

Muqarnas form efficiency in diffusing Sound waves within the Space

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Abstract

This study evaluates the sound performance of Muqarnas geometrical configuration and its effect on diffusing sound waves when exposed to sound source.

A description of Muqarnas design and its element's composition in addition to a description of different types of sound diffusers geometry is presented. Some Given models are selected in a practical study to evaluate their performance as Sound diffusers. This is done practically by choosing some forms of Muqarnas elements to create a Modular unit that are designed and analyzed as proposed case studies (different types of geometrical design of Muqarnas models) to test the behavior of sound wave spreading and its random reflections from its surfaces.

The objective is to study the Muqarnas form efficiency in scattering sound waves to treat some sound problems and improve the hearing environment within a space by giving the appropriate forms configuration of Muqarnas to act as a sound wave diffuser. This study concludes some useful findings depicting appropriate new forms of Muqarnas that act as efficient sound wave diffusers proposing their location in the closed space.

Keywords: *Architectural acoustics, Muqarnas design, Sound diffusion, Diffusers, Muqarnas forms*

1. Introduction

Muqarnas is considered as one of the traditional architectural elements of Islamic and Persian architecture which has been used as a smooth transition from the rectangular basis of the building to the domed ceiling. Muqarnas can be viewed as an ornamented element which is regarded as a sort of wall sculpture adorned with painting or tile mosaics. In its most basic form, Muqarnas is beautiful enough to be more decorated. See [figure 1].



Fig. 1 Muqarnas as an ornamented element (2)

In addition to its ornamental and sculptures role with a certain symbolic meaning, the Muqarnas can work as a specific construction function as well.

The construction function of Muqarnas is to distribute the structural mass load into several small elements using its elementary geometrical structure in rows. It also allows the intersection of flat

surfaces and curved ones to be converted from a rectangular to octagonal shape, and then to a circular dome. The study assumes that there is another function for the Muqarnas, which is the dispersion and propagation of the sound waves in closed space, by finding the relationship between the form of the Muqarnas and its efficiency in scattering the sound waves to improve the hearing state inside the space and to eliminate some sound problems.

The study Methodology consists of a brief theoretical background to indicate the effect of Muqarnas as a sound diffuser in a certain space. A hypothesis is set to study the impact of the Muqarnas form configuration on sound waves diffusion to treat one of the main factors affecting sound problems in a closed space like sound coloration and echo...etc. A computer geometrical form simulation of some case studies is applied to test the validity of the given hypothesis using simulated cases of different types of Muqarnas forms.

Results of this study indicate that the configuration forms of some merged Muqarnas elements act to disperse the reflected sound waves randomly and so, improve the hearing state.

Hypothesis- The geometrical configuration of Muqarnas forms exposed to the sound wave influence the paths of spreading sound waves and its random reflections which can improve the sound environment.

2. Muqarnas – Definition

Muqarnas is a structural and decorative architectural component that may appear in different designs and shapes. It consists of different layers of elements called tiers. Among these basic elements we can distinguish cells and intermediate elements. (2)

The cells look like small pieces of a vault, while the intermediate elements can be used to combine cells together. Those are formed out of small pointed niches with rhythmic modularity and infinite compositions to break down vaults and domes into multiple facets with the purpose of unifying a dome’s transitional zone into a compositional unity. Muqarnas designs are two-dimensional; that can be transferred to three-dimensional structures. See [figure 2]. (4)

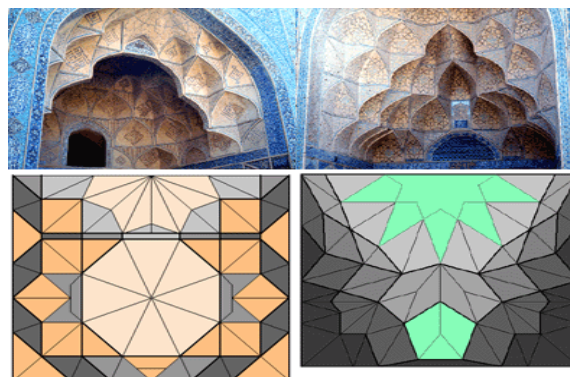


Fig. 2 Two and three dimensional Muqarnas structures (4)

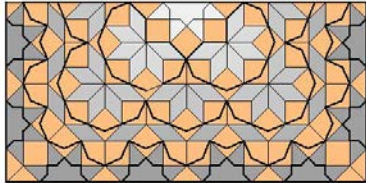
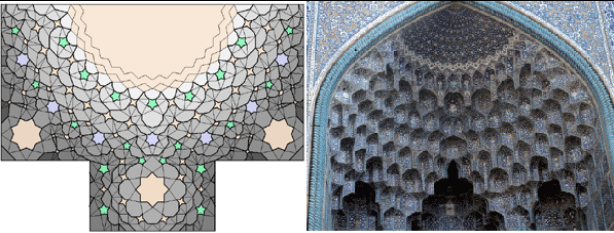

3. History of Muqarnas

In the middle of the tenth century Muqarnas began to develop in North Eastern Iran and central North Africa. Muqarnas spread throughout the Islamic world from the eleventh century on. The earliest mathematical approach to Muqarnas is given by al-Kashi. He distinguished four types of Muqarnas: the simple Muqarnas, the clay-plastered Muqarnas, the curved Muqarnas and the Shirazi Muqarnas. (5)

The top views of the simple, clay-plastered and curved Muqarnas consist of triangles and quadrilaterals, while Shirazi Muqarnas plan contains other polygons such as pentagons, hexagons, octagons and multipoint stars.

Shiro Takahashi classes Muqarnas into three types: the square lattice Muqarnas, the pole table Muqarnas and 'other style' Muqarnas, which do not belong to the two types, see [table 1].

Table 1: Types of Muqarnas / Shiro Takahashi (5)

Type of Muqarnas		Muqarnas Forms
Square lattice	Squares and 45 degree rhombuses.	 <p>. Square lattice: Design of a Muqarnas in Samarqand</p>
Pole table	The center of the pole and the star shaped elements.	 <p>Pole table: The design and a picture of the Shah Mosque.</p>
other style	Cannot be properly categorized.	 <p>Other style: suleymaniye Mosque, Istanbul, Turkey.</p>

4. Muqarnas designs

The three-dimensional Muqarnas can be projected into a plane since elements do not overlap. Hence, elements of a Muqarnas can be distinguished between cells and intermediate elements. The word cell is used as a translation for the Arabic word (*Bayt*). (5)

4-1. The element`s composition of Muqarnas:

- Muqarnas consists of cells, and those cells can be divided into facets and a roof. The facets are the straight planes perpendicular to the horizon and the roof is the upper part of the cell as shown in [figure 3].

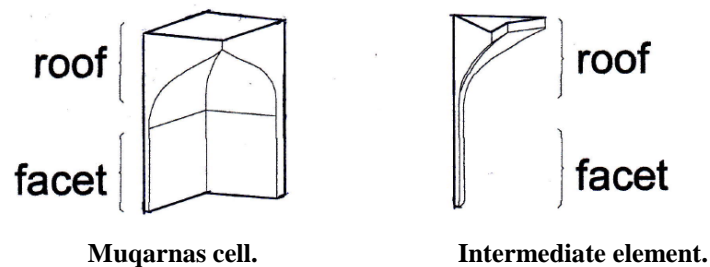


Fig. 3 Muqarnas cell and intermediate element components (5)

- Between the roofs of two adjacent cells a curved surface can be located in the form of either a triangle or two triangles. These intermediate elements can also be divided into facets and a roof, between two adjacent cells either one intermediate element (one triangle) can be located or two intermediate elements (two triangles). One intermediate element can thus connect the roofs of two cells or of one cell and another intermediate element. It is also possible that two cells lack the connection of an intermediate elem.
- Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier (*tabaqa*), they are constructed with the same unit of measure.
- Every element of a Muqarnas has two curved sides which all have the same measurements and shape. The short vertical line of the curved side, the front side of an element, is called the apex. The long vertical line is simply called the backside. In a cell the two curved sides join at the apex. The curved sides of a cell and an intermediate element. Side A is the apex; side B is the backside. See [figure 4]. (5)

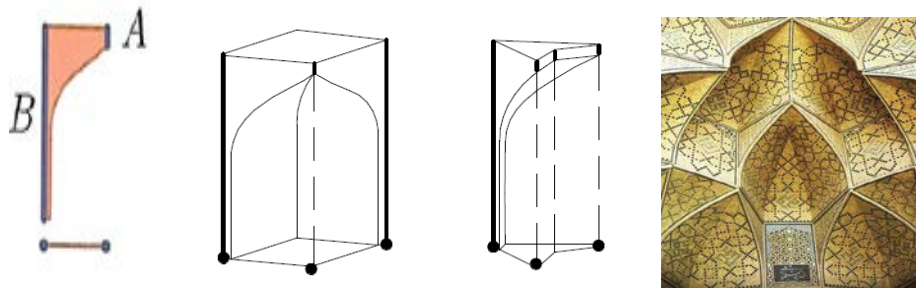
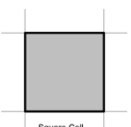
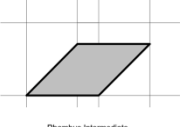
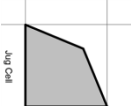
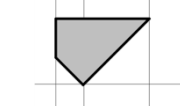
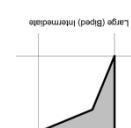
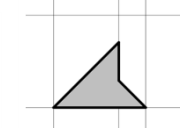

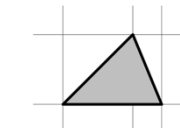


Fig. 4 Side A is the apex; side B is the backside (5)

4-2. Basis of the Elements

The top views of the elements are based on the square and the rhombus. We can merge two types or more of elements and intermediate elements to create new different forms. See [table 2].

Table 2: Forms of Muqarnas Elements (5)

Basic Elements			
Square element	Rhombus element		
square cell and an intermediate element	 Square Cell	 Rhombus Intermediate	rhombus cell and an intermediate element
Jug element cell	 Jug element cell	 Almond Cell	Almond Cell.
Large biped an intermediate element	 Large (biped) Intermediate	 Small (Biped) Intermediate	Curved sides an intermediate element
Half square Cell and an intermediate element.	 Half square cell	 Half Rhombus	Half rhombus cell and an intermediate element

4-3. Combining Muqarnas Elements

There are different ways to conjoin Muqarnas elements:

- Muqarnas as a structure built from elements as if they were in the steps of a stair which are called one tier (*tabaqa*).
- They can also meet other intermediate elements at other sides. The front of intermediate elements can meet at the apex.
- The backside of a cell can stand on the curved side of an element in a lower tier.
- The backside of a cell can stand on the front side of an intermediate element in a lower tier.
- An intermediate element can stand on the front of a cell in a lower tier.

4-4. Directing a Muqarnas graph

The direction of the top view of an element, can determine whether it is a cell or an intermediate cell and even in which tier it is located.

Every directed line (called arrow) represents a curved side and points to the apex of the associated element. See [figure 5].

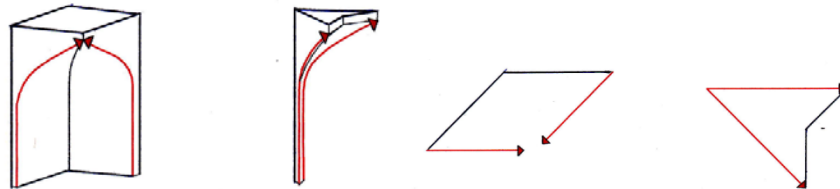
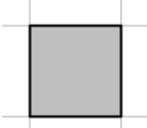

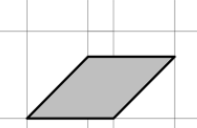
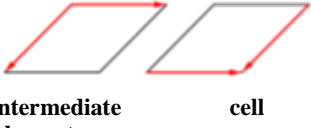
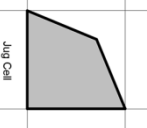
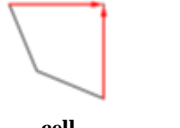
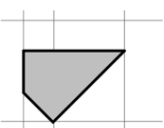


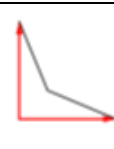
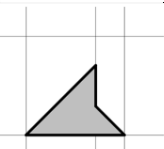

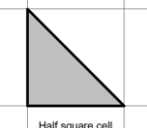
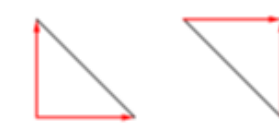
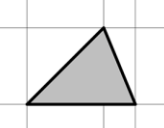



Fig. 5 Directing Muqarnas: The arrows represent a curved side. (5)

This means that an arrow often gives the position where two elements join at the curved side. The curved sides of a cell join at the apex. In contrast the curved sides of arrows of an intermediate element may join at the backside.

The directions to the edges represent the curved sides of an element that only appears in one way (either a cell or an intermediate element). Edges without a direction represent the plane backsides of a cell or the front sides of an intermediate element. [Table 3] is created to show this:

Table 3: The possible directions of the top views of the elements (author)

The directions of the top views of the elements (cell or an intermediate element)			
 <p>Square Cell</p>	 <p>Intermediate Element cell</p>	 <p>Rhombus Intermediate</p>	 <p>Intermediate element cell</p>
 <p>Jug Cell</p>	 <p>cell</p>	 <p>Almond Cell</p>	 <p>cell</p>
 <p>Large (Biped) Intermediate</p>	 <p>intermediate element</p>	 <p>Small (Biped) Intermediate</p>	 <p>intermediate element</p>
 <p>Half square cell</p>	 <p>intermediate element cell</p>	 <p>Half Rhombus</p>	 <p>intermediate element cell</p>

5. Sound diffusion

Sound diffusion is defined as the bending and scattering of sound waves around obstacles and surfaces in the space. It can be created by using surface irregularities and scattering elements.

Diffusers are hard surfaced acoustical treatments with either irregular surfaces that break up and scatter sound waves in multiple directions, or convex curved surfaces to gently split frequencies and deflect them in multiple directions. As a consequence, the diffusers enhance the hearing state within the room.(6)

The diffusers do not absorb the incident sound but it reflects it in as many directions as possible. Rather than removing any sound energy, diffusers are used to effectively reduce distinct echoes and reflections while still leaving a live sounding space compared to a reflective surface. This will lead to high-quality sound throughout the room.

The projections of the surface irregularities must be at least one- seventh of the wavelengths of diffused sound waves. So the obstacles should be too small compared to the wavelengths of sound waves. This is more effective for low than high-frequency sounds.

5-1. Acoustic parameters of sound diffusers

1. Diffusion coefficient

This value describes the degree of diffusion as a function of frequency. The degree of absorption exists since every real diffuser also absorbs part of the sound.

2. Sound absorption coefficient (α)

The sound absorption coefficient is the energy of the incident sound which is absorbed by the surface. Its value is between (0 and 1). Sound diffusers are usually made of hard materials, so that their sound absorption is minimal.

3. Sound scattering coefficient (S)

It defined as the ratio between the acoustic energy reflected in non-specular directions and the totally reflected acoustic energy from rough surface. A sound scattering surface is defined as a surface with $(S) \geq 0.5$. A sound scattering coefficient of (1.0) indicates that 100% of the reflected sound is scattered in all directions, whereas a value of (0.0) indicates that the sound is reflected in one specular direction. Likewise, a sound absorption coefficient of (1.0) indicates that the sound is totally absorbed, whereas (0.0) means that it is entirely reflected. See [figure 6]. (7)

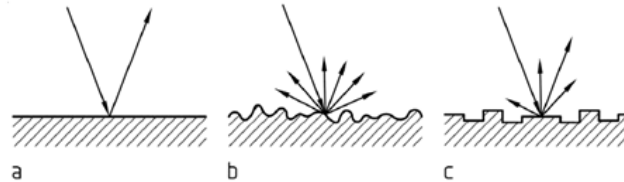


Fig. 6 Specular and diffuse reflection (7)
a= specular (regular) reflection, b,c = scattered reflection (diffused reflection)

5-2. Types of Diffusers

Diffusers come in many shapes and materials. Two-dimensional diffusers scatter sound in a hemispherical pattern by the creation of a grid, whose cavities have wells of varying depths. Generally, diffusers can be divided in two basic forms; Mathematical and Geometric, [Table4]. Geometric Diffusers break up sound reflections and disperse them more evenly throughout the listening space. Random dispersion of sound reflections improves sound quality and consistency for the listener. The types of Mathematical & Geometric diffusers can be classified as follows:

1. MLS (Maximum length sequence) diffusers
2. QRD (Quadratic Residue Diffusers)
3. Two-dimensional (hemispherical) diffusers
4. Simple curved surfaces

5. Irregular geometric structures
6. Periodic geometric structures
7. Art diffusers

The most common types of sound diffusers that have forms close to those of Muqarnas are the irregular, pyramidal and conical forms of geometric diffusers, such as those of type (5), (6) and (7) in particular. It will randomly diffuse the sound to prevent some sound problems, such as, echo and sound coloration...etc.).

The unique organic curvature of the Art Diffuser type control specular reflections above 4 kHz makes it preferable to other designs. The Art Diffuser deflects the sound hemi-spherically, as a result of the bi-cubic concentric rings, and the various quadratic well depths. See [figure 7]. (3)

These diffusers reduce the intensity of sound by scattering it over an expanded area. This prevents standing waves while maintaining the live intensity of the sound.

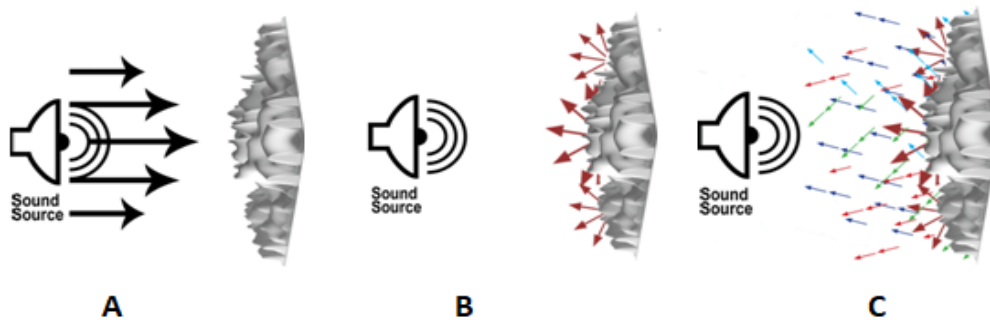


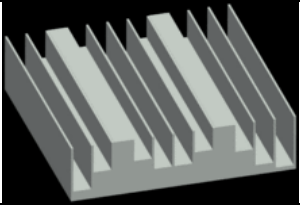
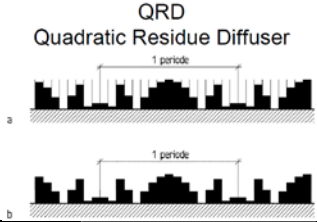

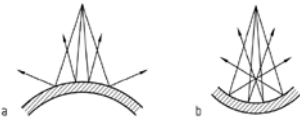
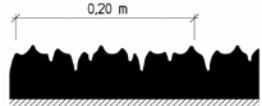
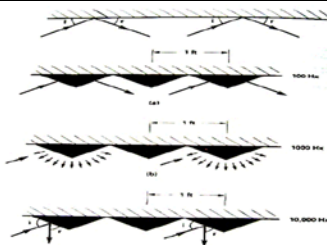
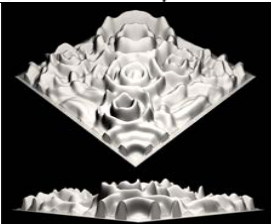
Fig. 7 Sound scattering by art diffuser (3)

To summarize the forms of sound diffusers that can be selected to act as a Muqarnas:

1. Pointed small cones
2. Prisms with horizontal section of (triangle or convex surface)
3. Irregular pyramidal and conical forms

Those forms of diffusers work for the scattering of sound waves to prevent sound problems and sound concentration from domes and vaults surfaces.

Table 4: Some of Mathematical & geometrical type of Muqarnas (3)


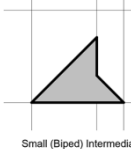

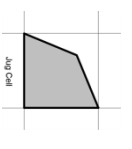
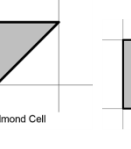
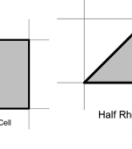

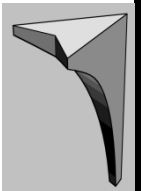

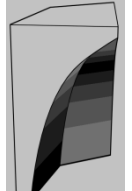
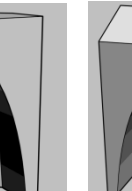
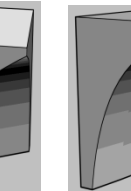
Type No.	Type of Mathematical & Geometric diffusers	Form configuration
1	<p>MLS - Maximum length sequence diffusers</p> <p>MLS are made of strips of material with two different depths. The width of the strips is smaller than or equal to half the wavelength of the frequency</p>	
2	<p>QRD - Quadratic-residue diffusers</p> <p>Combine the excellent diffusion characteristics of MLS designs with wider bandwidth called a quadratic-residue diffuser (Schroeder diffuser).</p>	<p>QRD Quadratic Residue Diffuser</p> 
3	<p>Two-dimensional (hemispherical) diffusers</p> <p>Two-dimensional diffusers scatter sound in a hemispherical pattern. This is done by the creation of a grid, whose cavities have wells of varying depth, according to the matrix.</p>	
4	<p>Simple curved surfaces</p> <p>Convex surfaces will scatter sound, concave surfaces can focusing of sound</p>	<p>Convex and concave surfaces</p> 
5	<p>Irregular geometric structures</p> <p>Consisting of concave, convex and pointed units (pyramidal and conical forms) arranged irregularly.</p>	<p>Irregular geometric structure</p> 
6	<p>Periodic geometric structures</p> <p>Pyramids or triangular have interesting acoustic properties, depending of the angle of the sides of the surface.</p>	
7	<p>Art diffuser</p> <p>Organic quadratic diffuser that improves sound clarity</p>	

6. Practical study

This study has used some of the basic forms of Muqarnas elements that can act as sound diffusers, some of them as cell and others as intermediate elements. See [table 5].

They can be arranged to compose different types of module unites by merging two or three elements to create new different forms of module units.

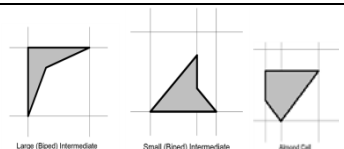
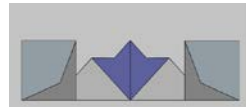
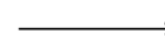
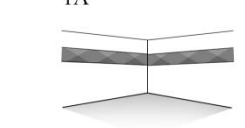
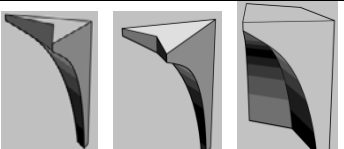
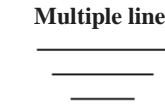
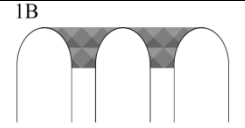
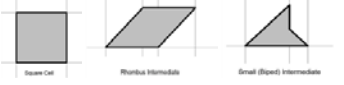


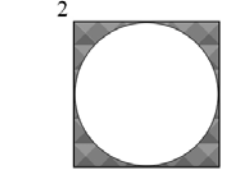
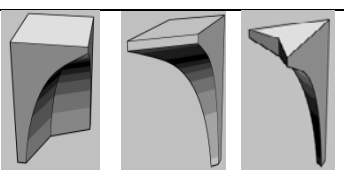
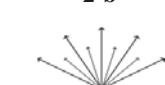
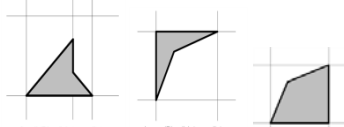
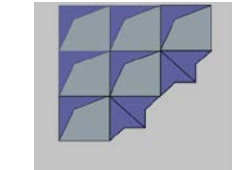

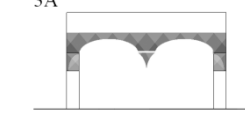
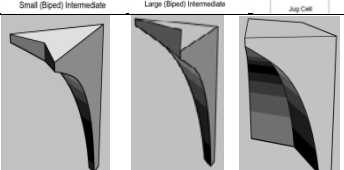

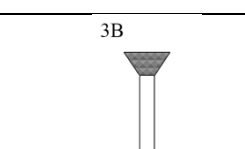
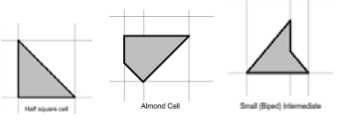
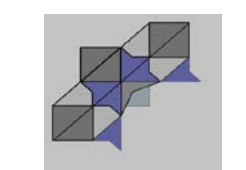

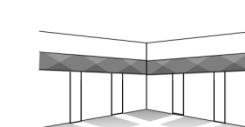
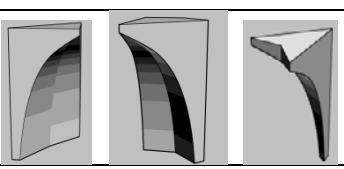
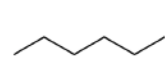
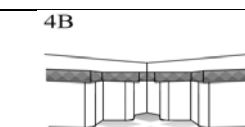
Table 5: Some of selected Muqarnas elements to create the studied module unit form

Selected types of Intermediate element and cell							
Intermediate element				cell			
							2D
							3D

These elements are re-configured to form a geometrical module unit with certain specifications and dimensions. They are considered as case studies whose forms and dimensions are chosen and arranged in different axes then located in different locations within the space to work as sound diffusers, see [table 6].

The elements have been combined with repetition and symmetry arrangements around vertical or diagonal axes to form the new module units with rhythmic modularity patterns and geometric formations that differ from the traditional patterns of the Muqarnas.

Table 6: Some of the Muqarnas element studied as a module unit to act as diffuser (author)

Type No.	Muqarnas Elements	Module units	The way of arrangement	Module unit location in a space
1	 <p>Large (Biped) Intermediate Small (Biped) Intermediate Almond Cell</p>		<p>1-a Straight Linear</p> 	<p>1A</p> 
			<p>1-b Multiple line</p> 	<p>1B</p> 
2	 <p>Square Cell Rhombus Intermediate Small (Biped) Intermediate</p>		<p>2-a Radial</p> 	<p>2</p> 
			<p>2-b</p> 	
3	 <p>Small (Biped) Intermediate Large (Biped) Intermediate Jug Cell</p>		<p>3-a Repetition & symmetrical</p> 	<p>3A</p> 
			<p>3-b</p> 	<p>3B</p> 
4	 <p>Half square cell Almond Cell Small (Biped) Intermediate</p>		<p>4-a Straight line</p> 	<p>4A</p> 
			<p>4-b A multi-directional line</p> 	<p>4B</p> 

Through the geometrical analysis of the sound wave paths incident on module surfaces, and by applying the reflection law to measure the incident and reflection angles, the sound waves

dispersion with randomly scattered sound wave propagation and reflections can be found. See figures in [Tables 7-1, and 7-2] which illustrate the behavior of scattered sound waves after their reflection from the surfaces of the model units.

Table 7-1: The effect of geometrical configuration of module units on sound diffusing
Type (1-2) (author)

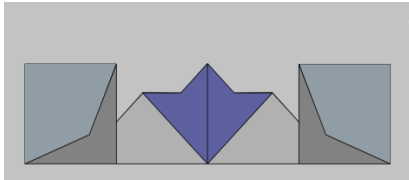
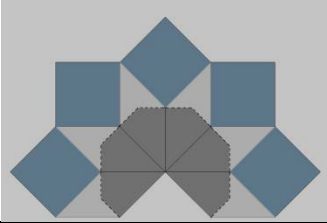
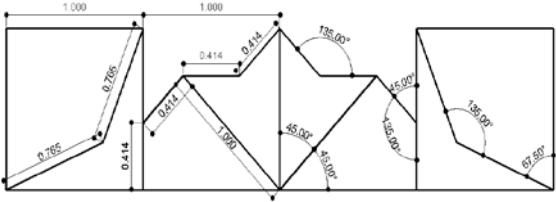
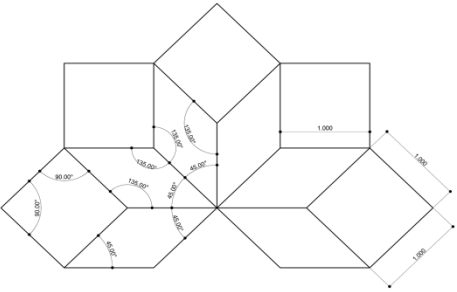
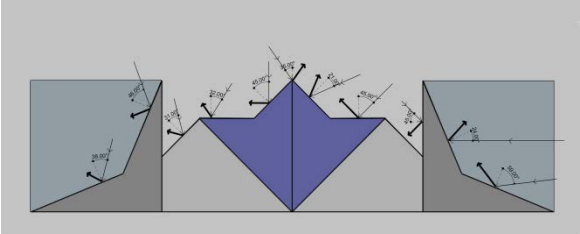
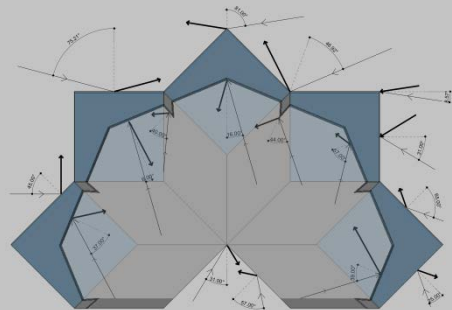
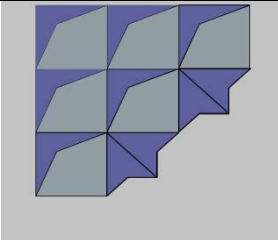
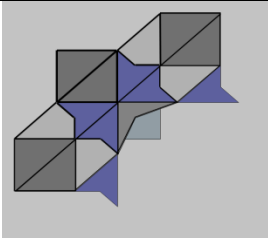
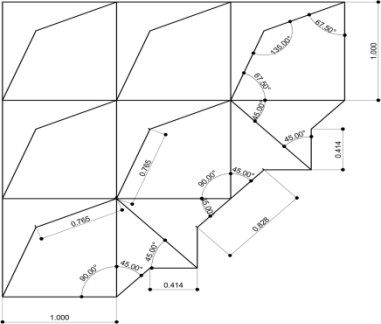
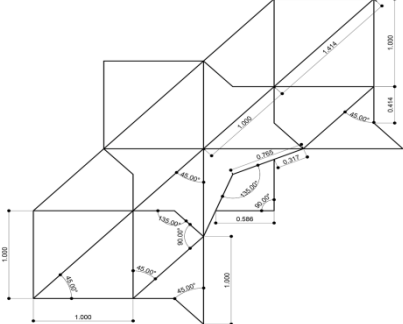
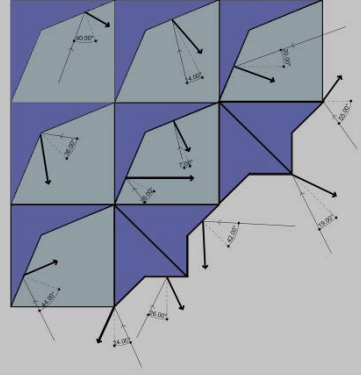
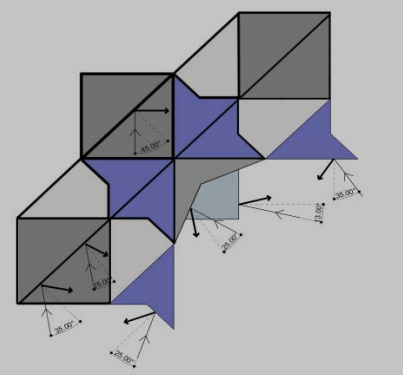
Type No.	1	2
Module unit		
Shape and dimension		
Module units as diffuser		

Table 7-2: The effect of geometrical configuration of module units on sound diffusing Type (3-4) (author)

Type No.	3	4
Module unit		
Shape and dimension		
Module units as diffuser		

It is possible to replicate and combine the modular units in different patterns to form a set of combined merging modules giving different types of geometric formations, which can be used in several places within the space and used as Muqarnas, as shown in [Table 6].

These geometric formations of combined merging modules were illustrated in two and three dimensions as shown in the figures of [Tables: 8-1, 8-2, 8-3, and 8-4].

Table 8-1: Two & three dimensional forms of combined modules- type (1) (author)

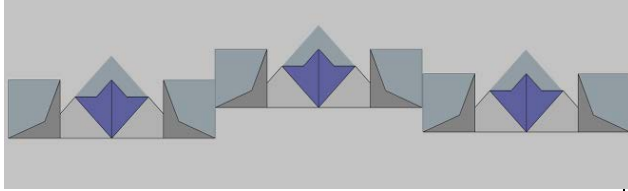
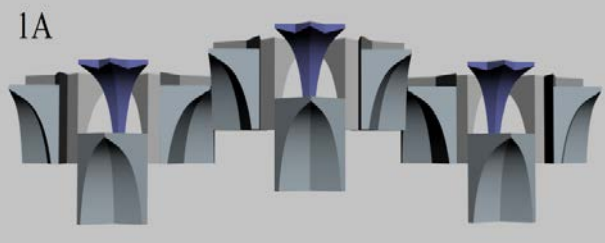
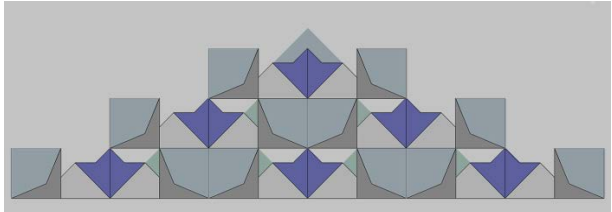
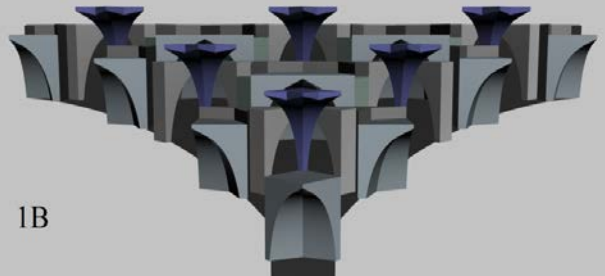
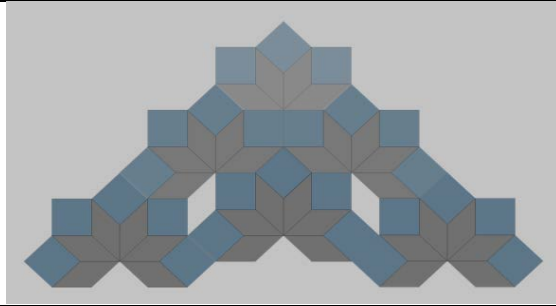
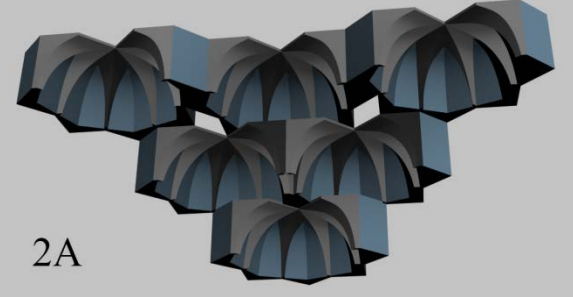
Type No.	1	
1-A		
combined merging module (2D)	combined merging module (3D)	
		
1-B		
		

Table 8-2: Two & three dimensional forms of combined modules- type (2)

Type No.	2	
2-A		
combined merging module (2D)	combined merging module (3D)	
		
2-B		

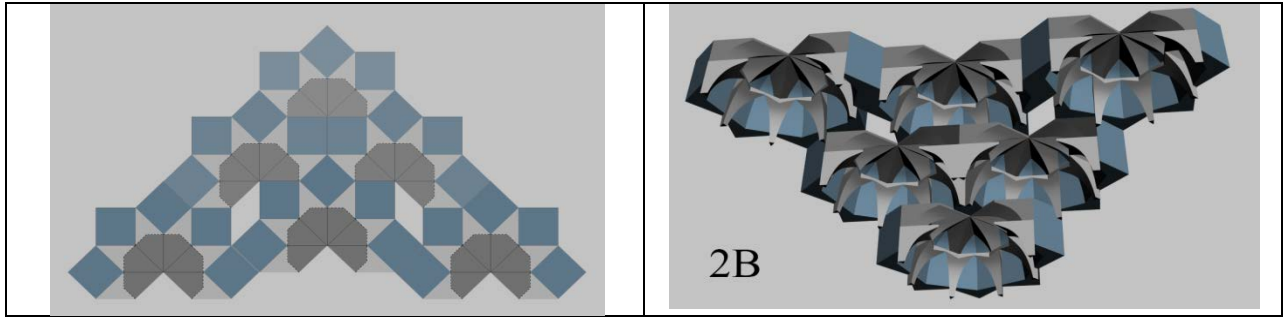


Table 8-3: Two & three dimensional forms of combined modules- type (3) (author)

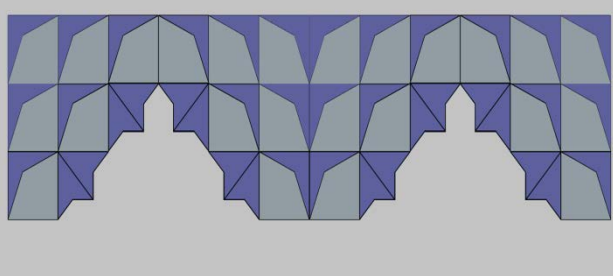
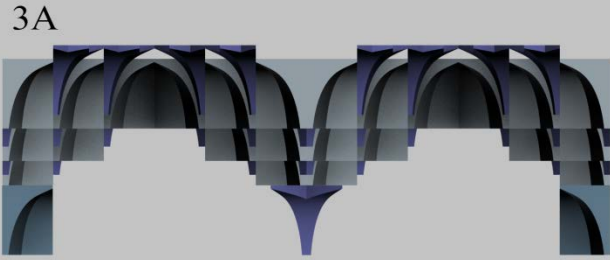
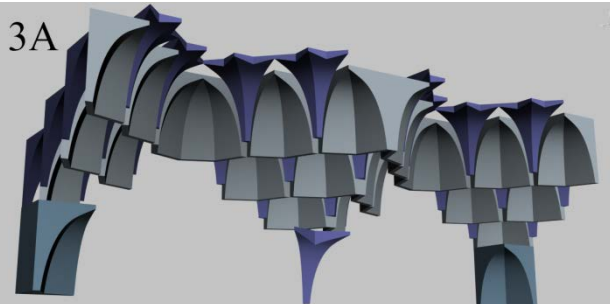
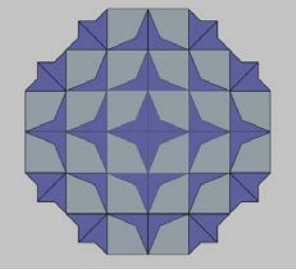
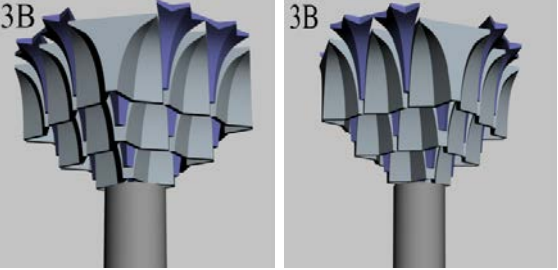
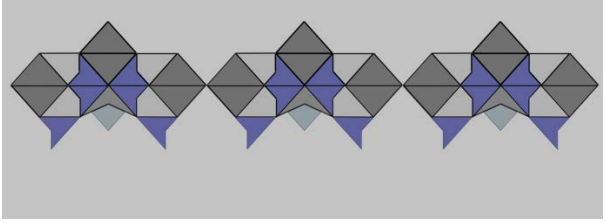
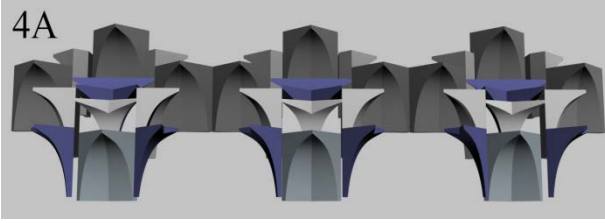
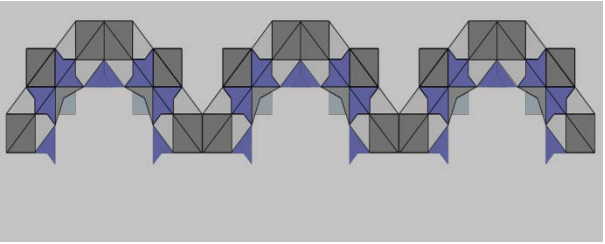
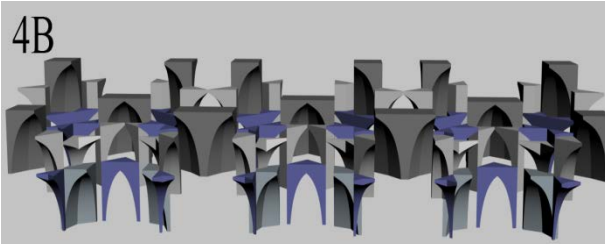
Type No.	3	
3 –A		
combined merging module (2D)	combined merging module (3D)	
		
		
3 –B		
		

Table 8-4: Two & three dimensional forms of combined modules- type (4) (author)

Type No.	4	
4-A		
combined merging module (2D)	combined merging module (3D)	
	4A 	
4-B		
	4B 	

A composed model can also be created by merging several types of combined module units together. Those can be formed symmetrically on both sides of certain axis or in random order.

This composed model can be used in the location of spherical triangles placed at the corners on the vertical structural elements around the domed square space. See [Table 9, and 10].

Table 9: The component of composed module (author)

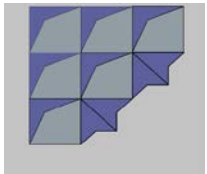

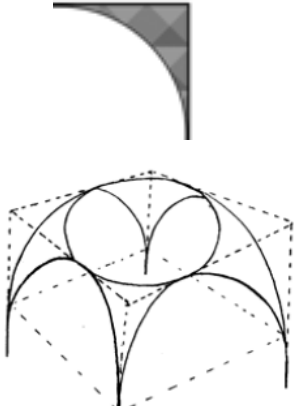

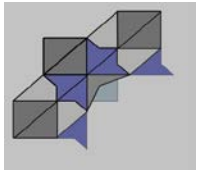
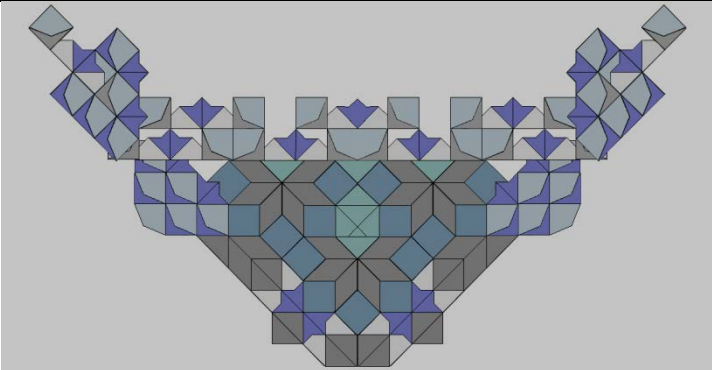
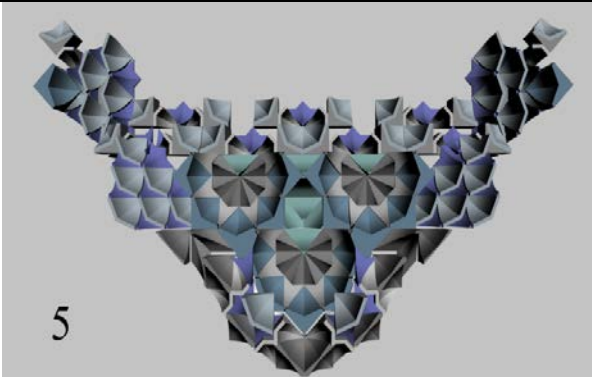
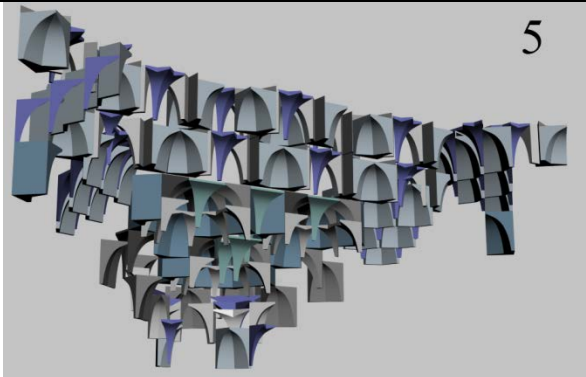
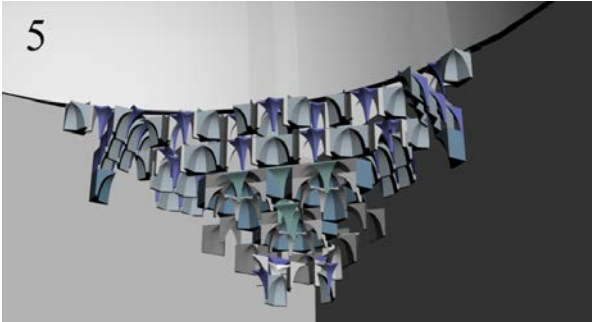
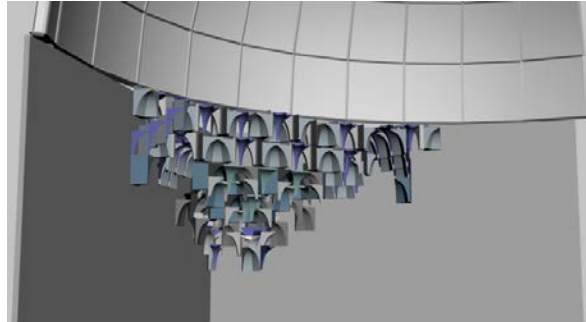
Combined models units		Module unit arrangement	Module unit location in a space
		Symmetrical on diagonal axis Of Spherical triangles of the dome	
			

Table 10: Two & three dimensional composed model from merging different combined models.

Composed model type Type – 5 (2D)	
	
(3D)	
	
	

7. Conclusions

The most common sound diffuser forms that can be used in Muqarnas are those having irregularly pyramidal, conical forms, and sharp angled triangular shape, as shown in [Table 4] of types (5), (6), and (7).

The Pyramidal form and the sharp angled triangular shape are the forms that exist in both the sound diffuser (such as; irregular geometrical structure, periodic geometrical structure, and art diffuser), with the forms of Muqarnas elements (such as: small biped, large biped, half rhombus, jug, and almond).

It was observed after measuring the dimensions of the modules units (case studies), and through the geometrical analysis of the incident and reflection sound wave paths as well as measuring the incident and reflection angles, that the sound waves dispersed randomly scattering the sound waves with multi-reflections from the module surfaces. This dispersion of the incident sound waves make it works as a good sound diffuser.

The proposed Muqarnas forms that act as sound diffusers can be used in several locations within the space, such as:

- Spherical triangles in the junction of the vertical walls with the dome, where the square shape of the space plan is transformed into the circle shape of the dome plan or its (podium), as shown in the type (5) in [table 10], and type (2A), (2B) in [table 8-2] respectively.
- column`s capital as shown in type (3B) in [table 8-3].
- Arcade collider such in type (3A) / [table 8-3], and between arches as shown in type (1B) / [table 8-3].
- Frieze on the upper part of surrounding walls like in type (1A) / [table 8-1], and type (4A) / [table 8-4].
- The curved wall as in type (4B) / [table 8-4] to treat some of the sound problems such as sound concentration and echo.....etc.

Acknowledgment

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8. References

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