IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 3 Issue 5, May 2016.
www.ijiset.com
ISSN 2348 – 7968

# Comparative Study of Asphalt Institute – Witczak 1-40D Dynamic Moduli for Polythene Bag Modified HMA Concrete Using Predictive Models

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## Abstract

Material characterization is fundamental in the Mechanistic-Empirical design of flexible pavement. One of such key material property is the stiffness of the pavement which influences tensile strain levels and also necessary for either the determination or prediction of fatigue cracking synonymous with pavement life. Research has also shown that dynamic modulus, E*, is one of the various forms of pavement stiffnesses that truly represent field conditions. However, different agencies and authors have developed predictive models to determine this one important parameter required as input in mechanistic design of flexible pavement. Though, results have also shown that the dynamic modulus from these various researchers vary greatly under different circumstances leaving behind one key question; which is the most appropriate model to be used. In an attempt to answering such question the present research focussed on a comparative study between the two most commonly used models "**Asphalt Institute and Witczak 1-40D Models** for determining dynamic modulus". The research was carried out for a polythene bag modified HMA concrete for loading frequencies of 0.1Hz, 5Hz and 10Hz respectively. Results obtained showed that the dynamic modulus from the Witczak 1-40D Models were more conservative in terms of design for all frequencies tested. Secondly, the comparative model developed showed that the relationship between the logarithm of the Asphalt Institute and Witczak 1-40D dynamic moduli is polynomic and satisfying the conditions of the general form of $Y = aX^2 + bX + c$ with $R^2$ 0f 0.924 for all cases.

**Key words: Comparism, Dynamic Modulus, Modification and HMA Concrete**

## 1. Introduction

One of the key elements of Mechanistic-Empirical (M-E) flexible pavement design is the characterization of material properties. One of such material property in particular is the stiffness which influences tensile strain levels; therefore it is necessary to investigate this property to successfully predict fatigue cracking. Research has shown that there are various forms of measuring elastic properties of asphalt concrete mixes such as young's modulus, bulk modulus, shear modulus, resilient modulus, elastic modulus and dynamic modulus. However, more recent research has posited that the dynamic modulus, E* closely simulate true field conditions and thus is more reliable for use in design [1].

E* can be determined directly by laboratory testing or it can be estimated using predictive equations as a function of mixture properties. The more recently developed M-E design program, the Mechanistic-Empirical Pavement Design Guide (MEPDG), offers both methods to characterize E*. Furthermore, in M-E pavement design, accurate representation of material characteristics is imperative to a successful and reliable design: in particular is the HMA dynamic modulus, E* which helps to define the visco-elastic nature of HMA by quantifying the effects of temperature and frequency on stiffness under dynamic loading. This is necessary to accurately predict the in-situ pavement responses to varying speeds, and temperatures throughout the pavement's cross-section. E*

ISSN 2348 – 7968

can be determined in the laboratory through the AASHTO TP-62 procedure or it can be predicted by one of many E* predictive models, the four most recent including: Asphalt Institute, Hirsch, Witczak 1-37A, and Witczak 1-40D [2] and [3]. To predict E* from one of these four models, no laboratory testing is required beyond viscosity testing, determination of gradation information and rudimentary volumetric testing. In addition, the dynamic modulus of an asphalt mixture which is a significant parameter that determines the ability of material to resist compressive deformation as it is subjected to cyclic compressive loading and unloading [4]; has been suggested by NCHRP Projects 9-19 and 9-29 as a simple performance test (SPT) to verify the performance characteristics of Super-pave mixture designs [5]. It has also been suggested as the potential quality control-quality assurance parameter in the field [6]. Dynamic modulus is also an input to the Mechanistic-Empirical Pavement Design guide (MEPDG) [7] and supports the predictive performance models developed as part of NCHRP project 1-37A [8].

At the highest degree of complexity for design, the MEPDG utilizes E* laboratory test results, and for lower levels of design, it utilizes one of the predictive equations [9]. Equipment to run E* testing is very costly ($75,000-$90,000) and many state DOTs in America do not currently have such equipment [10]. For example, the Alabama DOT (ALDOT) currently operates without such equipment since it is not needed for their current design framework. However, a pooled fund study, "**Implementation of the Asphalt Mixture Performance Tester (AMPT) for Super-pave Validation**," (Study No. TPF-5(178)) launched by the FHWA in 2008 makes this technology more economical for state transportation agencies [10]. As States contemplate implementing the MEPDG, it is necessary to assess the accuracy of the predictive models, in estimating E* in comparison to laboratory E* test results for a range of mix types.

On this basis the present research carried out a comparative study on dynamic modulus of polythene bag modified hot mix asphalt concrete (HMA) using predictive equations as posited by the Asphalt Institute and Witczak 1-40D models. The major focus was to ascertain the variations of dynamic modulus using these predictive models under varying polythene bag content and frequencies. In addition, develop a correlation between the Asphalt Institute and Witczak 1-40D dynamic moduli under these varying conditions. It is noteworthy that all considerations were for light traffic.

## 2. Materials and Methods

### *2.1. Sample collection*

The materials used for this study were waste polythene bag, asphalt, coarse and fine aggregates. The polythene bag used were obtained as wastes from the surrounding environment at the Rivers State University of Science and Technology, Port Harcourt while the aggregates used were obtained from market dealers at Mile 3 Diobu, in Port Harcourt City Local Government Area of Rivers State, Nigeria. On the other hand the asphalt used was collected from a private asphalt plant company H & H situated at Mbiama, in Ahoada West Local Government Area of Rivers State, Nigeria. After sampling of the materials, laboratory tests - specific gravity, grading of asphalt and sieve analysis of the aggregates used for mix-proportioning by straight line method - were carried out.

### *2.2. Sample preparation*

Samples were prepared using Marshal Design Procedures for asphalt concrete mixes as presented [11], [12] and [13]. The procedures involved the preparation of a series of test specimens for a range of asphalt (bitumen) contents such that test data curves showed well defined optimum values. Tests were scheduled on the bases of 0.5 percent increments of asphalt content with at least 3-asphalt contents above and below the optimum asphalt content. In order to provide adequate data, three replicate test specimens were prepared for each set of asphalt content used. During the preparation of the unmodified asphalt concrete samples, the aggregates were first heated for about 5 minutes before asphalt was added to allow for absorption into the aggregates. After which the mix was poured into a mould and compacted on both faces with 35 blows using a 6.5kg-rammer falling freely from a height of 450mm. Compacted specimens were subjected to bulk specific gravity test, stability and flow, density and voids analyses at a temperature of $60^0$C and frequencies of 0.1, 1, 5, 10 and 25Hz respectively as specified by [14]. The results obtained were used to determine the optimum asphalt content of the unmodified asphalt concrete. Waste polythene bags were then added at varying amounts (5 – 25 percent by weight of the asphalt at optimum) to the samples at optimum asphalt content and then re-designed using the same Marshal Design Procedures already stated above to produce polythene bag modified concretes having varying mix design properties particularly air voids content which greatly affects dynamic modulus. The varying values of air voids content obtained by inclusion of the waste polythene bag into the asphalt concrete was inputted into our Asphalt Institute and Witczak 1-40D model equations to obtain varying E* values used for comparism.

### *2.3. Theory*

The optimum asphalt content (O.A.C.) for the unmodified concrete was obtained using equation 1, according to the Marshal Design Procedure cited in [11] and [12] as follows:

$$O.A.C. = \frac{1}{3}\left(A.C._{max.\ stability} + A.C._{max.\ density} + A.C._{median\ limits\ of\ air\ voids}\right) \quad (1)$$

The Asphalt Institute predictive model used for the study in which the dynamic modulus is determined is as presented in Huang's Pavement Analysis and Design textbook [15]:

$$E^* = 100{,}000\ (10^{\beta_1}) \quad (2)$$

$$\beta_1 = \beta_3 + 0.000005\beta_2 - 0.00189\beta_2 f^{-1.1} \quad (3)$$

$$\beta_2 = \beta_4^{0.5} T^{\beta_5} \quad (4)$$

$$\beta_3 = 0.553833 + 0.028829\left(P_{200} f^{-0.1703}\right) - 0.03476V_a + 0.07037\lambda + 0.931757 f^{-0.02774} \quad (5)$$

$$\beta_4 = 0.483V_b \quad (6)$$

$$\beta_5 = 1.3 + 0.49825 \log f \quad (7)$$

Where;
E* = dynamic modulus (psi)
F =loading frequency (Hz)
T = temperature (°F)
$V_a$ = volume of air voids (%)
λ = asphalt viscosity at 77°F ($10^6$ poises)
$P_{200}$ = percentage by weight of aggregates passing No. 200 (%)
$V_b$ = volume of bitumen
$P_{77°F}$ = penetration at 77°F or 25°C

Similarly, the Witczak 1-40D predictive model used for the study to determine dynamic modulus is as presented below;

$$LogE^* = -1.249937 + 0.029232P_{200} - 0.001767(P_{200})^2 + 0.002841P_4 - 0.05809V_a - \frac{0.802208V_{beff}}{(V_{beff} + V_a)} \quad (8)$$
$$\frac{3.871977 - 0.0021P_4 + 0.003958P_{38} - 0.000017(P_{38})^2 + 0.00547P_{34}}{1 + e^{(-0.603313 - 0.31335\log f - 0.393532\log\eta)}}$$

Where,

ISSN 2348 – 7968

$|E^*|$ = asphalt mix complex modulus, in $10^5$ psi;
$\eta$ = bitumen viscosity, in $10^6$ poise;
$f$ = load frequency, in Hz;
$V_a$ = percent air voids in the mix, by volume;
$V_{beff}$ = percent effective bitumen content, by volume;
$P_{34}$ = percent retained on ¾-in. sieve, by total aggregate weight
$P_{38}$ = percent retained on 3/8-in. sieve, by total aggregate weight
$P_4$ = percent retained on No. 4 sieve, by total aggregate weight
$P_{200}$ = percent passing No. 200 sieve, by total aggregate weight

## 3. Results (see Tables 1-11)

Results obtained from preliminary laboratory tests are tabulated in the following tables as follows;

**Table 1: Laboratory test results of stated materials**

| Material | Waste Polythene Bag | Asphalt | Sand | Gravel |
|---|---|---|---|---|
| Specific gravity | 0.92 | 1.05 | 2.52 | 2.86 |
| Grade of binder material | - | 40/50 | - | - |
| Mix proportion (%) | - | - | 41 | 59 |
| Viscosity of binder (poise) | - | $1.45*(10^{-6})$ | - | - |
| Softening point | - | 50ºC | - | - |
| Penetration value | - | 53mm | - | - |

**Table 2: Mix design properties for unmodified asphalt concrete**

| Asphalt Content (%) | Stability (N) | Flow (0.25mm) | Density ($kg/m^3$) | Air voids (%) | VMA (%) |
|---|---|---|---|---|---|
| 6.0 | 2310 | 19.95 | 2071 | 3.5 | 29.8 |
| 5.5 | 2870 | 17.3 | 2120 | 3.6 | 26.2 |
| 5.0 | 3270 | 15.0 | 2260 | 3.9 | 21.78 |
| 4.5 | 3060 | 13.2 | 2240 | 4.3 | 22.1 |
| 4.0 | 2236 | 11.80 | 2050 | 4.9 | 28.8 |

**Table 3: Mix design properties for polythene bag modified asphalt concrete at 4.9% optimum asphalt content**

| Candle Wax Content (%) | Stability (N) | Flow (0.25mm) | Density ($kg/m^3$) | Air voids (%) | VMA (%) |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| 0.0 | 3,228 | 14.64 | 2,256 | 4.0 | 22.00 |
| 5 | 3,317 | 13.90 | 2,342 | 3.7 | 19.00 |
| 10 | 4,134 | 13.20 | 2,546 | 3.5 | 17.41 |
| 15 | 4,323 | 13.00 | 2,634 | 3.3 | 15.83 |
| 20 | 3,803 | 13.40 | 2,390 | 3.6 | 20.81 |
| 25 | 3,068 | 15.60 | 2,200 | 4.1 | 23.92 |

**Table 4: Schedule of Aggregates used for mix proportion (ASTM: 1951)**

| Sieve size (mm) | Specification limit | Aggregate A (Gravel) | Aggregate B (Sand) | Mix proportion (0.59A+0.41B) |
|---|---|---|---|---|
| 19.0 | 100 | 99.1 | 100 | 99.45 |
| 12.5 | 86-100 | 86.1 | 100 | 91.80 |
| 9.5 | 70-90 | 100 | 62 | 78 |
| 6.3 | 45-70 | 100 | 26 | 57 |
| 4.75 | 40-60 | 99 | 10 | 47 |
| 2.36 | 30-52 | 96 | 0 | 40 |
| 1.18 | 22-40 | 90 | 0 | 38 |
| 0.6 | 16-30 | 73 | 0 | 31 |
| 0.3 | 9-19 | 23 | 0 | 10 |
| 0.15 | 3-7 | 3 | 0 | 1.26 |
| 0.075 | 0 | 0 | 0 | 0 |

**Table 5: variation of Log E* with polythene bag content @ frequency of 0.1Hz**

| % Polythene Bag | Asphalt Institute (E*) | Log E* (Asphalt Institute) | Witczak 1-40D ( E*) | Log E* (Witczak 1-40D) |
|---|---|---|---|---|
| 0.00 | 62,611.23 | 4.796652236 | 14,165.32 | 4.15122639 |
| 5.00 | 64,132.81 | 4.807080269 | 14,231.36 | 4.153246405 |
| 10.00 | 65,167.68 | 4.81403226 | 14,255.48 | 4.153981845 |
| 15.00 | 66,219.26 | 4.820984323 | 14,262.19 | 4.154186218 |
| 20.00 | 64,648.18 | 4.810556303 | 14,245.51 | 4.153678002 |
| 25.00 | 62,112.11 | 4.793176283 | 14,135.94 | 4.150324693 |

**Table 6: variation of Log E* with polythene bag content @ frequency of 5Hz**

| % Polythene Bag | Asphalt Institute (E*) | Log E* (Asphalt Institute) | Witczak 1-40D ( E*) | Log E* (Witczak 1-40D) |
|---|---|---|---|---|
| 0.00 | 111,127.27 | 5.045820645 | 38,374.40 | 4.584041598 |
| 5.00 | 113,827.88 | 5.056248647 | 38,553.31 | 4.58606167 |
| 10.00 | 115,664.65 | 5.063200648 | 38,618.64 | 4.586796976 |
| 15.00 | 117,531.06 | 5.070152653 | 38,636.83 | 4.587001487 |
| 20.00 | 114,742.59 | 5.059724649 | 38,591.65 | 4.586493347 |
| 25.00 | 110,241.38 | 5.042344641 | 38,294.81 | 4.583139919 |

**Table 7: variation of Log E* with polythene bag content @ frequency of 10Hz**

| % Polythene Bag | Asphalt Institute (E*) | Log E* (Asphalt Institute) | Witczak 1-40D ( E*) | Log E* (Witczak 1-40D) |
|---|---|---|---|---|
| 0.00 | 126,355.27 | 5.10159336 | 46,836.02 | 4.670579983 |
| 5.00 | 129,425.95 | 5.112021361 | 47,054.38 | 4.672600055 |
| 10.00 | 129,425.95 | 5.112021361 | 47,134.12 | 4.673335403 |
| 15.00 | 133,636.58 | 5.125925353 | 47,156.32 | 4.673539906 |

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| | | | | |
|---|---|---|---|---|
| 20.00 | 130,466.00 | 5.115497347 | 47,101.18 | 4.673031787 |
| 25.00 | 125,347.99 | 5.098117374 | 46,738.89 | 4.669678394 |

## 4. Developing Model fit Between Asphalt Institute and Witczak 1-40D Dynamic Modulus

The researcher has assumed that the model fit between the asphalt Institute predictive model and Witczak 1-40D predictive model for determining dynamic modulus is related by a general power series of the form as shown below which was validated as would be seen later;

$$Y = aX^2 + bX + c \tag{9}$$

Where;
Y = Logarithm of Asphalt Institute Dynamic Modulus at any Frequency of Test (Log E*)
X = Logarithm of Witczak 1-40D Dynamic Modulus at same Frequency of Test as in Y (Log E*)
a, b and c are statistically determined co-efficients using SPSS

### Non Linear Model Syntax

A non linear model is one in which at least one of the parameters appear nonlinearly [16], [17] and [18]. More formally, in a nonlinear model, at least one derivative with respect to a parameter should involve that parameter. To solve the non linear regression using SPSS the variables were first collated into different cells in the "**DATA VIEW**" dialogue box. Next these variables were stringed and coded into another dialogue box called the "**VARIABLE VIEW CELL**". Finally, model syntax was developed that satisfies the condition of the initially assumed general form of the proposed comparative model. The non linear model syntax is of the form as shown below;

$$Y = a*(X**2) + (b*X) + c \tag{10}$$

Equation 10 is the non linear syntax model that corroborates the general form of the assumed model fit between the Asphalt Institute dynamic modulus and the Witczak dynamic modulus used for comparism in the SPSS program. Furthermore, the command (**) means raising a variable to the power of the co-efficient in the same bracket while the command (*) means multiplication.

## 5. Results

By applying equation 10 in the SPSS program the statistical co-efficients a, b and c were determined for the various frequencies of tests as follows; (see Appendix A: Table 1-3 for iteration history of co-efficients)

**Table 8: Model Co-efficients and $R^2$**

| Co-efficients | 0.1hz | 5hz | 10hz |
|---|---|---|---|
| a | -5.434 | -4.972 | -6.684 |
| b | 51.437 | 51.905 | 68.116 |
| c | -115.096 | -128.416 | -167.224 |
| $R^2$ | 0.924 | 0.924 | 0.924 |

By inputting the resulting co-efficients into equation (9) above for the various frequencies of test we have the resulting comparative models;

__For 0.1Hz__;

$$Y = -5.434X^2 + 51.437X - 115.096 \tag{11}$$

__For 5Hz__;

$$Y = -4.972X^2 + 51.905X - 128.416 \tag{12}$$

__For 10Hz__;

$$Y = -6.684X^2 + 68.116X - 167.224 \tag{13}$$

## 6. Validation of Results

The comparative models developed was validated by inputting the logarithm values of the Witczak dynamic modulus represented as X to obtain Y values representing the logarithm of the

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 3 Issue 5, May 2016.
www.ijiset.com
ISSN 2348 – 7968

Asphalt Institute dynamic modulus. The resulting Y's from the comparative models were compared to

| Polythene Bag Content (%) | 0.1Hz | | 5Hz | | 10Hz | |
|---|---|---|---|---|---|---|
| | Asphalt Institute Model | Comparative Model Asphalt Institute Model | Asphalt Institute Model | Comparative Model Asphalt Institute Model | Asphalt Institute Model | Comparative Model Asphalt Institute Model |
| 0 | 4.796652236 | 4.788245762 | 5.045820645 | 5.03986853 | 5.10159336 | 5.11032877 |
| 5 | 4.807080269 | 4.800993057 | 5.056248647 | 5.052617702 | 5.112021361 | 5.121774871 |
| 10 | 4.81403226 | 4.805623035 | 5.063200648 | 5.057248326 | 5.112021361 | 5.125927946 |
| 15 | 4.820984323 | 4.806908626 | 5.070152653 | 5.058535288 | 5.125925353 | 5.127081646 |
| 20 | 4.810556303 | 4.803710898 | 5.059724649 | 5.05533686 | 5.115497347 | 5.124214069 |
| 25 | 4.793176283 | 4.782541292 | 5.042344641 | 5.034164713 | 5.098117374 | 5.105202594 |

the original Y's obtained using the Asphalt institute model equation. The results showed that both have very high correlation of 0.924. See table 9 and appendix A, Tables 1-3.

**Table 9: validation Schedule at Varying Frequencies**

Note: $R^2$ = 0.924 for all amount of polythene bag content

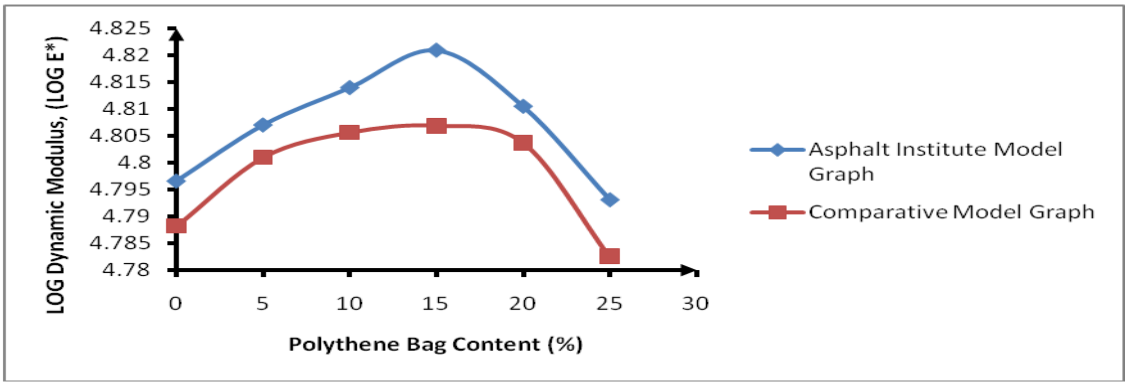


Fig 1: Asphalt Institute vs Comparative Model Graph at 0.1 Hz

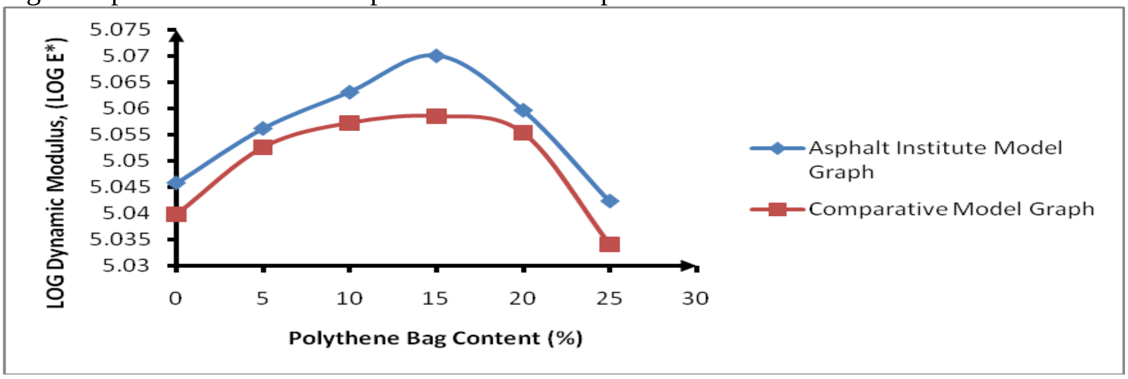


Fig 2: Asphalt Institute vs Comparative Model Graph at 5 Hz

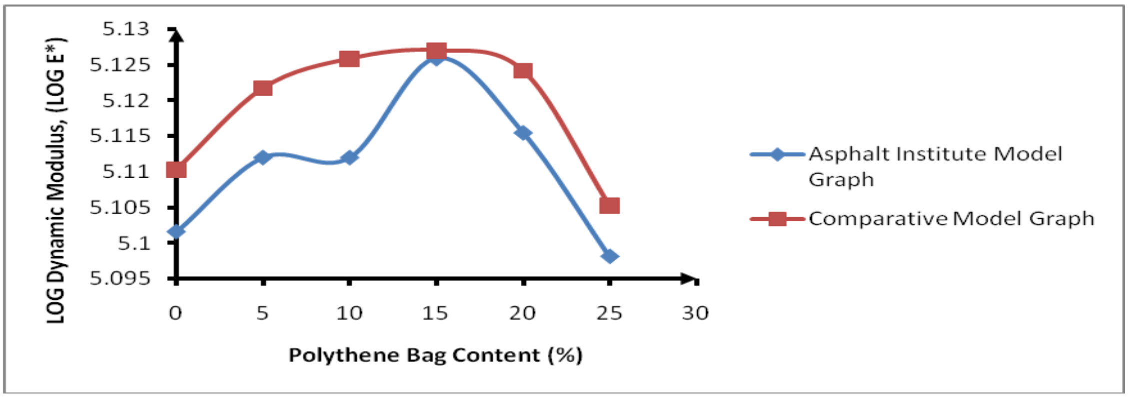


Fig 3: Asphalt Institute vs Comparative Model Graph at 10 H

## 7. Conclusion

The overall conclusions from the study are that;

1. For polythene bag modified hot mix asphalt concrete (HMA) results showed that the Witczak 1-40D dynamic modulus is more conservative than the Asphalt Institute dynamic modulus. See tables 5-7.
2. The proposed Comparative Model has shown to have very good correlation between the Asphalt Institute and Witczak 1-40D dynamic moduli having polynomic behaviour of the general form as in equation (9).
3. Asphalt Institute dynamic modulus can safely be predicted from Witczak 1-40D dynamic modulus models without necessarily carrying out tests associated with the asphalt institute methods for polythene bag modified hot mix asphalt concretes.

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## Appendix A: Iteration History

**Table 1: Iteration History for polythene bag modified HMA Concrete @ 0.1 Hz**

| Iteration Number | Residual Sum of Squares | Parameter | | |
|---|---|---|---|---|
| | | a | b | c |
| 0.1 | 53.618 | -5.166 | 49.800 | -115.900 |
| 1.1 | .000 | -5.003 | 49.839 | -115.891 |
| 2.1 | .000 | -5.206 | 50.594 | -115.515 |
| 3.1 | .000 | -5.299 | 50.938 | -115.344 |
| 4.1 | .000 | -5.434 | 51.438 | -115.096 |
| 5.1 | .000 | -5.434 | 51.437 | -115.096 |

Derivatives are calculated numerically.
a Major iteration number is displayed to the left of the decimal, and minor iteration number is to the right of the decimal.
b Run stopped after 5 iterations. Optimal solution is found.

**Table 2: Parameter Estimates**

| | Parameter | Estimate | Std. Error | 95% Trimmed Range | |
|---|---|---|---|---|---|
| | | | | Lower Bound | Upper Bound |
| Asymptotic | a | -5.434 | 1252.719 | | |
| | b | 51.437 | 10403.298 | | |
| | c | -115.096 | 21598.735 | | |
| Bootstrap | a | -5.434 | .000 | -5.434 | -5.433 |
| | b | 51.437 | .000 | 51.437 | 51.438 |
| | c | -115.096 | .000 | -115.096 | -115.096 |

a Based on 60 samples.
b Loss function value equals 4.22E-005.

**Table 3: ANOVA**

| Source | Sum of Squares | df | Mean Squares |
|---|---|---|---|
| Regression | 138.649 | 3 | 46.216 |
| Residual | .000 | 3 | .000 |
| Uncorrected Total | 138.649 | 6 | |
| Corrected Total | .001 | 5 | |

Dependent variable: ASPHALT INSTITUTE R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .924.