

Response of Rectangular Tank under Seismic Load by Coupling BEM and FEM

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Abstract:

In this paper fluid–structure interaction analysis of plate structures with compartments partially filled with a liquid. The compound plate was a simplified model of a fuel tank. The plate considered to be thin and Kirghoff–Lave line earth potheses are applied. The liquid is ideal and incompressible. It's properties and the filling levels may be different in each compartment. The plate vibrations coupled with liquid sloshing under the force of gravity were considered. The plate and sloshing modes were analyzed simultaneously. The coupled problem is solve during a coupled BEM and FEM. The tanks structure is modeled by FEM and the liquid sloshing In the fluid domain is described by BEM. The method relies on determining the fluid pressure from the system of singular integral equations. It numerical solution, the boundary element method was applied. The boundary of the liquid computational domain is discretized by nine-node boundary elements. Greens function was used to find pressure hydrodynamic ,wave sloshing base moment and shear base of tanks. the results of FEM and BEM were compered and the agreement ratio between them was very good.

Introduction:

Liquid sloshing, displaying a free-surface fluctuation in partially filled containers under external excitations, is a known physical phenomenon existing in a variety of engineering applications such as liquid oscillation in large storage tank by earthquake, the motion of liquid fuel in aircraft and spacecraft, the liquid motion in tank, and the water flow on the deck of ship. Such liquid motions may lead to the unexpected instability and failure of engineering structural system. Especially when the excitation stroke is large or the excitation frequency is close to the natural frequency of the container, the caused damage may be a tremendous loss of human, economic, and environmental resources. Thus, it is still urged the need for further research on the understanding of the complex sloshing behavior and the technique for sloshing suppression. Numerous studies related to liquid sloshing have been carried out numerically, theoretically, and experimentally in the past several decades. For example, Faltinsen [1] derived the linear analytical solution of a horizontally excitation tank in 2D. Frandsen [2] obtained the linear and second-order analytical solutions of a fixed tank with initial free surface being cosine curve. Faltinsen et al. [3] performed sloshing experiments in a square base tank. Akyildiz and Erdem U˘nal [4] made sloshing experiments in a moving tank with different baffle arrangements. Okamoto and Kawahara [5] performed the experiments of liquid sloshing in rectangular and hexagonal tanks. Faltinsen [6] and Nakayama and Washizu [7] adopted the boundary element method to model large amplitude sloshing in a 2D rectangular container subjected to a horizontal excitation. Wu et al. [8] gave an account of both 2D and 3D sloshing problems based on finite element method. Frandsen and Borthwick [9] simulated the sloshing motion in fixed and vertically excited containers using a finite difference solver. Wang and Khoo [10] and Sriram et al. [11] numerically simulated 2D sloshing waves due to random excitation. Eswaran et al. [12] numerically and experimentally studied the sloshing motions in baffled and unbaffled tank subjected to horizontal excitation. Cho et al.

[13] investigated the resonant sloshing response in 2D baffled tank by using FEM. Biswal et al. [14] used FEM to compute the nonlinear sloshing response of liquid in a rigid rectangular and cylindrical tank with horizontal baffle. Panigrahy et al. [15] experimentally researched hydrodynamic pressure developed on the tank with horizontal and vertical baffles subjected to horizontal excitation. Liu and Lin [16] adopted VOF method to simulate liquid sloshing in 3D rectangular baffled tank. Belakroum et al. [17] studied the vibratory behaviors of three different configurations of tanks equipped with baffles to predict the damping effect of baffles on sloshing in tank. In addition, Ortiz et al. [18] founded a powerful closed-form model to study the coupling of nonlinear liquid sloshing and the container motion, in which the fluid is modeled using a 2D BEM based on potential flow theory with modified Rayleigh damping, the container boundary can be arbitrarily curved, and the container motion can be induced by a flexible multi body system. As an extension of the previous studies, the liquid sloshing in a 2D tank with and without a vertical baffle, subjected to the coupled horizontal and vertical excitations, are numerically investigated by a fully nonlinear higher-order boundary element method. This paper is organized as follows. The mathematical formulation for rectangular tank by boundary element (Greens function) is firstly given in Section 1. Then, in Section 2, the present numerical results including wave surface and hydrodynamic pressure by BEM are compared with FEM results.

Mathematical Formulation

Referring to Figure 1, liquid sloshing in a rectangular tank with length L and water depth H is considered in the present study. Two Cartesian coordinate systems, including a space-fixed

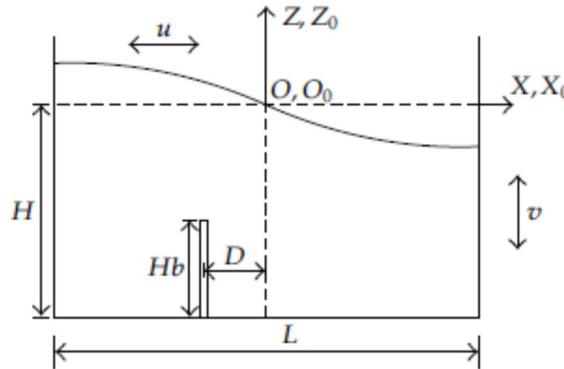


Figure 1: Definition sketch.

coordinate $O_0X_0Z_0$ and a tank-fixed coordinate OXZ , are defined with Z and Z_0 positive upwards. The two systems coincide with each other when the tank is at rest. The displacement of tank is defined as $X_b(t)$, $z_b(t)$ in coordinate $O_0X_0Z_0$. Under the assumptions that fluid motion is irrotational and fluid is incompressible and inviscid, the fluid motion is, therefore, governed by Laplace equation:

where ϕ is the velocity potential. The following condition is satisfied on the interface of tank and liquid:

$$\nabla^2 \phi = 0, \tag{2.1}$$

where $U \frac{dX_b}{dt}$ is the velocity of the tank and n is the outward vector normal to the tank walls. On the instantaneous free surface $z_0 = \eta_0(x_0, t)$, both the fully nonlinear dynamic and kinematic boundary conditions are satisfied:

$$\frac{\partial \phi}{\partial n} = U \cdot n, \tag{2.2}$$

where η_0 is free surface and g is gravity acceleration. By using the following 2.4 and 2.5, the 2.3 can be transformed into 2.6 and 2.7 in the moving system OXZ :

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + g \eta_0 &= 0, \\ \frac{\partial \eta_0}{\partial t} + \frac{\partial \phi}{\partial x_0} \frac{\partial \eta_0}{\partial x_0} - \frac{\partial \phi}{\partial z_0} &= 0, \end{aligned} \tag{2.3}$$

where $\eta = \eta_0 - z_b$ is the free surface in coordinate system OXZ for fixed x . A semi-mixed Eulerian- Lagrange an technique is adopted to change the potential with time on the free surface as follows:

$$\nabla_{x_0 z_0} = \nabla_{xz}, \tag{2.4}$$

$$\left(\frac{\partial}{\partial t} \right)_{x_0 z_0} = \left(\frac{\partial}{\partial t} - \frac{dX_b}{dt} \cdot \nabla \right)_{xz}, \tag{2.5}$$

$$\frac{\partial \phi}{\partial t} - \nabla \phi \cdot \frac{dX_b}{dt} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + g(\eta + z_b) = 0, \tag{2.6}$$

$$\frac{\partial \eta}{\partial t} + \left(\frac{\partial \phi}{\partial x} - \frac{dx_b}{dt} \right) \frac{\partial \eta}{\partial x} - \left(\frac{\partial \phi}{\partial z} - \frac{dz_b}{dt} \right) = 0, \tag{2.7}$$

$$\frac{\delta\phi[x, \eta(x, t), t]}{\delta t} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial z} \frac{\partial\eta}{\partial t}. \tag{2.8}$$

φ

$$\frac{\delta\phi}{\delta t} + \frac{\partial\phi}{\partial z} \frac{\partial\eta}{\partial t} - \nabla\phi \cdot \frac{dX_b}{dt} + \frac{1}{2} \nabla\phi \cdot \nabla\phi + g(\eta + z_b) = 0. \tag{2.9}$$

$$\nabla^2\varphi = 0,$$

$$\frac{\partial\varphi}{\partial n} = 0,$$

$$\frac{\partial\eta}{\partial t} = -\frac{\partial\varphi}{\partial x} \frac{\partial\eta}{\partial x} + \frac{\partial\varphi}{\partial z}, \tag{2.11}$$

$$\frac{\delta\varphi}{\delta t} = \frac{\partial\varphi}{\partial z} \frac{\partial\eta}{\partial t} - \frac{1}{2} \nabla\varphi \cdot \nabla\varphi - g\eta - x \frac{du}{dt} - \eta \frac{dv}{dt}.$$

Thus dynamic boundary condition is written as:

$$\varphi(x, z, 0) = -xu(0) - zv(0). \tag{2.12}$$

The initial free surface $\eta_-, x, 0_-$ is fixed according to practical simulation. In addition, hydrodynamic pressure acting on tank can be obtained by Bernoulli equation in space-fixed coordinate OX0Z0:

$$P = -\rho \left(\frac{\partial\varphi}{\partial t} + \frac{1}{2} \nabla\varphi \cdot \nabla\varphi + g\eta + x \frac{\partial u}{\partial t} + z \frac{\partial v}{\partial t} \right). \tag{2.14}$$

$$P = -\rho \left(\frac{\partial\phi}{\partial t} + \frac{1}{2} \nabla\phi \cdot \nabla\phi + g\eta \right). \tag{2.13}$$

Then, the force acting on tank can be obtained by integration and the time derivative of velocity potential, that is, first term in _2.14_, is obtained by the acceleration potential method. The direct boundary integral equation is derived to solve prescribed boundary value problem by using the second Green’s theorem [19]. Applying the Greens function

satisfying the impermeable condition on the sea bed, integral equation can be written as where p and q are source and field points, and C is the solid angle which can be conveniently and economically computed by the indirect method in the present study [20]. Γ is liquid domain boundary including free surface boundary and solid boundary and G is a simple Green function and is written as follows:

$$C(p)\varphi(p) = \int_{\Gamma} \left(\varphi(p) \frac{\partial G(p, q)}{\partial n} - G(p, q) \frac{\partial \varphi(q)}{\partial n} d\Gamma \right), \quad (2.15)$$

$$G(p, q) = \frac{1}{2\pi} (\ln r_1 + \ln r_2), \quad (2.16)$$

Then the boundary surface is discretized with a number of three-node line elements. The geometry of each element is represented by the quadratic shape functions, thus the entire curved boundary can be approximated by a number of higher-order elements. Within the boundary elements, physical variables are also interpolated by the same shape function that is, the elements are Iso parametric. Then the discretized integral equation can be written as linear equation systems:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \varphi|_{\text{on solid surfaces}} \\ \frac{\partial \varphi}{\partial n}|_{\text{on free surface}} \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix}. \quad (2.17)$$

Numerical Results:

The numerical solution was obtained by using BEM as It was described beforehand. In the present numerical simulation we used 60 boundary elements along the bottom,60 along the width of the free surface. Fig. 3 shows the first three modes of liquid sloshing at the free surface in the rigid rectangular By using FEM. The procedure is then repeated at each time step until the analysis is complete. Fig. 2 shows in a flowchart format the procedure for producer to BEM and shows how the data is transferred between rectangular tank wall and fluid.

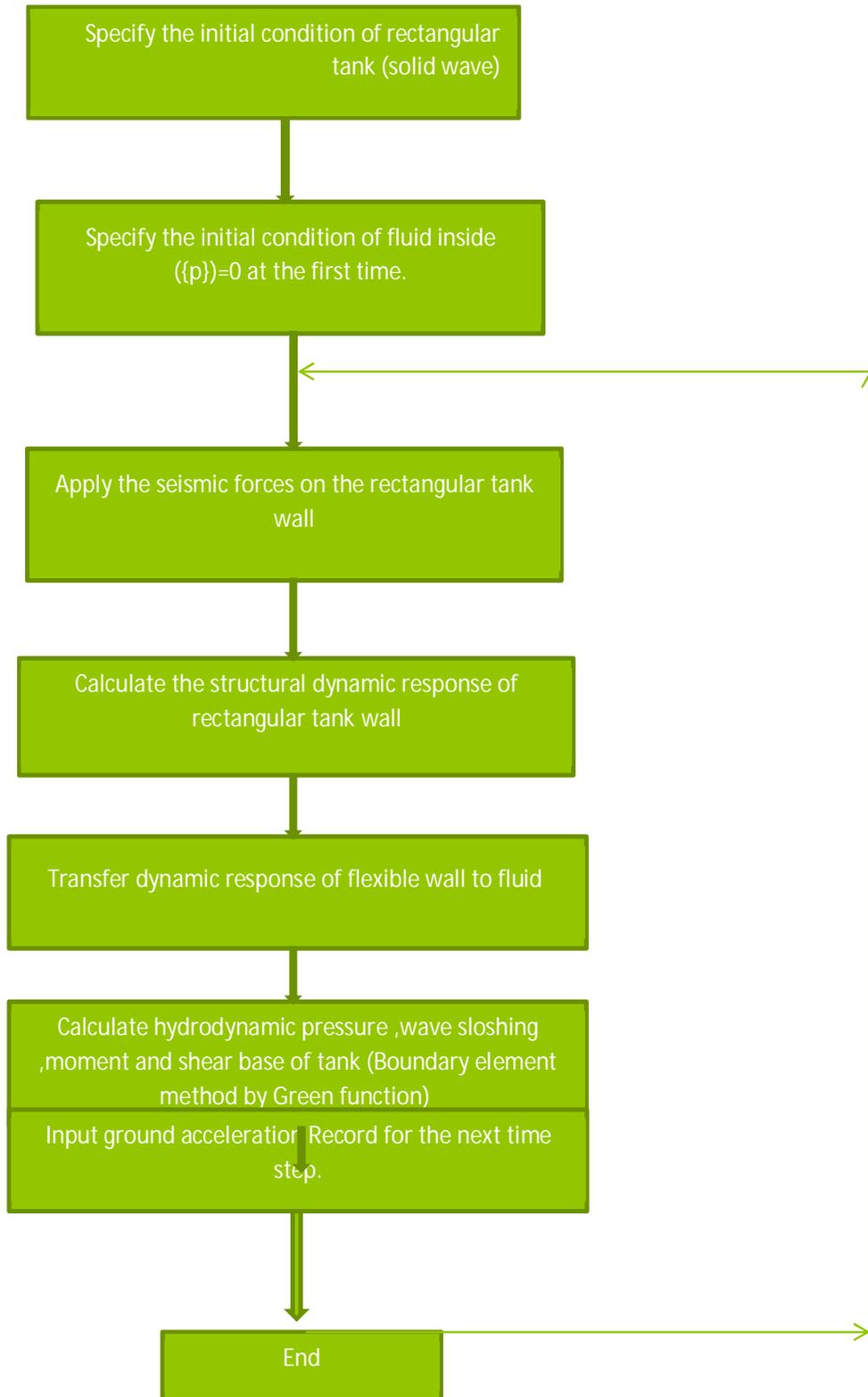


Fig.2 Procedure of Numerical solution by BEM

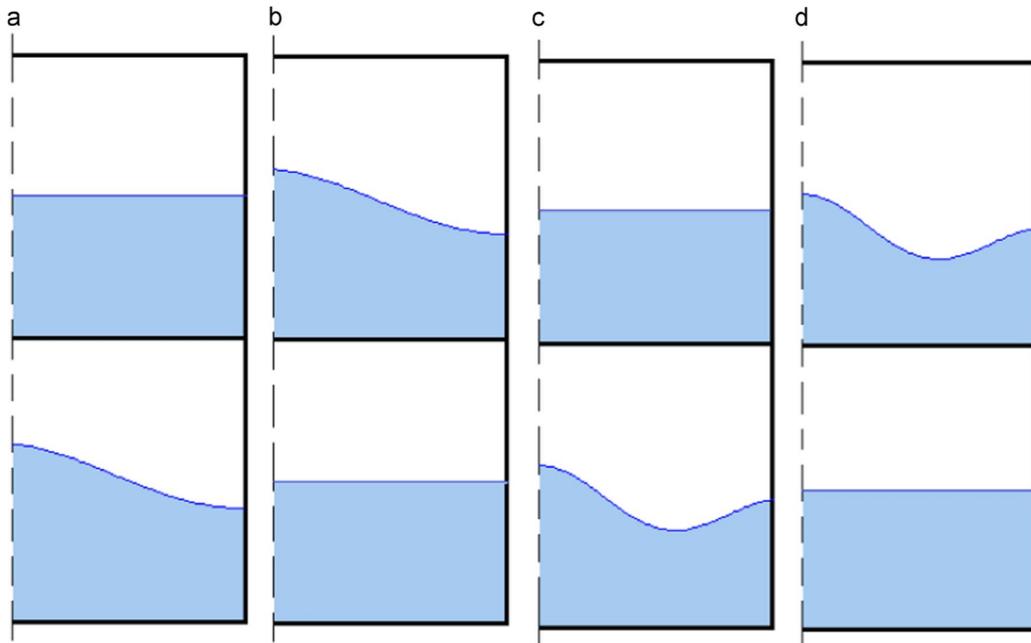


Fig.3 modes of liquid sloshing at the free surface in the rigid rectangular By using FEM.

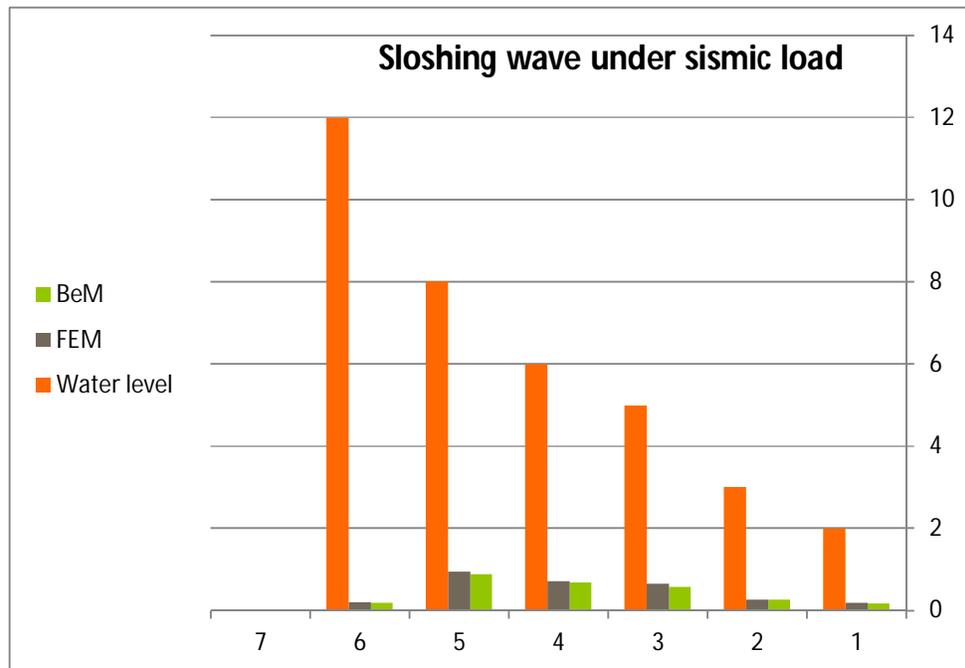


Fig.4 Sloshing wave under seismic load by using BEM and FEM.

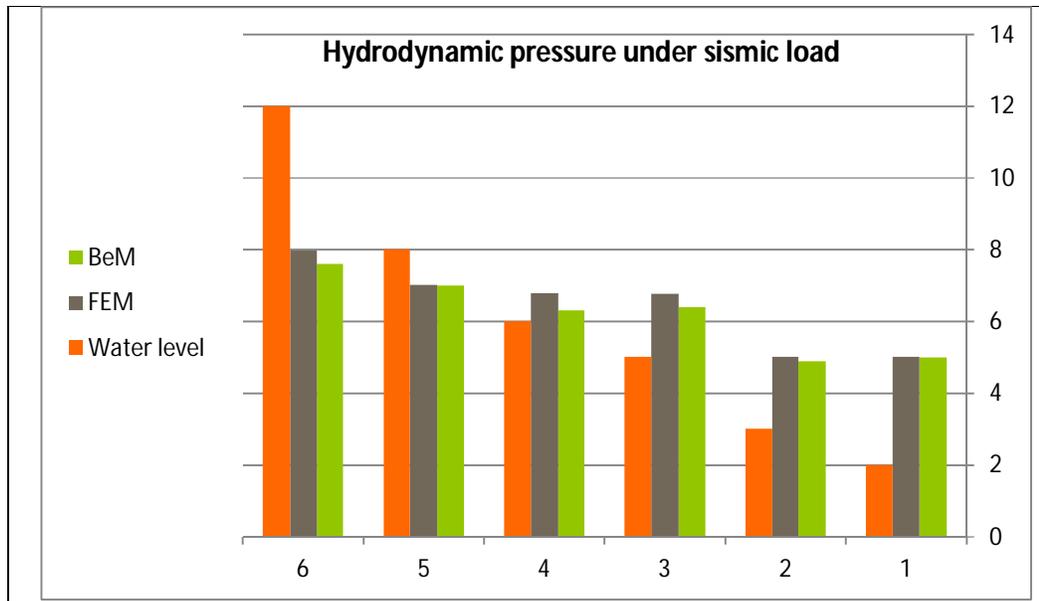


Fig.5 Hydrodynamic pressure under seismic load by using BEM and FEM.

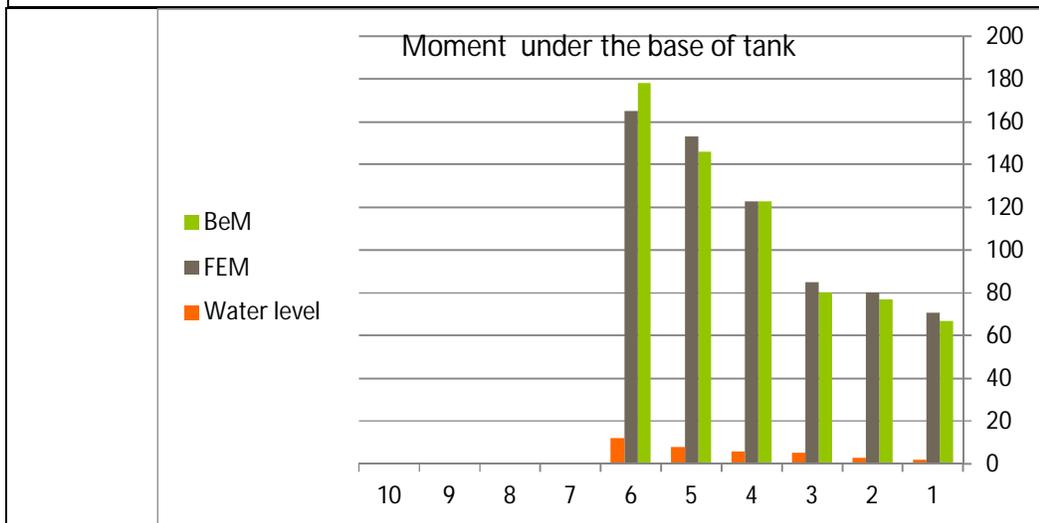


Fig.6 Moment under the base of tank by using BEM and FEM.

Conclusion:

The numerical procedure based on the coupling finite element formulation and boundary element method is developed for numerical analysis of fluid–structure interaction for rectangular tank. We introduce their presentation of the velocity potential as the sum of two potentials, one of them corresponds to problem of the fluid seismic load in the rigid plate and another one corresponds to problem of elastic plate with fluid without including the gravitational component. Integration(Greens function) by the fluid volume is accomplished using BEM based fluid flow solver. The spectrum of re- quences for double tank was analyzed.

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