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Some Results on Prime Labeling Of Graphs

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Abstract

A graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we have proved that some classes of graphs such as the flower pot, coconut tree, umbrella graph, shell graph, carona of a shell graph, carona of a wheel graph, carona of a gear graph, butterfly graph, Two copies of cycle C_n having a common vertex and carona of a alternative triangular snake are prime graphs.

Keywords: Prime Labelings, Graphs

1. Introduction

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. Whereas for notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [7]. Two integers a and b are said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graph. Fu and Huany [3] have proved that the path P_n on n vertices is a prime graph. Deretsky *et. al.*, [2] have proved that the cycle C_n on n vertices is a prime graph.

Around 1980 Roger Etringer conjectured that all trees have prime labeling which is not settled till today. Meena. and Kavitha [1 & 11]has proved that the *fusion*, duplication in butterfly graphs is prime and also prove that star, cycle, *H* – graph, crown graph are strongly prime Graphs. The prime labeling for planar grid is investigated by Sundaram *et. al.*, [6] and Lee. *et.al*., [4] has proved that the wheel W_n is a prime graph if and only if n is even.

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Definition: 1.1. Let $G = (V(G),E(G))$ be a graph with p vertices. A bijection $f : V(G) \to \{1,2,\ldots p\}$ is called a prime labeling if for each edge $e = uv$, $\gcd\{f(u), f(v)\}=1$. A graph which admits prime labeling is called a prime graph.

Definition: 1.2. A coconut tree $CT(m,n)$ is the graph obtained from the path P_m by appending n new pendent edges at an end vertex of P_m.

Definition: 1.3. An umbrella graph $V(m,n)$ is the graph obtained by joining a path P_n with the central vertex of a fan f_m.

Definition: 1.4. For every positive integer $n > 1$ there is a prime p such that $n < p < 2n$

Definition: 1.5. A shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graph are denoted as $C(n, n-3)$. A shell S_n is also called fan f_{n-1}.

Definition: 1.6. The wheel W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as apex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges.

Definition: 1.7. The gear graph G_n is obtained from the wheel by subdividing each of its rim edge.

Definition: 1.8. A double shell consists of two edge disjoint shells with a common apex.

Definition: 1.9. A double shell in which each shell of any order is a Bow graph.

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Definition: 1.10. The join of two graphs G and H, denoted by $G+H$, is the graph where $V(G) \cap V(H) = 0$ and each vertex of G is adjacent to all vertices of H. When $H=K1$, this is the corona graph $K1 \odot G$.

Definition: 1.11. Butterfly graph as a bow graph with exactly two pendant edges at the apex.

Definitions: 1.12. An alternate triangular shake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots. u_n$ by joining u_i and u_{i+1} (alternatively) to a new vertex v_i. That is every alternate edge of path is replaced by C_3.

2. Main Results

Theorem: 2.1. The flower pot graph is a prime graph

Proof:

Let G be the flower pot graph. Let $\{v_1, v_1 \ldots. v_n, u_1, u_2, \ldots. u_m\}$ be the vertices of the flower pot. Then the vertex set $V(G) = \{v_i / 1 \le i \le n\} \cup \{u_j / 1 \le j \le m\}$ and the edge set $E(G) = \{u_1 v_j / 1 \le j \le m\} \cup \{u_i u_{i+1} / 1 \le i \le m-1\} \cup \{u_m u_1\}$ then we note that $|V(G)| = n + m$, where n, m are any positive integers.

Now define a labeling $f : V \to \{1, 2, \ldots. m+n\}$ as follows.

$f(u_i) = i$ for $1 \le i \le m$,

$f(v_j) = m + j$ for $1 \le j \le n$

Clearly vertex labels are distinct.

Then for any edge $e = u_1 v_j \in G$, $\gcd(f(u_1), f(v_j)) = \gcd(1, f(v_j)) = 1$ for $1 \le j \le n$ and for the edge $e = u_1 u_m \in G, \gcd(f(u_1), f(u_m)) = \gcd(1, f(u_m)) = 1$ also for any edge $e = u_i u_{i+1} \in G$, $\gcd(f(u_i), f(u_{i+1})) = \gcd(i, i+1) = 1$ for $1 \le i \le m-1$ since they are consecutive positive integers. Thus the labeling defined above gives a prime labeling for graph G. Thus G is a prime graph.

Theorem: 2.2. The coconut tree $CT(m,n)$ is a prime graph for all positive integers n and $m \ge 2$.

Proof:

Let $CT(m,n) = G$ be the coconut tree.

Let $u_1, u_2, u_3 \ldots. u_m, v_1, v_2, \ldots. v_n$ be the vertices of coconut tree. Then the vertex set $V(G) = \{u_1, u_2, u_3 \ldots. u_m, v_1, v_2, \ldots. v_n\}$. Let $u_1, u_2 \ldots. u_m$ be the vertices of the path P_m and $v_1, v_2, \ldots. v_n$ be the pendant vertices attached with the end vertex of the path P_m. Now the edge set $E(G) = \{u_i, u_{i+1} / 1 \le i \le m-1\} \cup \{u_m v_j / 1 \le j \le n\}$ then

We note that $|V(CT(m,n))| = m + n$.

Define a labeling $f : V \to \{1, 2 \ldots. m+n\}$ as follows

$f(u_i) = i + 1$ for $1 \le i \le m-1$;

$f(u_m) = 1$;

$f(v_j) = m + j$ **for** $1 \le j \le n$;

Clearly vertex labels are distinct. Then for any edge $e=u_m v_j \in CT(m,n), \gcd f(u_m), f(v_j) = \gcd(1, f(v_j)) = 1,$ and for any edge $e=u_i u_{i+1} \in CT(m,n) \gcd(f(u_i), f(u_{i+1})) = (i+1, i+2) = 1$ for $1 \le i \le m-2$. Since these are consecutive positive integers. Thus the labeling defined above gives prime labeling. Hence $CT(m,n)$ is a prime graph for all n and $m \ge 2$.

Theorem: 2.3. The umbrella graph $u(m,n)$ is a Prime graph.

Proof:

Let $G = U(m,n)$ be the umbrella graph.

Let $u_1, u_2, ...u_n, v_1, v_2, ...v_m$ be the vertices of umberlla graph. Let $u_1, u_2, ...u_n$ be the vertices of the path P_n and $v_1, v_2, ...v_m$ be the vertices of the fan f_m where u_1 is the central vertex. Now the vertex set
$V(G) = \{u_1, u_2, ...u_n, v_1, v_2, ...v_m\}$ and the edge set
$E(G) = \{v_i v_{i+1} / 1 \le i \le m-1\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le m\}$ we noted $|V(G)| = m+n$.
Define a labeling $f : V(G) \to \{1, 2....m+n\}$ by

$f(u_1) = 1$

$f(u_i) = i + m$ **for** $2 \le i \le n$

$f(v_i) = i + 1$ for $1 \le i \le m$

Clearly vertex labels are distinct.

Then for any edge for $e = v_i v_{i+1} \in U(G)$, $\gcd(f(v_i), f(v_{i+1})) = \gcd(i+1, i+2) = 1$ for $1 \le i \le m-1$. Since they are consecutive integers and for any edge $e = u_i u_{i+1} \in U(G)$, $\gcd(f(u_i), f(u_{i+1})) = \gcd(m+i, m+i+1) = 1$ for $2 \le i \le m-1$ since these are consecutive integers. Also for edge $e = u_1 v_i \in U(G)$, $\gcd(f(u_1), f(v_i))$
$= \gcd\ (1, f(v_i)) = 1$. Thus the labeling defined above gives the prime labeling. Hence G is a prime graph.

Theorem: 2.4. Shell graphs are prime graphs, for $n \ge 5$

Proof:

Let G be a shell and let $v_1, v_2, ...\ v_n$ be the vertices of G. Now the vertex set. $V(G) = \{v_1, v_2, ...\ v_n\}$ and the edge set
$E(G) = \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_n v_1\} \cup \{v_1 v_i / 3 \le i \le n-1\}$ $|V(G)| = n.$

We define a labeling $f : V(G) \to \{1, 2, 3...n\}$ as follows.

$f(u_i) = i$ for $1 \le i \le n$

Clearly vertex labels are distinct for any edge $e = u_i u_{i+1} \in G,$ for $1 \le i \le n-1,$ then $\gcd(f(u_i), f(u_{i+1})) = \gcd(i, i+1) = 1$ for $1 \le i \le n-1$. Since they are consecutive integers and for any edge. $e = u_i u_i \in G,$ for $3 \le i \le n$ then $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$ for $3 \le i \le n$. Thus the labeling defined above give a prime labeling. Hence G is a prime graph.

Theorem: 2.5. Carona of a Shell is a prime graph, for $n \ge 5$

Proof:

Let S_n be the shell graph with vertices $v_1, v_2, ...v_n$ Let $u_1, u_2, ... u_n$ be the corresponding new vertices, join $u_i v_i$ for $1 \le i \le n$, then resulting graph $G = S_n \odot K_1$ with vertex set $V(G) = \{u_i, v_i / 1 \le i \le n\}$ and the edge set $E(G) = \{u_i v_i / 1 \le i \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_1 u_n\} \cup \{u_1 u_i / 3 \le i \le n-1\}$. Here $|V(G)| = 2n.$

Now we define the labeling $f : V(G) \to \{1, 2, 3 ... 2n\}$ as follows.

$f(u_i) = 2i - 1$ for $1 \le i \le n$

$f(v_i) = 2i$ for $1 \le i \le n$

Clearly vertex label are distinct for any edge $e = u_i v_i \in G.$ $\gcd(f(u_i), f(v_i)) = \gcd(2i-1, 2i) = 1$. Since they are consecutive integers and for any edge. $e = u_i u_{i+1} \in G, \gcd(f(u_i), f(u_{i+1})) = \gcd(2i-1, 2i+1) = 1$. Since these are consecutive odd positive integers and also for any edge $e = u_1 u_i \in G$ for $3 \le i \le n$. gcd $(f(u_1), f(u_i)) =$ $\gcd(1, f(u_i)) = 1$.

Thus the labeling defined above give a prime labeling. Hence G is a prime graph.

Theorem: 2.6. The graph $G \odot K_1$ is a prime graph where $G = W_n$ for all integers $n \ge 4.$

Proof:

Let W_n be the wheel graph with apex vertex v_0 and rim vertices $v_1, v_2, ...v_n$. now let $v_0^1, v_1^1, v_2^1, ..., v_n^1$ be the new vertices which are attached to $v_0, v_1, v_2, ...v_n$ respectively. Then the resultant graph $G_1 = G \odot K_1$ with vertex set $V(G) = \{v_i, v_i^{'} / 0 \le i \le n\}$ and the edge set $E(G) = \{v_0 v_1 / 1 \le i \le n\} \cup \{v_0 v_0^{'}\} \cup \{v_i v_i^{'} / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_n v_1\}$. We note that $|V(G_1)| = 2(n+1)$ then define a labeling $f : V(G_1) \to \{1, 2, ...2(n+1)\}$ by

$f(v_i) = 2i + 1$ for $0 \le i \le n$

$f(v_i^{'}) = 2i + 2$ for $0 \le i \le n$

Clearly vertex label are distinct.

Now for any edge $e = v_i v_i^1 \in G_1,$ $\gcd(f(v_i), f(v_i^1)) = \gcd(2i+1, 2i+2) = 1,$ for $0 \le i \le n$ Since these are all consecutive positive integers, and for any edge $e = v_i v_{i+1} \in G$ $\gcd(f(v_i), f(v_{i+1})) = \gcd(2i+1, 2i+3) = 1$ for $1 \le i \le n-1$ since these are consecutive odd positive integers and also for any edge $e = v_0 v_i \in G_1, \gcd(f(v_0), f(v_i)) = \gcd(1, f(v_i)) = 1$ for $1 \le i \le n$. Thus the labeling defined above gives prime labeling. Hence G_1 is a prime graph.

Theorem: 2.7. The graph $G \odot K_1$ is a prime graph where $G = G_n$ for all integers $n \ge 4$.

Proof:

Let W_n be the wheel with apex vertex v_0 and rim vertices $v_1, v_2, ...v_n$. To obtain the gear graph G_n subdivide each rim edge of wheel by the vertices $u_1, u_2, ...u_n$ where each u_i is added between v_i and v_{i+1} for $i = 1, 2, ...n-1$ and u_n is added between v_1 and v_n. Now let $v_i^{'}, v_i^{'}, ...v_n^{'}, u_i^{i}, u_2^{'}, ...u_n^{'}$ be the new vertices which are attached to $v_0, v_1, v_2, ...v_n$ and $u_1, u_2, ...u_n$ respectively. Then the resulting graph $G = G_n \odot K_1$ with vertex set

$V(G)=\{v_i, v_i^1/0\le i\le n\}\cup\{u_i, u_i^1/1\le i\le n\}$ and the edge set $E(G)=\{v_iv_i^1/0\le i\le n\}\cup\{u_iu_i^1/1\le i\le n\}\cup\{v_0v_i/1\le i\le n\}$ $\cup\{v_iu_i/1\le i\le n\}\cup\{u_iv_{i+1}/1\le i\le n-1\}\cup\{v_1u_n\}$ we note that $|V(G)|=2(2n+1)$ then define a labeling $f:V(G)\to\{1,2,3,....2(2n+1)\}$ as follows.

$f(v_0)=1$

$f(v_0^{'})=2$

$f(v_1)=4$

$f(v_1^{'})=3$

$f(u_i)=4i+1$ for $1\le i\le n$

$f(u_i^1)=4i+2$ for $1\le i\le n$

$f(v_i)=4i-1$ for $2\le i\le n$

$f(v_i^1)=4i$ for $2\le i\le n$

Here for any edge $e=v_iv_i^{'},\ v_0v_0^{'}, v_1v_1^{'}, u_iu_i^{'}\in G,$

$\gcd(f(v_i), f(v_i^{'}))=\gcd(4i-1,4i)=1$ for $2\le i\le n$

$\gcd(f(v_0), f(v_0^{'}))=\gcd(1, f(v_0^{'}))=1$

$\gcd(f(v_1), f(v_1^{'}))=\gcd(4,3)=1$

$\gcd(f(u_i), f(u_i^{'}))=\gcd(4i+1,4i+2)=1$ for $1\le i\le n$.

Since these are all consecutive positive integers and for any edge $e=u_iv_i, u_iv_{i+1},\ u_1v_1, u_nv_1\in G$ then

$\gcd(f(u_1), f(v_1))=\gcd(5,4)=1$

$\gcd(f(v_1), f(u_n))=\gcd(4, f(u_n))=1$ $f(u_n)$. Since $f(u_n)$ is odd positive integer.

$\gcd(f(v_1), f(u_i))=\gcd(4i-1,4i+1)=1$ for $2\le i\le n$ and

$\gcd(f(u_i), f(v_{i+1}))=\gcd(4i+1,4i+3)=1$ for $1\le i\le n-1$. Since these are odd consecutive positive integers.

Clearly all the vertex labels are distinct. Thus the labeling defined above gives a prime labeling. Hence G is a prime graph.

Theorem: 2.8. All butterfly graphs with shell orders m and n are prime graphs for all integers $m,n\ge 3$.

Proof:

Let G be a butterfly graph with shells of order m and n excluding the apex (Note that the shell order excludes the apex). Let the number of vertices in G be N. Now we describe the graph G as follows: In G, the shell that is present to the left of the apex called as the left wing and the shell that is present to the right of the apex is considered as the right wing. Let m be the order of the right wing of G and n is the order the left wing of G. The apex of the butterfly graph is denoted as v_0. Denote the vertices in the wing of the butterfly are denoted from top to bottom as $u_1, u_2, u_3, ...u_n$. The vertices in the right wing of the butterfly are denoted from top to bottom as $v_1, v_2, ...v_m$. The pendant vertices are here the vertex set $V(G) = \{v_1, u_j, w_1, w_2, v_0 / 1 \le i \le m, 1 \le j \le n\}$ and the edge set $E(G) = \{v_0 w_i / i = 1, 2\} \cup$ $\{v_0 v_i, v_0 u_j / 1 \le i \le m, 1 \le j \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{v_i v_{i+1} / 1 \le i \le m-1\}$.

Clearly $|V(G)| = m + n + 3$.

Now define a labeling $f : V(G) \to \{1, 2, ...m + n + 3\}$ by

$f(v_0) = 1$

$f(v_i) = i + 1$ for $1 \le i \le m$

$f(u_i) = m + i + 1$ for $1 \le i \le n$

$f(w_i) = m + n + 1 + i$ for $i = 1, 2$

Then for any edge $e = v_0 v_i, v_0 u_j, v_0 w_k \in G$, $\gcd(f(v_0), f(v_i)) = \gcd(1, f(v_i)) = 1$ for $1 \le i \le m$ for $\gcd(f(v_0), f(u_j)) = \gcd(1, f(u_j)) = 1$ for $1 \le j \le n$ $\gcd(f(v_0), f(w_k)) = \gcd(1, f(w_k)) = 1$ for $i = 1, 2$ and for any edge $v_i v_{i+1}, u_j u_{j+1} \in G,$ $\gcd(f(v_i)), (f(v_{i+1})) = \gcd(i+1, i+2) = 1$ for $1 \le i \le m-1$ and $\gcd(f(u_j), f(u_{j+1})) = \gcd(m+i+1, m+i+2) = 1$ for $1 \le j \le n-1$.

Since these are all consecutive positive integers. Clearly vertex labels are distinct. Thus the labeling defined above gives prime labeling. Hence G is a prime graph.

Theorem: 2.9. Two copies of cycle C_n sharing a common vertex is a prime graph $n \ge 3$ n is any positive integer.

Proof:

Let the cycle C_n be $u_1, u_2, ...u_n, u_1$ consider two copies of cycle C_n . Let G be a graph obtained from two copies of cycle C_n sharing common vertex with $|V(G)| = 2n - 1$ and $|E(G)| = 2n$, let us take u_1 be the common vertex between two copies of C_n.

Now vertex set $V(G) = \{u_1, u_2, ...u_{2n-1}\}$ and the edge set

$E(G) = \{u_1 u_i / i = 2, n, n+1, 2n-1\} \cup \{u_i u_{i+1} / 2 \le i \le n-1\} \cup \{u_i u_{i+1} / n+1 \le i \le 2n-2\}$

Define a labeling $f : V(G) \to \{1, 2, ...2n-1\}$ as follows.

$f(u_i) = i$ for $1 \le i \le 2n-1$

Clearly vertex labels are distinct.

Then for any edge $e = u_i u_i \in G, \gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$, for $i = 2, n, n+1, 2n-1$ and for any edge $e = u_i u_{i+1} \in G,$ $\gcd(f(u_i), f(u_{i+1})) = 1$ for $1 \le i \le n-1$ and $n+1 \le i \le 2n-2$.Since it is consecutive positive integers.

Thus labeling defined above gives a prime labeling for a graph G. Thus G is a prime graph.

Theorem: 2.10. The graph $G \odot K_1$ is a prime graph where $G = AT_n$ for all integers $n \geq 2$ and n is odd.

Proof:

Let $G = A(T_n)$ be alternative triangular snake obtained from a path $u_1, u_2, ...u_n$ by joining u_i and u_{i+1} (alternately) to new vertex v_i where $1 \leq i \leq n-2$. Now let x_i be the vertex which is joined to u_i for $1 \leq i \leq n$, Let y_i be the vertex which is joined to v_i for $1 \leq i \leq n-2$. Then the resulting graph is $G_1 = G \odot K_1$ where $G = AT_n$.

Now the vertex set $V(G_1) = \left\{ v_1, v_2, v_3, ...v_{\frac{n-1}{2}}, u_1, u_2, ...u_n, x_1, x_2, ...x_n, y_1, y_2, ..., y_{\frac{n-1}{2}} \right\}$. and the edge set

$$E(G_1) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i x_i / 1 \leq i \leq n\} \cup \left\{ v_i y_i \middle/ 1 \leq i \leq \frac{n-1}{2} \right\}$$
$$\cup \left\{ u_i v_j \middle/ i = 1, 3, 5, ...n-2, j = 1, 2, ... \frac{n-1}{2} \right\} \cup \left\{ v_j u_i \middle/ j = 1, 2, ... \frac{n-1}{2}, i = 2j \right\}$$

Here $|V(G_1)| = 3n-1$

Now define a labeling $f : V(G) \rightarrow \{1, 2, ...3n-1\}$ as follows.

$f(u_i) = 3i-2$ for $i = 1, 3, 5, ...n$

$f(u_i) = 3i-1$ for $i = 2, 4, 6, ...n-1$

$f(x_i) = 3i-1$ for $i = 1, 3, ...n$

$f(x_i) = 3i$ for $i = 2, 4, ...n-1$

$f(v_i) = 6i-3$ for $i = 1, 2, 3, ... \frac{n-1}{2}$

$f(y_i) = 6i-2$ for $i = 1, 2, 3, ..., \frac{n-1}{2}$

Clearly vertex labels are distinct.

Then for any edge $e = u_i x_i, v_i y_i \in G_1$,

$\gcd(f(u_i), f(x_i)) = 1$ for $i = 1, 2, 3, ...n$ and $\gcd(f(v_i), f(y_i)) = 1$ for $i = 1, 2, ... \frac{n-1}{2}$

Since there are consecutive positive integers, for any edge $e = u_i v_i, v_i u_{2i}, u_i u_{i+1} \in G_1$, $\gcd(f(u_i), f(v_j)) = 1$ for $i = 1, 3, 5....n$ and $j = 1, 2, ...n.$

$\gcd(f(v_i), f(u_{2i})) = 1$ for $i = 1, 2, 3.... \frac{n-1}{2}$

$\gcd(f(u_i), f(u_{i+1})) = 1$ for $i = 2, 4, 6, ...n-2$

Since they are consecutive odd positive integers and for any edge $e = u_i u_{i+1} \in G_1$,

$\gcd(f(u_i), f(u_{i+1})) = 1 \gcd(3i-2, 3i+2) = 1$ for $i = 1, 3, 5, ...n-2$ Since their differences are 4 but they are not divisible by 4.

Thus the labeling defined above give s a prime labeling for a graph G_1, Hence G_1 is a prime graph.

Theorem: 2.11. The graph $G \odot K_1$ is a prime graph where $G = AT_n$ for all integer $n \geq 2$ and n is even.

Proof:

Let $G = A(T_n)$ be alternative triangular snake obtained from a path $u_1, u_2, ...u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex u_i where $1 \le i \le n-1$. Now let x_i be the vertex which is joined to u_i for $1 \le i \le n$, let y_i be the vertex which is joined to u_i $1 \le i \le n-1$ then the resulting graph is $G_1 = G\square K_1$, where $G = A(T_n)$ graph.

Now the vertex set $V(G) = \left\{u_1, u_2, ...u_n, v_1, v_2, ...v_{\frac{n}{2}}, x_1, x_2, ...x_n, y_1, y_2, ...y_{\frac{n}{2}}\right\}$

If n is even and the edge set

$E(G_1) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i x_i / 1 \le i \le n\} \cup \left\{v_i y_i \Big/ 1 \le i \le \frac{n}{2}\right\} \cup \left\{u_i v_j \Big/ i = 1, 3, 5, ...n-1, j = 1, 2, ...\frac{n}{2}\right\}$

$\cup \left\{v_j u_i \Big/ j = 1, 2, ...\frac{n}{2}, i = 2j\right\}$.

Here $|V(G_1)| = 3n$

Now define a labeling $f : V(G) \to \{1, 2, ...3n\}$ as follows.

$f(u_i) = 3i - 2$ for $i = 1, 3, 5, ...n-1$

$f(u_i) = 3i - 1$ for $i = 2, 4, 6, ...n$

$f(x_i) = 3i - 1$ for $i = 1, 3, ...n-1$

$f(x_i) = 3i$ for $i = 2, 4, ...n$

$f(u_i) = 6i - 3$ for $i = 1, 2, 3, ..., \frac{n}{2}$

$f(y_i) = 6i - 2$ for $i = 1, 2, 3, ..., \frac{n}{2}$

Clearly vertex labels are distinct, then for any edge $e = u_i x_i, u_i y_i \in G$,

$\gcd(f(u_i), f(x_i)) = 1$ for $i = 1, 2, ...n$ and

$\gcd(f(v_i), f(y_i)) = 1$ for $i = 1, 2, ...\frac{n}{2}$

Since these are consecutive positive integers. For any edge $e = u_i v_i, v_i u_{2i}, u_i u_{i+1} \in G_1$,

$\gcd(f(u_i), f(v_j)) = 1$ for $i = 1, 3, 5, ...n-1$ $j = 1, 2, ...\frac{n}{2}$.

$\gcd(f(v_i), f(u_{2i})) = 1$ and $i = 1, 2, ...\frac{n}{2}$

$\gcd(f(u_i), f(u_{i+1})) = 1$ for $i = 2, 4, 6....n-2$

Since these are consecutive odd positive integers and for any edge $e = u_i u_{i+1} \in G_1$,

$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i-2, 3i+2) = 1$ for $i = 1, 3, 5, ...n-1$.

Since these difference are 4 but it is not divisible by 4.

Thus the labeling defined above gives a prime labeling for graph G. Hence G is a prime graph.

3. Conclusion

Labeled graph is the topic of current interest due to its diversified application. We investigate eleven new results on prime labeling. It is an effort to relate the prime labeling and some graph operations. This approach is novel as it provides prime labeling for the larger graph resulted due to certain graph operations on a given graph.

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