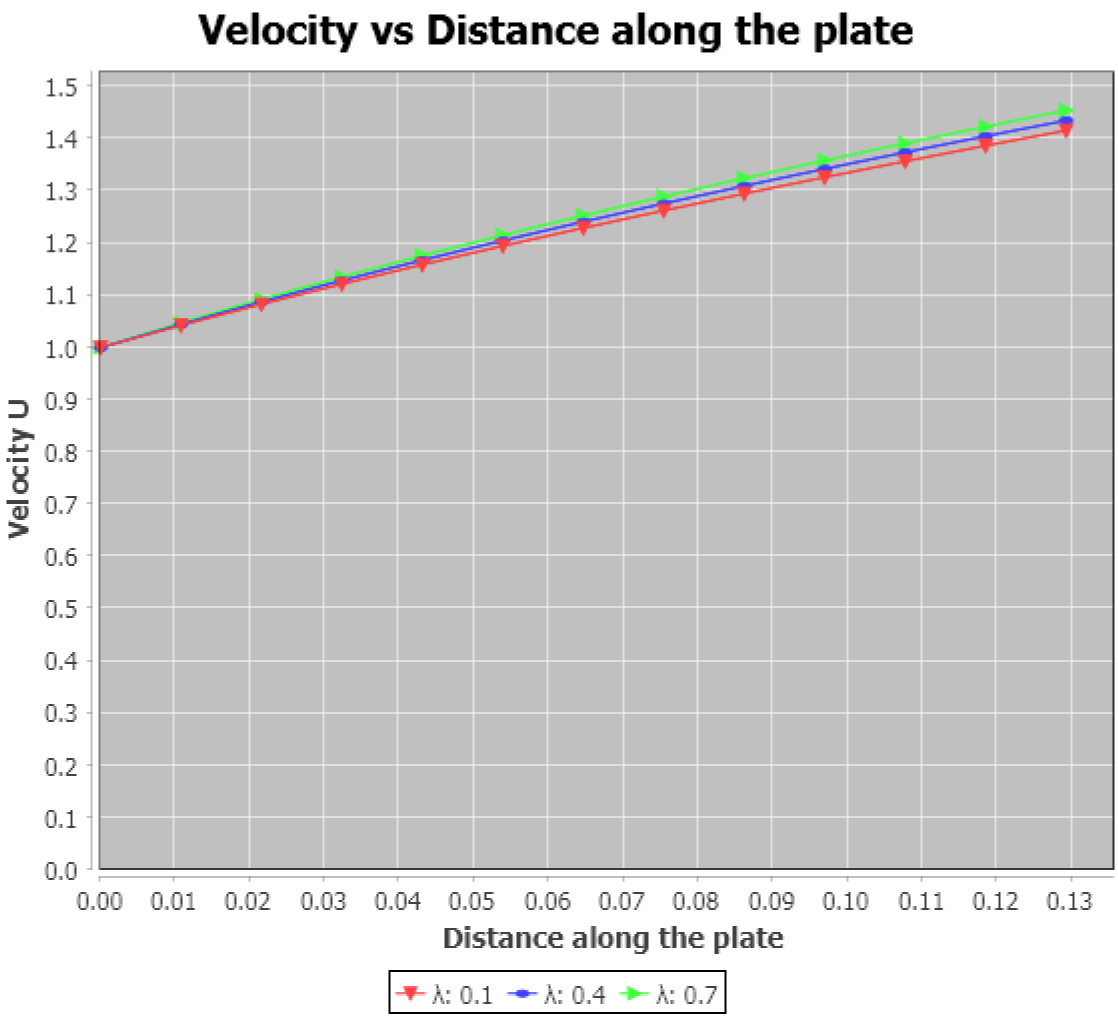

Figure 4: Velocity profiles varying velocity ratio for ***Pr = 0.71, Re = 5, Ec = 0.4, dP/dx = -5, ε = 0.2***

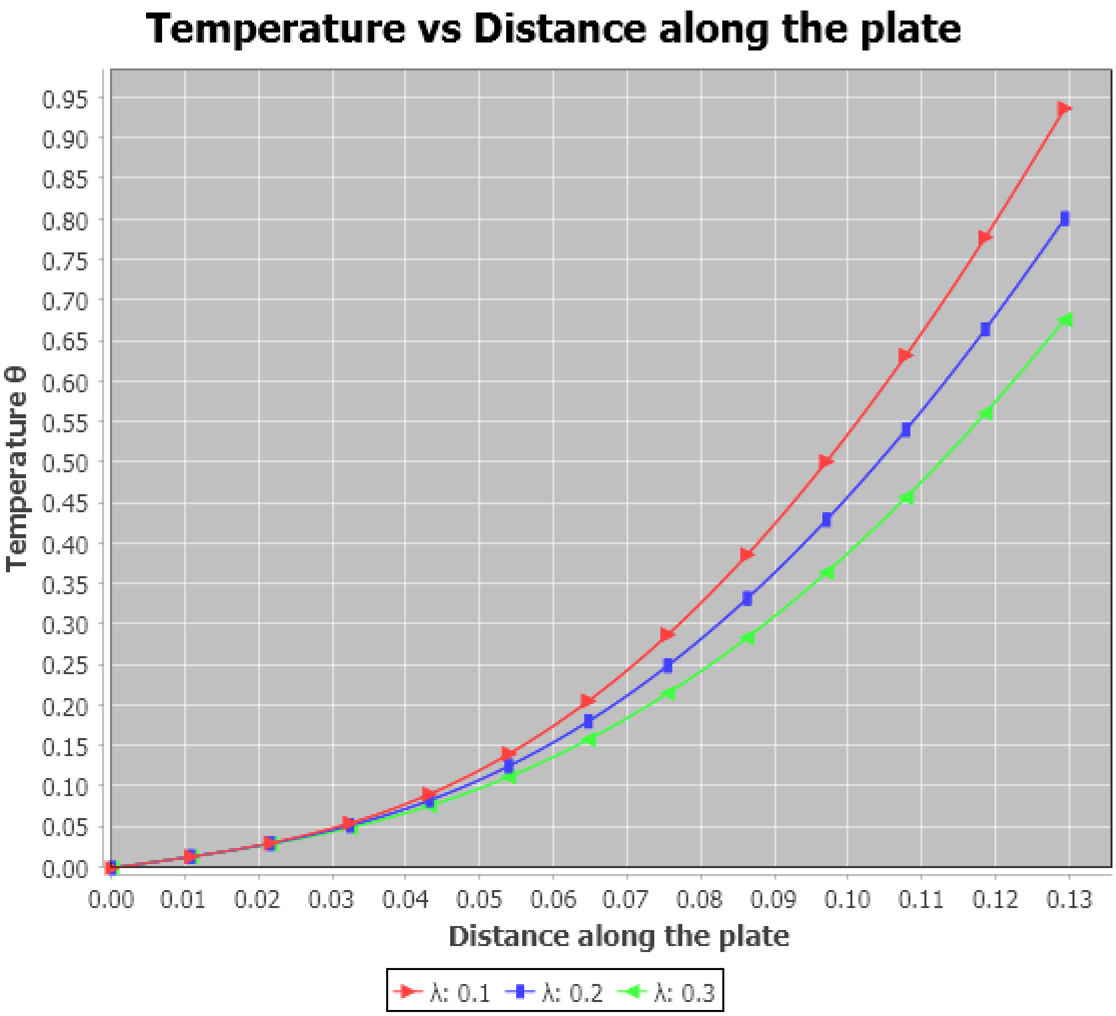

Figure 5: Temperature profiles varying velocity ratio for ***Pr = 0.71, Re = 5, Ec = 0.4, dP/dx = -5,*** $\varepsilon = 0.2$

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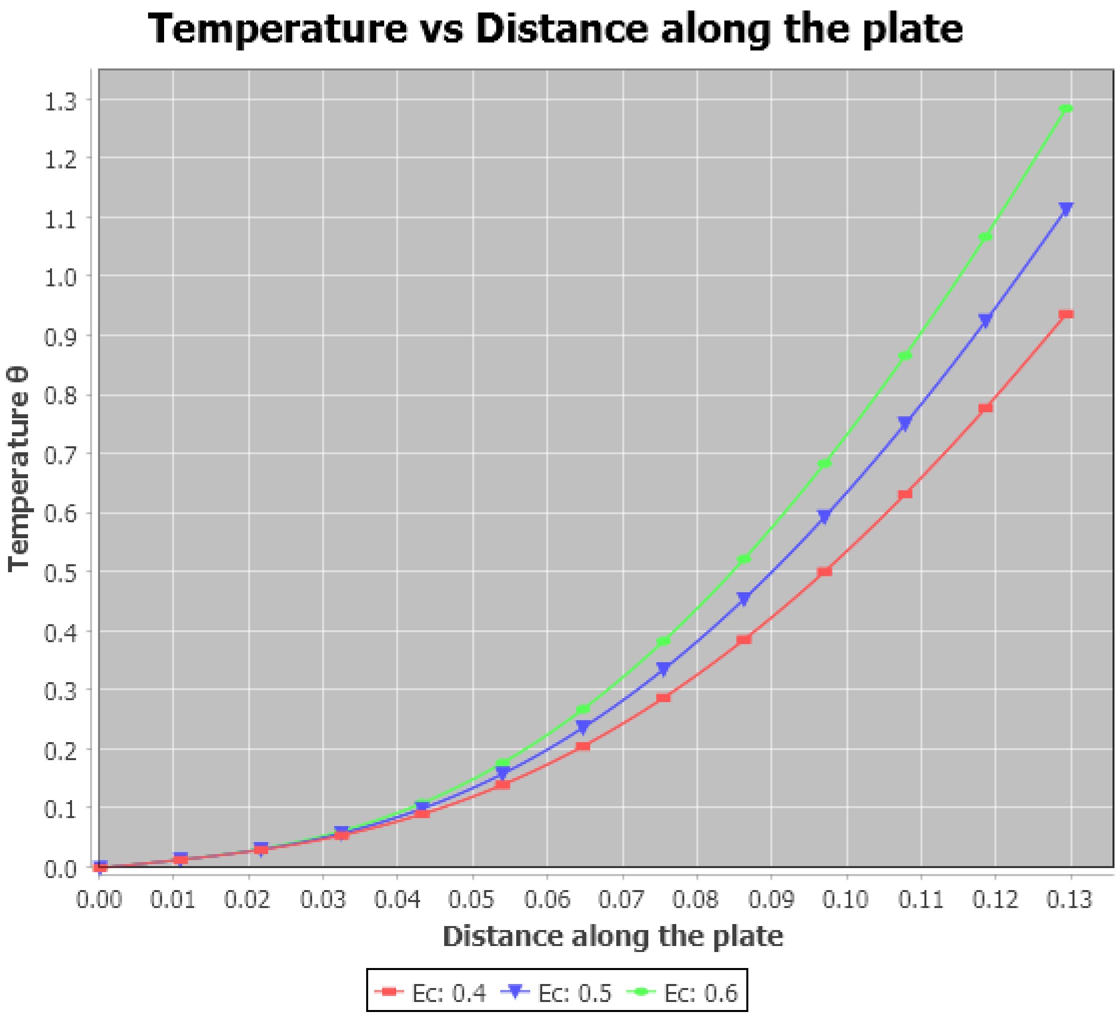

Figure 6: Temperature profiles varying viscous dissipation parameter for ***Pr = 0.71, Re = 5, dP/dx = -5,*** **ε *= 0.2* , $\lambda = 0.1$**

5. Conclusions

In this paper, the effects of temperature dependent viscosity and viscous dissipation on fluid flow over a moving isothermal flat plate have been determined. Numerical results have been obtained using finite difference numerical method. From the study, the following conclusions have been made;

1. Increase in viscosity parameter (ε) causes an increase in velocity profile whereas it decreases temperature profile.
2. Increase in velocity ratio (λ) increases velocity profile but decreases temperature profile.
3. Increasing viscous dissipation parameter (Ec) increases temperature profile.

Nomenclature

Symbols	Quantity
C	Velocity of light, ms^{-1}
C_P	Specific heat capacity at constant pressure, $Jkg^{-1}k^{-1}$
Ec	Eckert number $\left(= \frac{U^2}{C_p(T_W - T_\infty)}\right)$
i, j	Integer Variables
K	Thermal Conductivity, $Wm^{-1}k^{-1}$
L	Characteristic Length, m
P	Pressure of the fluid, Nm^{-2}
P^*	Dimensionless pressure
Pr	Prandtl number $\left(= \frac{\mu C_p}{K}\right)$
$\vec{q}$	Velocity Vector of the field, ms^{-1}
Re	Reynolds number $\left(= \frac{\rho u l}{\mu}\right)$
T	Dimensional temperature, K
T_W	Temperature at the wall, K
T_∞	Free stream temperature, K
u, v	Dimensional velocity vector components, ms^{-1}
$u,^* v^*$	Dimensionless velocity component
U_W	Plate velocity, ms^{-1}
U_∞	Free stream velocity, ms^{-1}
x, y	Cartesian coordinate in dimensional form, m
x^*, y^*	Cartesian coordinate in dimensionless form
$\Delta x, \Delta y$	Distance intervals, m
ε	Measure of variable viscosity in dimensionless form
λ	Surface velocity in dimensionless form
μ	Coefficient of viscosity, $kgm^{-1}s^{-1}$
μ_∞	Coefficient of viscosity at free stream temperature, $kgm^{-1}s^{-1}$

μ_W	Coefficient of viscosity at surface temperature , $kgm^{-1}s^{-1}$
ρ	Fluid density, kgm^{-3}
$\emptyset$	Viscous dissipation function, s^{-2}
θ	Dimensionless temperature

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