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New Lorentz Field Dependent Lorentz Transformation Due to Photon Direction Change

[1]Elnabgha Mohamed Nageeb Mohamed Ali, [2]Mubarak Dirar Abdallah, [3]Sawsan Ahmed Elhouri Ahmed, [4]Fatma Medani & [5]Ahmed Elfaki

[1]Kordofan University- Faculty of Education - Department of physics- Al-Obied-Sudan

[2]Sudan University of Science &Technology-College of Science-Department of Physics- Khartoum- Sudan & International University of Africa- College of Science-Department of Physics Khartoum-Sudan

[3]University of Bahri-College of Applied & Industrial Sciences-Department of Physics-Khartoum - Sudan

[4]Taif University- College of Applied Medical Science- Department of physics- Taif, KSA

[5]Sudan University of Science &Technology-College of Science-Department of Physics- Khartoum

Abstract

New Lorentz transformation accounting for the effect of the field on space time and mass was derived. This transformation is based on the effect of the field on the photon trajectory by preserving the photon speed but changing its direction. According to this version the photon is accelerated by the field due to the change of its direction. This transformation shows that length, space and mass are affected by the field. It also shows the beauty of Einstein curved space concept and its advantage to describe fields more correctly than Newton's laws.

Keywords: Lorentz Transformation; Field; Curved Space

Introduction

One of the important theories in physics is Einstein's special relativity. It is radical theory that made modifications to concept of space, time and energy [1]. It is formed from two hypotheses; first, the homogeneity of the space and other is invariability of light speed in vacuum as maximum velocity of transforming any kind of signal. In Newtonian mechanics the concept of space, time and mass are absolute and they have the same value at any references frames, while they are not absolute in special relativity, they are relative quantities according to the velocities of the observers in inertial frames of references. The hypothesis of invariability of light speed is come from Einstein's conclusion for the results of Micholson-Morly experiments. Einstein used the above hypothesizes and

Lorentz transformation obtained the expressions of time, length, mass and energy [2]. Special relativity theory passed many tests that confirmed its accurate result and it prove its self as extended theory for Newtonian mechanics in high speed near light speed [3]. It succeeded to explaining many physical phenomena like pair production, photoelectric effect and meson decay [4]. But special relativity suffers from noticeable setbacks, some of these setbacks is related to the fact that classical limit of energy expression in special relativity does not coincide with Newtonian energy expression, where the term of potential energy been missing. That means that the energy expression by special relativity does not satisfy the correspondence principle. Also the special relativity can't explain the gravitation red shift for photon due to gravitation field [5]. The photon mass is proportional to its frequency, that conflict with fact that the mass in special relativity is not a function of the field potential. The same holds for the time, length and mass expressions in special relativity, which does not recognize the effect of gravitational field in the weak limit, which is not in conformity with that of general relativity where time and length are effected by gravitational field. However, Einstein's general relativity is generally covariant theory of gravity. Many offers were made to correct and modify special relativity to include the effect of gravity and other fields. These offers concentrated to the mass notion and energy without considering the effect of both motion beside fields on time, length and mass. Some attempts were made to involve the curvature effect of space time on energy and momentum [6,9], but their energy expression is incomplete because they deal with equation of motion instead of Lorentz transformation. These drawbacks motivate searching for new model accounting for the effect of fields.

Lorentz Transformation for Accelerated Photon

Consider the Lorentz transformation

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) (1)$$

$$\grave{x} = \gamma\left(x - vt + \frac{at^2}{2}\right) (2)$$

Consider the two frames $(x.t)$ and $(\grave{x}.\grave{t})$ have their origin coincide at $= \grave{t} = 0$. If a pulse of light is received from a source S then its position in the two frames becomes at t and $\grave{t}$ respectivity

$$x = ct \quad (3 \cdot a)$$

$$\grave{x} = c\grave{t} \quad (3 \cdot b)$$

Substitute $(3.a)$ & $(3.b)$ in (1) yields

$$ct = \gamma\left(c\grave{t} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) = \gamma\left((c+v)\grave{t} - \frac{a\grave{t}^2}{2}\right)$$

$$t = \gamma\left(\left(1 + \frac{v}{c}\right)\grave{t} - \frac{a\grave{t}^2}{2c}\right)$$

$$t = C_1\grave{t} + C_2\grave{t}^2 (4)$$

Where

$$C_1 = \gamma\left(1 + \frac{v}{c}\right)(5\cdot a)$$

$$C_2 = -\frac{\gamma a}{2c}(5\cdot b)$$

Substituting $(3.a)$ & $(3.b)$ in (2) gives

$$c\grave{t} = \gamma\left(ct - vt + \frac{at^2}{2}\right)$$

$$\grave{t} = \gamma\left(\left(1 - \frac{v}{c}\right)t + \frac{at^2}{2c}\right)$$

$$\grave{t} = C_3 t + C_4 t^2 (6)$$

Where

$$C_3 = \gamma\left(1 - \frac{v}{c}\right)(7\cdot a)$$

$$C_4 = \frac{\gamma a}{2c}(7\cdot b)$$

Substitute (6) in (4) to get

$$t = C_1(C_3 t + C_4 t^2) + C_2(C_3 t + C_4 t^2)^2 (8)$$

$$t = C_1 C_2 t + C_1 C_4 t^2 + C_2 C_3^2 t^2 + 2C_2 C_3 C_4 t^3 + C_2 C_4^2 t^4 (9)$$

Comparing the coefficients of $t.t^2.t^3$and t^4 on both sides gives

$$C_1 C_3 = 1 \quad (10)$$

$$C_1 C_4 = -C_2 C_3^2 (11)$$

$$2C_2 C_3 C_4 = 0 \quad (12)$$

$$C_2 C_4^2 = 0 \quad (13)$$

From $(5.a)$ and $(7.a)$, (10) becomes

$$\gamma^2\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) = 1$$

$$\therefore \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}(14)$$

From $(5.a)$ and $(7.b)$, (11) becomes

$$\gamma^2\left(1 + \frac{v}{c}\right)\frac{a}{2c} = \frac{\gamma^3 a}{2c}\left(1 - \frac{v}{c}\right)(15)$$

From $(5.b)$, $(7.a)$ and $(7.b)$, (12) becomes

$$-\frac{2\gamma^3 a^2}{4c^2}\left(1-\frac{v}{c}\right)=0 \quad (16)$$

From $(5.b)$ and $(7.b)$, (13) becomes

$$-\frac{\gamma^3 a^3}{8c^3}=0 \quad (17)$$

In view of equation (14) γ take the same special relativity form. However, equations (15),(16) and (17) shows that the Lorentz transformation (1) and (2) gives consistent results only when $(a=0)$. This requires trying another transformation to take care of effect of fields. One can assume that the light is accelerated due to the effect of field on photon trajectory. It is well known in mechanics that any particle can be accelerated if its magnitude of velocity v is constant when it change its direction.This happens for particles having constant speed v and moving in a circular orbit, thus changing its direction regularly and possessing an acceleration

$$a=\frac{v^2}{r} \quad (18)$$

towards the centre of a circular orbit. According to general relativity (GR) the photon move in a curved trajectory in a gravitational field, although the magnitude of photon speed c is constant, but it is accelerated due to the change of photon direction, since the change of photon direction decreases its speed in the original direction. For example if the photon change its direction by $\Delta\emptyset$ during time intervalΔt, its acceleration becomes

$$a=\frac{\Delta c}{\Delta t}=\frac{c-c\sin\Delta\emptyset}{\Delta t}\approx\frac{c(1-\Delta\emptyset)}{\Delta t} \quad (19)$$

This means that SR and GR are not in conflict with each other, this shows how beauty is Einstein relativity compared to Newton's laws. The photon acceleration can be found by using the relation between work done and energy change according to gravity red shift. The change in photon energy is given by

$$\Delta E=h\grave{f}-hf=V \quad (20)$$

Where V is the field potential. Here one assume that V is potential of any field; not gravity field only.The change of energy is equal to the work done, again assuming constant mass and constant acceleration, one gets

$$F\cdot x=max=V \quad (21)$$

The photon displacement can be found by using the expression for photon interval in a curved space, to get

$$0=c^2d\tau^2=g_{00}c^2dt^2-g_{xx}dx^2 \quad (22)$$

Assuming that the photon obeys static isotropic constraints $g_{00}=g_{xx}$, one gets

$$dx^2 = g_{00}^2 c^2 dt^2 = \left(1+\frac{2\varphi}{c^2}\right)^2 c^2 dt^2 \quad (23)$$

$$dx = \left(1+\frac{2\varphi}{c^2}\right) cdt$$

Thus integrating both sides yields

$$x = \left(1+\frac{2\varphi}{c^2}\right) ct \quad (24)$$

Similar relation can be obtained by finding the photon acceleration by assuming$x = ct$ to get

$$a = \frac{V}{mx} = \frac{\varphi}{x} = \frac{\varphi}{ct} (25)$$

In view of equation (3.a) and (25) the position is given by

$$x = ct - \frac{at^2}{2} = ct - \frac{\varphi t^2}{2ct} = ct - \frac{\varphi}{2c}t \quad (26)$$

Similarly, equation (3.b) and (25) gives

$$\grave{x} = c\grave{t} + \frac{a\grave{t}^2}{2} = c\grave{t} + \frac{\varphi \grave{t}^2}{2\grave{x}} = c\grave{t} + \frac{\varphi}{2c}\grave{t} (27)$$

Consider the Lorentz transformation

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) (28)$$

Where the average velocity v_m is given by

$$v_m = \frac{v+v_0}{2} = \frac{v+v-a\grave{t}}{2} = v - \frac{a\grave{t}}{2} \quad (29)$$

Thus

$$\grave{l} = v\grave{t}^2 - \frac{a\grave{t}^2}{2} = \left(v - \frac{a\grave{t}}{2}\right)\grave{t} = v_m\grave{t} (30)$$

Thus

$$x = \gamma(\grave{x} + v_m\grave{t}) (31)$$

Similarly

$$\grave{x} = \gamma(x - v_m t) (32)$$

For static source in a frame S the photon is not accelerated, thus

$$x = ct \quad (33)$$

But the observer in $\grave{S}$ sees the source S is accelerated and the photon moves in curved space, thus (see equation (14))

$$\grave{x} = c\grave{t} + \frac{\varphi}{2c}\grave{t} \quad (34)$$

By substituting (33) and (34) in (31) yields

$$ct = \gamma\left(c + \frac{\varphi}{2c} + v_m\right)\grave{t} (35)$$

Similarly if the source is at rest in frame $\grave{S}$ the photon position is given by

$$\grave{x} = c\grave{t} (36)$$

Since $\grave{S}$ is accelerated with resprct to S due to the field effect, therefore the photon move in a curved space , thus it is accelerated, hence

$$x = ct - \frac{\varphi}{2c}t \qquad (37)$$

Substitute (36), (37) in (32) to get

$$c\grave{t} = \gamma\left(c - \frac{\varphi}{2c} - v_m\right)t \qquad (38)$$

From (34) and (37)

$$\grave{t} = \frac{\grave{t}}{c}\gamma^2\left(c + \frac{\varphi}{2c} + v_m\right)\left(c - \frac{\varphi}{2c} - v_m\right) \qquad (39)$$

So

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{\varphi}{2c^2} + \frac{v_m}{c}\right)^2}} \qquad (40)$$

Thus the generalized special relativistic energy is given by

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{\varphi}{2c^2} + \frac{v_m}{c}\right)^2}} (41)$$

Neglect the term consisting of c^2, yields

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_m^2}{c^2}}} (42)$$

Where

$$v_m = \frac{v + v_0}{2} \qquad (43)$$

But when the particle moves against the field

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi$$

$$\therefore v_0^2 = v^2 + 2\varphi \qquad (44)$$

by Assuming that $v$and v_0 represent the average values that related to maximum valuesv_{max} and v_{0max} according to relations $v = \frac{v_{max}}{\sqrt{2}}$ and $v_0 = \frac{v_{0max}}{\sqrt{2}}$

$$\therefore v_{0max}^2 = v_{max}^2 + 4ax \qquad (45)$$

then

$$v_m^2 = \left(\frac{v_{max} + v_{0max}}{2}\right)^2 = \frac{v_{max}^2 + 2v_{max}v_{0max} + v_{0max}^2}{4}$$

$$= \frac{v_{max}^2 + 2v_{max}\sqrt{v_{max}^2 + 4ax} + v_{max}^2 + 4ax}{4}$$

$$v_m^2 = \frac{2v_{max}^2 + 2v_{max}^2\sqrt{1+\frac{4ax}{v_{max}^2}} + 4ax}{4}$$

$$\approx \frac{2v_{max}^2 + 2v_{max}^2\left(1+\frac{2ax}{v_{max}^2}\right) + 4ax}{4} \quad (46)$$

$$\therefore v_m^2 = \frac{4v_{max}^2 + 8ax}{4} = v_{max}^2 + 2ax = v_{max}^2 + 2\varphi \quad (47)$$

But from equation (38) for $\frac{v_m^2}{c^2} < 1$ then

$$\gamma = 1 + \frac{v_m^2}{2c^2} = 1 + \frac{v_{max}^2 + 2\varphi}{2c^2} (48)$$

$$\therefore \gamma = 1 + \frac{1}{C^2}\left(\frac{v_{max}^2}{2} + \varphi\right) \quad (49)$$

Thus equation (41) and (38) gives

$$E = \gamma m_0 c^2 = m_0 c^2 + \frac{1}{2} m_0 v_{max}^2 + m_0 \varphi = m_0 c^2 + T + V \quad (50)$$

Thus the generalized special relativity energy relation satisfies the Newtonian limit. This is since the energy include kinetic beside potential energy term.

The gravitational red shift of photons can also be explained by using GSR. Assuming photon in free space so its potential energy $V = 0$, by using (50) and plank hypothesis , one can get

$$hf = m_0 c^2 + T \quad (51)$$

if the photon enters gravitational field its frequency (51) changes also to $\grave{f}$. Thus equation (50) gives

$$h\grave{f} = m_0 c^2 + T + V = hf + V \quad (52)$$

thus fortunately equation (48) explains the gravitational red shift.

Discussion

Equations (15),(16) and (17) can only be satisfied if the acceleration a vanishes. This means that Lorentz transformation (1) and (2) are not suitable for describing the behavior of particles in fields by using the acceleration concept and assuming a to be invariant. However if one uses the concept of potential φ and assume it to be invariant, one can find useful expression for γ similar to that obtained by other researchers**[8]**. This expression explains the gravitational red shift and satisfies the Newtonian limit. The fact that the potential invariance concept is suitable for Lorentz transformation of fields may be related to the fact that

$$g_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) \rightarrow \varphi = -\frac{c^2}{2}(g_{00} + 1)$$

Which means that φ reflects space deformation. Inserting this relation in equation (23) gives

$$x = ct - \frac{\varphi}{2c}t = ct + \frac{c}{4}(g_{00} + 1)t = \frac{5}{4}ct + \frac{c}{4}g_{00}t$$

Which means that the field deform the space. This makes the relation

$$l = vt - \frac{at^2}{2}$$

which describes the motion in Euclidean space invalid. The fact that the field deforms space in generalized special relativity conforms completely with GR, which shows also that the gravity field deforms the space. The space deformation shows also that the photon acceleration is not due to the change of light speed but due to the change of photon (light) direction. This result shows that GSR and GR are not in conflict with each other but they integrate each other. This shows how beauty is the Einstein space deformation concept.

Conclusion

The Lorentz transformation based on photon motion in a curved space relation, resembles that obtained by others. It also satisfies Newtonian limit and predict gravitational red shift. This model is in agreement with previous GSR models.

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