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Similarity Solution of Laminar Natural Convection Flow of Non-Newtonian Viscoinelastic Fluids

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Abstract

General group symmetry analysis is applied to analyze the natural convection boundary layer flow of some Non-Newtonian viscoinelastic fluids over a vertical flate plate. The velocity distribution, the slope of velocity and temperature variations for two different non-Newtonian fluids namely, Prandtl-Eyring fluid and Williamson fluid are studied through the extended form of numerical method 'Satisfaction of asymptotic boundary conditions technique due to Nachtsheim and Swigert. The mutual comparison between these two different fluids is made with their graphical representation. It is observed that velocity and skin friction in Williamson fluids are considerably higher than those in Prandtl –Eyring.

Keywords: Non-Newtonian Fluid, Similarity solution, Powell- Eyring fluid, Williamson fluid, Linear Group of Transformation

Introduction

The formulation of the group-theoretic method, also called symmetry analysis, is contained in the general theories of continuous transformation groups that were introduced and treated extensively by Lie [1], Oberlack [2]. The class of solution known as similarity solution plays an important role to find out non-Newtonian effects especially in the boundary layer flows. The similarity method involves the determination of similarity variables which reduce the system of partial differential equations governing such flow situation in to ordinary differential equations. Indeed similarity solution is only the class of exact solution for the governing differential equations.

The mixed convection boundary layer flow of non-Newtonian fluid in the presence of strong magnetic field has wide range of application in nuclear engineering and industries. In astrophysical and geophysical studies, the MHD boundary layer flows of an electrically conducting fluid through porous media have also enormous applications. Ghosh and Shit [3] presented an interesting result on mixed convection MHD flow of Viscoelastic fluid in a porous medium past a hot vertical plate.

Lai and Kulacki [4] had shown an interesting problem regarding free convective flow of Newtonian fluid through a vertical porous medium with varying permeability. Rahman, Mamun and Azim [5] presented an interesting result on a natural convection flow past a vertical plate considering the temperature dependent thermal conductivity and heat conduction. Using implicit finite difference scheme Laganathan, Ganesan and Iranian [6] solved a coupled non-linear momentum and energy equations for the investigation of the effect of thermal conductivity on the free convective flow over a semi-infinite vertical plate under the influence of transverse magnetic field. Singh and Gorla [7] carried out the boundary layer analysis of free convective flow of conducting Newtonian fluid over an infinite vertical porous plate.

The studies of free-convective flow along a vertical cylinder are important due to its applications in the field of geothermal power generation, drilling operations, geological formulation. Pop, Ingham and Cheng [8] investigated the growth of the free convection boundary-layer on an isothermal horizontal circular cylinder embedded in a porous medium. El-Shaarawi and Sarhan [9] have considered the fully developed free convective flow in the vertical annuli with one boundary isothermal and opposite adiabatic boundary. Bhadram [10] have considered the combined free and forced convective flow and heat transfer in vertical annulus when a radial magnetic field is applied. Singh and Jha [11] studied the fully developed natural convective flow in the presence of a radial magnetic field by obtaining a unified solution when the thermal boundary condition at the inner cylinder is of mixed kind while outer one is kept on constant temperature. Kumar and Singh [12] studied the Effect of induced magnetic field on natural convection in vertical concentric annuli heated/cooled asymmetrically. Timol and Kalthia [13] is probably first to develop systematic analysis of natural convection flows of all non-Newtonian visco-inelastic fluids characterized by the special functional relationship of stress strain components.

Many researchers have studied the transient laminar natural convection flow past a vertical porous plate for the application in the branch of science and technology such as in the field of agriculture engineering and chemical engineering. In petroleum refineries, movement of oil, water and gas through porous media for purification and filtration are bright applied areas of research with the advancement of science and technology, MHD study on any fluid flow phenomenon exhibits some results which have constructive application for the design of devices. MHD heat transfer has great importance in the liquid metal flows, ionizes gas flow in a nuclear reactor and electrolytes. Research works on radiation of heat in natural convection flow are very limited, though these have many modern applications as missile technology used in army, nuclear power plant, parts of aircraft and ceramic tiles.

The classical theory of fluid mechanics is based upon the hypothesis of a linear relationship between two tensor components , shearing stress and rate of strain ,

$$\tau = -\mu \frac{\partial u}{\partial y} \qquad (1)$$

The fluids with properties different from that described by equation (1), called Non-Newtonian fluids.

Jain, Darji and Timol [14] presented an interesting result on natural convection boundary layer flow of non-Newtonian Sutterby fluids. So motivated by this, we produce similarity analysis via group transformation technique for steady laminar natural convection heat transfer of all time independent non-Newtonian fluids over a non-isothermal vertical flat plate, characterized by the property that its stress tensor component τ_{ij} can be related to the strain rate component e_{ij} by the arbitrary continuous function of the type

$$\mathcal{F}\left(\tau_{ij}, e_{ij}\right) = 0 \qquad (2)$$

The similarity equations obtained are more general and systematic along with auxiliary conditions. Recently this method has been successfully applied to various non-linear problems by Abd-el-Malek et al [15], Darji and Timol [16, 17, 18] and Adnan et al [19].

Mathematical formations

Considering the two dimensional equations of steady incompressible, laminar natural convection flows over vertical flat plate with a Cartesian coordinate system. The stress-strain relation, under the boundary layer assumption can be found in the form of arbitrary function only non-vanishing component τ_{yx}.

Then the relation (2) can be given by

$$\mathcal{F}\left(\tau_{yx}, \frac{\partial u}{\partial y}\right) = 0 \tag{3}$$

Following Timol and Darji [20], the basic equations of continuity, momentum, heat transfer of two dimensional steady incompressible, laminar natural convection flows over a vertical flat plate with a Cartesian coordinate system in usual notations are:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\tau_{yx}\right) + g\beta\theta \tag{5}$$

Energy Equation

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

With the stress-strain relationship

$$\mathcal{F}\left(\tau_{yx}, \frac{\partial u}{\partial y}\right) = 0 \tag{7}$$

With boundary conditions

$$y = 0, \quad u = v = 0, \quad \theta = \theta_w$$
$$y = \infty, \quad u = \theta = 0 \tag{8}$$

The above equation can be made dimenstionless using following quantities,

$$x^* = \frac{G_r}{L}x,\ y^* = \frac{y}{L}\left(\frac{R_e}{3}G_r\right)^{\frac{1}{2}},\ u^* = \frac{u}{U_\infty},\ R_e = \frac{U_\infty L}{v}$$

$$v^* = \frac{v}{U_\infty}\left(\frac{R_e}{3G_r}\right)^{\frac{1}{2}},\ \tau^*_{yx} = \frac{\tau_{yx}}{\rho U_\infty^2}\left(\frac{R_e}{3G_r}\right)^{\frac{1}{2}},\ \theta^* = \frac{\theta}{(T_w - T_\infty)}$$

$$\theta^*_w = \frac{\theta_w}{(T_w - T_\infty)},\ P_r = \frac{U_0 L}{\alpha R_e},\ G_r = \frac{L}{U_\infty^2}g\beta(T_w - T_\infty) \tag{9}$$

Substitute the values in equation (4) to (8) and the asterisks (for simplicity),

We get

Continuity Equation:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{10}$$

Momentum Equation

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\rho}\frac{\partial}{\partial y^*}\left(\tau^*{}_{y^*x^*}\right) + \theta^* \tag{11}$$

Energy Equation

$$u^*\frac{\partial \theta^*}{\partial x^*} + v^*\frac{\partial \theta^*}{\partial y^*} = \frac{1}{3P_r}\frac{\partial^2 \theta^*}{\partial y^{*2}} \tag{12}$$

With the stress-strain relationship

$$\mathcal{F}\left(\tau^*{}_{yx}, \frac{\partial u^*}{\partial y^*}\right) = 0 \tag{13}$$

With boundary conditions

$$y = 0, \quad u^* = v^* = 0, \quad \theta^* = \theta_w \tag{14}$$

$$y = \infty, \quad u^* = \theta^* = 0 \tag{15}$$

Introducing stream function ψ such that,

$$u^* = \frac{\partial \psi}{\partial y} \quad , v^* = -\frac{\partial \psi}{\partial x} \tag{16}$$

Equation of continuity (10) gets satisfied identically, equation (11-15) become

$$\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\tau^*_{yx}\right) + \theta^* \tag{17}$$

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$$\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}=\frac{1}{3P_r}\frac{\partial^2\theta^*}{\partial y^{*2}} \quad (18)$$

$$\mathcal{F}\left(\tau^*_{yx},\frac{\partial^2\psi}{\partial y^2}\right)=0 \quad (19)$$

With boundary conditions,

$$y=0,\quad \frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial x}=0,\quad \theta^*=\theta_w \quad (20)$$

$$y=\infty,\quad \frac{\partial\psi}{\partial y}=0\,,\quad \theta^*=0 \quad (21)$$

Group Theoretic Method

The group theoretic method which is used to find the similarity transformation is based on the concepts derived from continuous transformation of groups. Recently, this technique is found to give most adequate treatment of boundary layer equation. The basic concept of this method was first introduced by Lie [21] in the later part of the century. For the present problem we introduce one parameter group of transformation given below:

$$\bar{x}^*=P^{\alpha_1}x^*\,,\quad \bar{y}^*=P^{\alpha_2}y^*,\quad \bar{\tau}^*_{yx}=P^{\alpha_3}\tau^*_{yx}$$

$$\bar{\psi}^*=P^{\alpha_4}\psi^*\,,\quad \bar{\theta}^*=P^{\alpha_5}\theta^* \quad (22)$$

Where $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ and P are constant

From equation (22) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\alpha_1}}=\left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\alpha_2}}=\left(\frac{\bar{\tau}^*_{yx}}{\tau^*_{yx}}\right)^{\frac{1}{\alpha_3}}=\left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\alpha_4}}=\left(\frac{\bar{\theta}^*}{\theta^*}\right)^{\frac{1}{\alpha_5}}=P \quad (23)$$

Introducing the linear transformation, given by equation (23), into the Equations (17-19) result in

$$P^{\alpha_1+2\alpha_2-2\alpha_4}\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}\frac{\partial^2\bar{\psi}^*}{\partial\bar{x}^*\partial\bar{y}^*}-P^{\alpha_1+2\alpha_2-2\alpha_4}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*}\frac{\partial^2\bar{\psi}^*}{\partial\bar{y}^{*2}}=P^{\alpha_2-\alpha_3}\frac{\partial}{\partial\bar{y}^*}\left(\bar{\tau}^*_{yx}\right)+P^{-\alpha_5}\bar{\theta}^* \quad (24)$$

$$P^{\alpha_1+\alpha_2-\alpha_4-\alpha_5}\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}\frac{\partial\bar{\theta}^*}{\partial\bar{x}^*}-P^{\alpha_1+\alpha_2-\alpha_4-\alpha_5}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*}\frac{\partial\bar{\theta}^*}{\partial\bar{y}^*}=\frac{1}{3P_r}P^{2\alpha_2-\alpha_5}\frac{\partial^2\bar{\theta}^*}{\partial\bar{y}^{*2}} \quad (25)$$

and

$$\mathcal{F}\left(p^{-\alpha_3}\bar{\tau}^*_{yx}\,,\quad p^{2\alpha_2-\alpha_4}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right)=0 \quad (26)$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$\alpha_1+2\alpha_2-2\alpha_4=-\alpha_5 \quad (27)$$

$$\alpha_2-\alpha_3=-\alpha_5 \quad (28)$$

$$\alpha_1+\alpha_2-\alpha_4-\alpha_5=2\alpha_2-\alpha_5 \quad (29)$$

$$2\alpha_2-\alpha_4=0 \quad (30)$$

$$\alpha_3=0 \quad (31)$$

By solving above Equations with $\alpha_1\neq 0$, we get

$$\frac{\alpha_2}{\alpha_1}=\frac{1}{3}\,,\quad \frac{\alpha_3}{\alpha_1}=0,\quad \frac{\alpha_4}{\alpha_1}=\frac{2}{3}\,,\quad \frac{\alpha_5}{\alpha_1}=-\frac{1}{3} \quad (32)$$

Introducing equation (32) into equation (23) result in

$$\eta=\frac{y^*}{x^{*\frac{1}{3}}}\,,\quad \psi^*=f(\eta)x^{*\frac{2}{3}},\quad \theta^*=G(\eta)x^{*-\frac{1}{3}}\quad and\quad \tau^*_{yx}=H(\eta) \quad (33)$$

With the boundary conditions, Eqs (20-21) becomes

$$\eta=0\,,\qquad G(0)=1 \quad (34)$$

$$\eta\to\infty\,,\qquad G(\infty)=0 \quad (35)$$

Introducing equations (33) in equation (17)-(19), we get following similarity equation

$$f'^2(\eta)+2f(\eta)f''(\eta)-3H'(\eta)-3G(\eta)=0 \quad (36)$$

$$2f(\eta)G'(\eta)+f'(\eta)G(\eta)+\frac{1}{Pr}G''(\eta)=0 \quad (37)$$

With stress-strain relationship is given by,

$$\mathcal{F}(H,f''')=0 \quad (38)$$

With the boundary condition

$$\eta=0\,,\quad f'(0)=0\,,f(0)=0\,,G(0)=1\ subject\ to\ \theta_w=1$$

$$\eta\to\infty\,,\quad f'(\infty)=0,\quad G(\infty)=0 \quad (39)$$

Numerical Solution for Prandtl-Eyring and Williamson Model:

Non-Newtonian fluid models based on functional relationship between shear-stress and rate of the strain, shown by (3). Most research work is so far carried out on power-law fluid model; this is because of its mathematical simplicity. On the other hand fluid models other than Power-law model are mathematically more complex and the natures of partial differential equations governing these flows are too non-linear boundary value type and hence their analytical or numerical solution is bit difficult. For the present study the partial differential equation model, although mathematically more complex, is chosen mainly due to two reasons. Firstly, it can be deduced from kinetic theory of liquids rather than the empirical relation as in power-law model. Secondly, it correctly reduces to Newtonian behavior for both low and high shear rate. This reason is somewhat opposite to pseudo plastic system whereas the power-law model has infinite effective viscosity for low shear rate and thus limiting its range of applicability.

Prandtl-Eyring Model

Mathematically, the Prandtl-Eyring Model can be written as (Bird et al [22], Skelland [23], and Wilkinson [24])

$$\tau_{yx} = A \sin h^{-1}\left(\frac{1}{B}\frac{\partial u}{\partial y}\right) \tag{40}$$

Introducing the dimensionless quantities from (9) into equation (40) and then substituting it into the first equation of (36), we get

$$f''' = \frac{1}{\delta}\left(f'^2 + 2ff'' - 3G\right)\left(1 + \lambda F''^2\right)^{\frac{1}{2}} \tag{41}$$

Where $\delta = \frac{A}{\mu B}$; $\lambda = \frac{\rho u_\infty^3 G_r}{3\mu L B^2}$ are dimensionless numbers and can be referred as a flow parameters with same energy equation and boundary conditions.

Williamson Model

The Williamson Model is given by

$$\tau_{yx} = \left(\frac{A}{B + \frac{\partial u}{\partial y}} + \mu_\infty\right)\frac{\partial u}{\partial y} \tag{42}$$

Following the same procedure as described above, we get

$$f''' = \frac{\left(f'^2 + 2ff'' - 3G\right)\left(1 + \sqrt{\lambda} f''^2\right)^2}{\delta\left(1 + \sqrt{\lambda} f''^2\right)^2} \tag{43}$$

Where $\delta = \frac{A}{\mu B}$; $\lambda = \frac{\rho u_\infty^3 G_r}{3\mu L B^2}$ are dimensionless numbers and can be referred as a flow parameters with same energy equation and boundary conditions.

A numerical solution of (41) and (43) is obtained using Method of Satisfaction of Asymptotic Boundary Condition (MSABC) due to Nachtsheim and Swigert [25] with the energy equation together with boundary condition (39).The detail of this technique is recently presented by Patel and Timol [26]. Controlling the non-dimensional numbers $\delta = 10$ and then for $\lambda = 5 \times 10^3$; $\lambda = 5 \times 10^4$; $\lambda = 5 \times 10^5$ the velocity profile, the slope of velocity profile and temperature profile for Pr=0.7 are generated.(see figure-1 ,figure-2,figure-3)

Prandtl-Eyring

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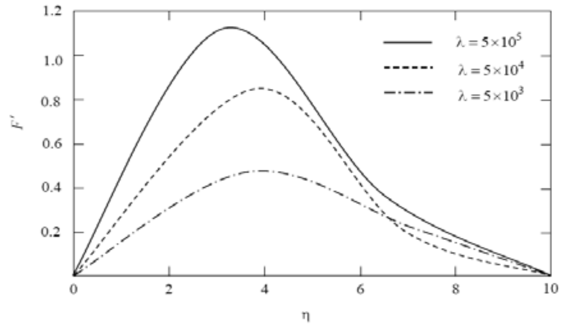

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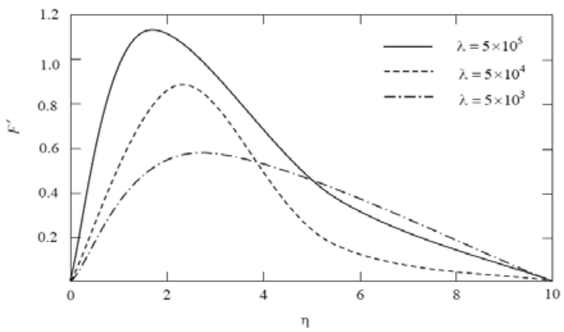
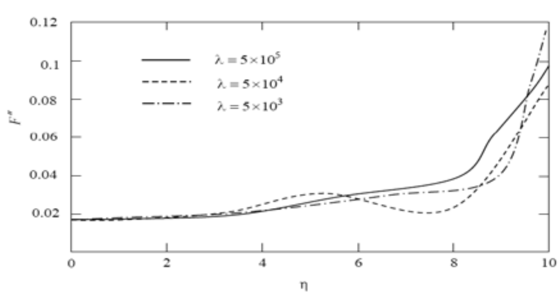


Figure-1: Velocity Profile

Prandtl-EyringModel



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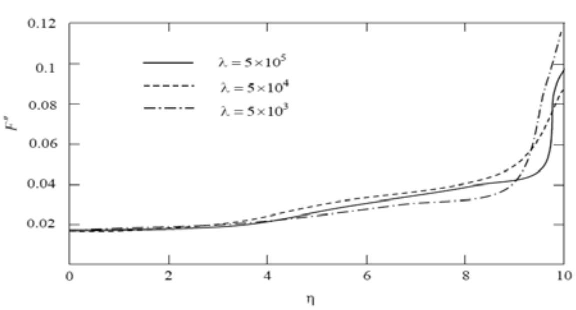
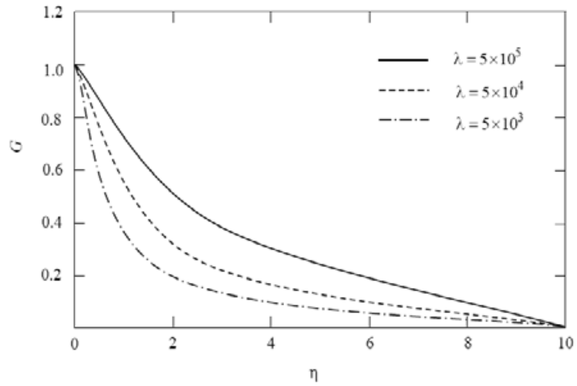


Figure-2: Slope of Velocity (Skin friction) profile

Prandtl-EyringModel



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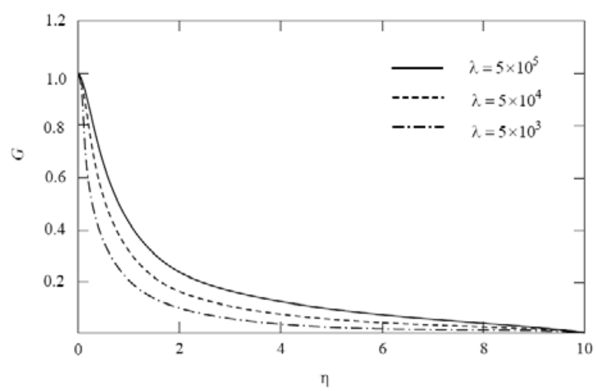

Figure -3: Temperature profile for Pr=0.7

Conclusions

The similarity analysis for natural convection boundary layer flow of a class of Non-Newtonian viscoinelastic fluids past vertical flat surface are derived using one parameter linear group transformation technique. Non-Newtonian fluids considered are characterized by the mathematical property that its shearing stress tensor component is related to rate of deformation component by some arbitrary continuous function. The similarity equations, from highly non linear partial differential equations are derived for two particular models namely Prandtl-Eyring and Williamson along with its proper boundary conditions. Further these similarity equations are solved numerically using shooting method namely MSABC.

At the end it is observed that,

1. The velocity variation in Williamson fluid is quite higher than the Prandtl-Eyring fluid.
2. The skin friction at the wall in Prandtl-Eyring fluid is higher than Williamson fluid.
3. The temperature variation for Pr=0.7, in Williamson fluid is little slower than that of Prandtl-Eyring fluid.

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