IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 4 Issue 6, June 2017
ISSN (Online) 2348 – 7968 | Impact Factor (2016) – 5.264
www.ijiset.com

Convergent and Divergent Thinking-Based Instructional Strategies to Students' Achievement in Trigonometry at Advanced Level Mathematics

Tsododo Misheck.
Mutare Teachers' College, Department of Mathematics, Science and Technology, P. O. Box, 3293 Paulington, Mutare , Zimbabwe.
Corresponding Author
E-mail: mx3tsododo@hotmail.com , tsododom@mutaretechers.ac.zw and mictsod2@gmail.com
Cell: +263 777 438209

Mutoto Cosmas.
Pamushana High School, Department of Mathematics, P. Bag 514,
Nyika, Zimbabwe.
E-mail: mutotocosmas@yahoo.com

Teaching and learning are complicated processes which are influenced by many different factors, of which one of the most fundamental is the approach adopted to present and teach content. This study seeks for the most effective and efficient instructional approach (divergent or convergent based) which leads to the improvement of achievements of students learning trigonometry at Advanced Level mathematics. Three groups of 10 members from 60 students at Pamushana High School participated in this study. They took part in a pre-test and a post-tests on trigonometry from June 2014 to April 2015. The data of the study was analyzed using the ANOVA, t-Test and Spearman's Rank correlation method. Data(scores) obtained from the post-tests by convergent group and divergent group significantly supported what had been hypothesized at the outset of the study, that divergent based instructional approach proved more effective than the convergent based instructional approach in improving the students' understanding of trigonometric concepts.

Keywords: *Trigonometry, Correlation, Conceptualiser, Instructional strategies, Convergent thinking, Divergent thinking*

1. Introduction

The teaching/learning process is becoming more and more complex. Instructors at different learning institutes are facing difficulties in solving the puzzle on the selection of the most suitable instructional strategies for a particular learning session. However at their disposal are research findings which have always dwelt too much on instructional methods such as discovery method, lecture methods and heuristic methods. Not much research has been done concerning other abstract instructional strategies such as the divergent or convergent-based instructional strategies. This research is looking at instructional strategies which are either convergent or divergent based and their effects on the achievement of the objectives of the learning process particularly in trigonometry at Advanced Level mathematics.

One of the critical factors that impact the teaching and learning process is how to design effective instruction that can address diverse learning styles and academic backgrounds. Learning is evidenced not only in changes in behaviour but also changes in cognitive processes. Effective student learning occurs as a result of effective teaching strategies, as well as teacher knowledge of the subject matter. Additionally, a teacher's clarity, stimulation of interests, and openness to opinions are other significant factors to manage learning efficiency and effectiveness (Lyons et al 2003). Underlying all of this is the importance of the teacher's critical understanding of learning theories and how to apply them to the cognitive, motivational, and psychological learning process associated with academic success. Any productive teaching methods or activities are governed by learning theories, a distinct area in curriculum development and educational practice. In order to develop effective lesson plans to bring about the attainment of desired objectives, Brown and Green (2006 p.46) said that, teachers "must possess a variety of skills and have a solid understanding of different concepts, ideas, and theories."

In open and flexible learning contexts, instructional materials have the capacity to cater for individual needs while enabling collaborative forms of learning. At the outset, when designing materials for a given group of learners, instructional designers typically carry out a needs analysis or profile of the learners in order to ascertain the prior knowledge, motives, background interests, attitudes and experiences of learners. The rationale for such investigations is that individual dispositions will somehow affect and often influence learners' readiness to gain from the instruction that is offered, and influence academic progress. Instructional designers customarily acknowledge individual differences in their designs and often plan to adapt instruction to the needs of individual learners. Instructional practice and information gained from such needs analysis enables the design of learning resources tailored closely to the needs of the learners.

For instructional designers, an often neglected source of information on individual differences is the growing body of research on learning styles and strategies, which explains how individuals learn, process new knowledge and represent information. It is suggested that current research literature in the area of learning styles and strategies can provide instructional designers with insights into individual differences in learning and performance that can be factored into the design process. This study sought to add empirical research to this field of study by examining the effects of divergent based instructional strategies and convergent based instructional strategies on problem-solving performance in trigonometry and Advanced Level.

1.1 Background to the study

In a second language context, ways in which critical thinking might be interpreted and taught have become highly debated concerns for second language learning scholars and practitioners in recent years (Thompson, 2008).Convergent and divergent teaching methods are derived from these ways of education. Convergent thinking is a term proposed by Guilford in 1967 as the opposite of divergent thinking. Convergent thinking is the process of finding a single best solution to a problem that we are trying to solve (Williams, 2003). Many tests that are used in schools, such as multiple-choice tests, spelling tests, math quizzes, and standardized tests, are measures of convergent thinking.

Divergent thinking is the process to create several unique solutions intending to solve a problem. The process of divergent thinking is spontaneous and free-flowing, unlike convergent thinking, which is systematic and logical. When using convergent thinking, we use logical steps in order to choose the single best solution. By using divergent thinking, instead of only choosing among appointed options, we search for new options. Convergent thinking stands firmly on logic and less on creativity, while divergent thinking is mostly based on creativity. We use divergent thinking mostly in open-ended problems to which creativity is a fundamental part (Williams, 2003).

1.1.1 Student Challenges in Trigonometry at Pamushana High

In an endeavour to reveal the background to the study, the researcher highlighted the main challenges which are currently being faced by students at Pamushana High in trigonometry. Fig 1.1 shows some of the challenges. The researcher became aware of such challenges during the attachment teaching practise from May to June 2014. Trigonometry was one of the topics taught during that time.

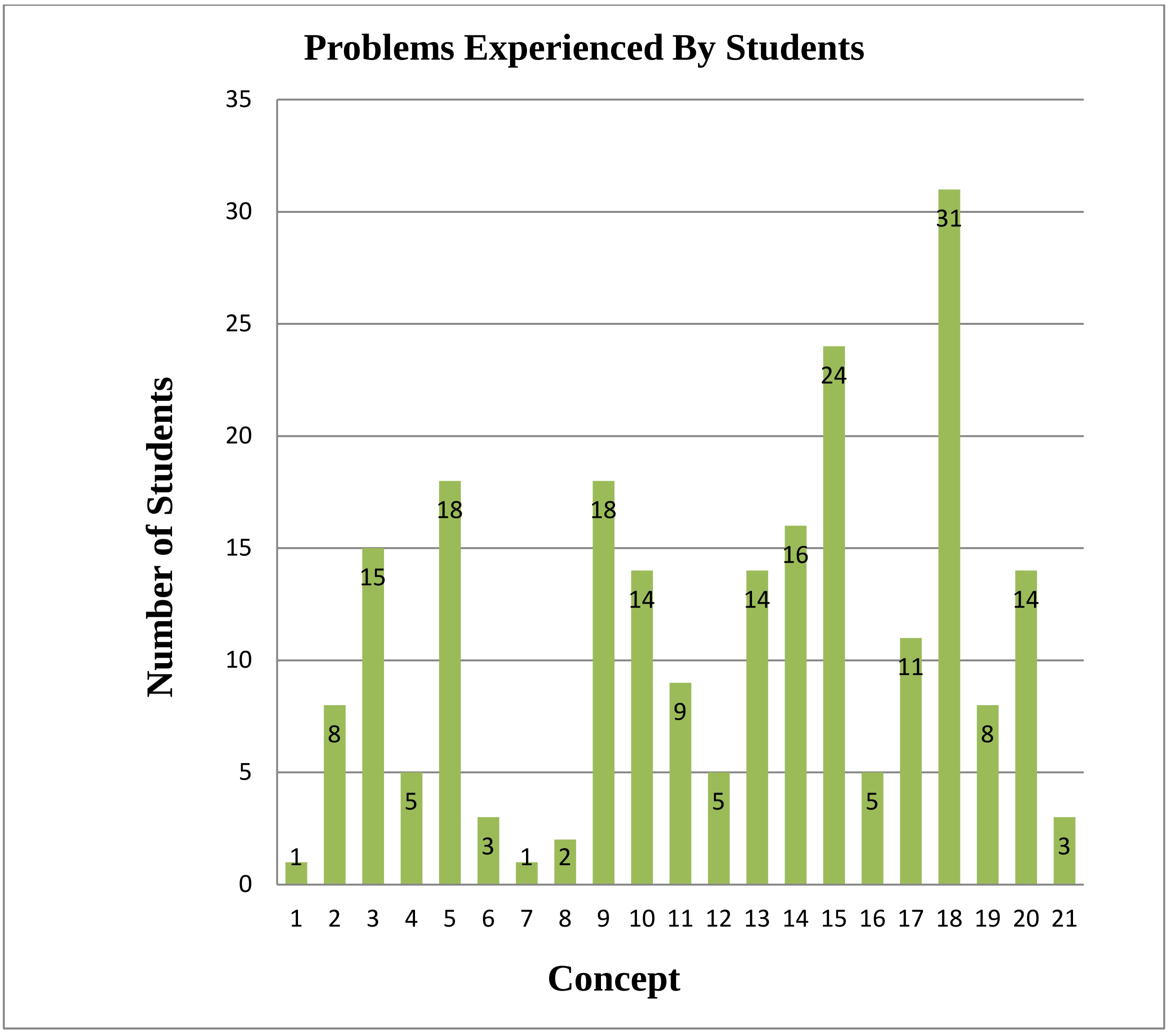

Fig 1.1: Number of Students Experiencing Problems in Corresponding Trigonometric Concepts

Key to Fig 1.1

Concept Number	Concept Description
1	Apply properties to find missing sides and angles
2	Definitions for trigonometric functions to find reciprocal identities

3	Identify a quadrant of an angle
4	Apply Pythagorean identities
5	Apply reference angles to non acute angles
6	Calculate radian measure
7	Find area of a sector
8	Find perimeter of a sector
9	Express shaded area in terms of sine or cosine or multiples of such functions
10	Recognize periodic functions
11	Analyze sine, cosine and tangent functions
12	Graph sine , cosine and tangent functions
13	Apply transformations of sine, cosine and tangent functions
14	Graph secant, cosecant, and cotangent functions
15	Apply transformations of secant, cosecant and cotangent functions
16	Solving trigonometric equations
17	Proving trigonometric identities
18	Maximum and minimum values of complex trigonometric functions
19	Differentiation of trigonometric functions
20	Integration of trigonometric functions
21	Convert between degrees and radians

The following is an elaboration of some of the problems experienced in trigonometry by students at Pamushana High school which was gained from the end of topic (Trigonometry) test, students' response to questionnaire (fig 1.1) and teachers' response to questionnaire.

1) Application of Reference Angle To Non Acute Angles

With reference to the test dossier constructed for the end of topic (trigonometry) test, 12 out of 60 pupils got the concept correct [write down tan227° as a trigonometric ratio of an acute angle?] 48 pupils did not even attempt it. The question has a difficult index of 0.2 which implies that it was one of the difficult questions in that exam. This concept also was one of those questions classified under bad discriminators since its discrimination index was 0.13. The concept is also linked with the concept of generating other possible solutions of an equation after getting the principal value from the calculator. Same students indicated that they confuse on the proper signs and correct quadrants in which angles must be reflected during the process of generating other solutions within a specific range. Analysis (bar graph above) of the student questioner shows that 18 out of 35 students have problems with the concept .The possible course of this problem might be the instructional methods used to deliver it. Hence the need to consider teaching the same concept, both by using convergent or divergent based instructional strategies and analyse their bearing to student acquisition of the concept.

2) Expressing Shaded Area In Terms Of Sine Or Cosine Functions

From the topic test question 3 requires students to show that the ratio of the shaded area to the area of triangle is ($4\pi\sqrt{3}$ -9): 9. Only 19 out of 60 pupils got it correct with only 25 attempting it, the rest did not attempt it. From the test dossier, its difficult index was 0.32 a relatively low index indicating that it was a fairly difficult question. The discrimination index was 0.37 which makes it a good discriminator. From the student questionnaire, 18 out of 35 students indicated that they face challenges on this concept. Some students pointed out that such question is asked with a variety of diagram same of which they cannot even recognise dimensions attached to the required shaded area. This concept is made difficult since it requires other concepts like area of sector (A=$\frac{1}{2}r^2\theta$) ; and area of segment and area of triangle (A=$\frac{1}{2}ab\sin c$) which are also problem areas to same students. These problems may result from rote learning. Thus the need to consider applying either convergent or divergent based instructional strategies.

3) Sketching Graphs Of Sine, Cosine, Tangent, Secant, Cosent And cotangent Function And Their Transformation

With reference to the end of topic test, pupils were required to describe transformation which the graph of y=sin x is mapped to graph of (a) y=sin(x + 90)° and (b) y=sin($\frac{x}{2}$)° and to sketch the graphs. Such questions are most difficult to pupils since they are awarded **B** marks and those **B** marks are only given for a correct result or statement independent of the method mark .Some pupils argued that they did not even know the proper co-ordinates to plot on the x-axis and y-axis for a given question, for example if required to sketch y=3$\sqrt{2}$ ($\cos x + \sin x$) in the interval 0 $\leq x \leq$ 360. Pupils could not realise that y should be expressed in the form Rcos($x-\theta$); R> 1 and $0 < \theta < 90$, so without realising this, many students will not attempt the question. On the same note students noted that they are not familiar with locating asymptotes for inverse functions before transformation($y = \frac{1}{\cos x}$) and after transformations (y=$\frac{1}{2\cos x}$). Many students said that the fact transformations of the form $y = f(x) + a$ and $y = f(x + a)$ confused them while those of the form y=f($\frac{x}{a}$) and y= $af(x)$ also confused them. Students also noted that they did not understand the symbols used for these transformations let alone how to distinguish them. In the test dossier number 4i (b) of describing transformations, the difficult index was 0.2 a low index indicating that it was a difficult item. On the same note its discrimination index was 0.13 indicating that it was a bad discriminator. From the bar graph above, 24 out of 35 students indicated the concept as difficult. Teachers also commented that students face challenges in this concept since they are not even good at drawing and transforming graphs of polynomials. What more trigonometric of functions? However, this concept should be taught with alternative instructional methods. Hence, the need to carry out this research

4) Finding The Maximum And Minimum Values Of Complex Trig Functions

This concept proved to be the most difficult in trigonometry as shown from the teachers' questionnaire. Most teachers mentioned it under concepts that paused problems to students. As evidenced from the test, students were required to find the greatest and least values of $\frac{39}{f(x)+14}$ and state the value of x at which they occur where $f(x) = (\cos x - \sin x)(17\cos x - 7\sin x)$; only 2 out of 60 pupils got it correct and 55 did not even attempt the question. Hence this question has the lowest difficult index (0.03) which indicated that it was the most difficult item. From the dossier, the same item had the smallest discrimination index (0.07) which showed that it was a bad discriminator. Students argued that since transformation of graphs is a pre-requisite of this concept and because they have challenges in transformation thus finding maximum or minimum values of trigonometric function becomes even more challenging. Hence, the need to look at other instructional strategies like divergent and convergent approach and their bearing to student achievement .From the bar graph above, 31 out of 35 students had difficulties with the concept.

5) Proving Trigonometric Identities

From student bar graph above 11 out of 35 students showed that they faced challenges in proving trigonometric identities. They indicated that proving an identity was a laborious task and others claimed that they were not aware of where and when to use those trigonometric identities in formula booklets. Some even had a feeling that those formula booklets must be re-written to include other useful identities which were currently not found in the booklets. Students condoned the fact that teachers only give them the identities without showing them how to derive them. To students, the identities are given as the "bible truth" which they cannot question or argue, about, thus they have a feeling that they don't use and apply them as "theirs" but apply them as something imposed on them. That had a negative impact on the learning process since students should be part of the process not observers. With reference to the test dossier, more than half of the pupils who wrote the test did not attempt the concept of proving that $\frac{\cos x}{1-\sin x} \equiv \sec x + \tan x$ and $\sec 2\theta + \tan 2\theta \equiv \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$. This concept has a tendency to penalise students in terms of time in the exam since their first attempt always leads them to hit a wall and they will consider another way which if unfortunate will hit another wall till they use the proper procedure which reduces either the right hand side to be equal to the left hand side or vice- versa. Such concepts which tend to have many ways of reaching the solutions have necessitated the need to carry out this project which looks at divergent and convergent instructional strategies and their impact in knowledge acquisition.

The lack of knowledge and skills in trigonometry also has a negative bearing in topics such as differentiation of trigonometric functions, integration of even or odd powers of sine, cosine or tangent functions, the use of the t-formula, application of deMoivre's theorem, angle between two vectors, resolution of forces and trajectory of a projectile to mention just a few. It is not surprising that many of the maths students at Pamushana due to the fact that they have challenges in trigonometry , they also fail to prove one of the least difficult standard integral, that is to show that $\int \tan x dx = \ln|secx| + c$. This problem emanates from poor manipulation with trigonometric identities. Trigonometric identities such as $cos^2x=\frac{1}{2}(1+\cos 2x)$ are required in the integration of even powers of trigonometric functions while $cos^2x+sin^2x = 1$ is used for integrating odd powers of trigonometric functions. In the derivation of the t-formula for $\cos x$, the identity $cos^2\frac{x}{2}= \frac{1+\cos x}{2}$ is used. The deMoivre's theorem makes use of trigonometric identities such as $\sin(x+y) = \sin x \cos y + \cos x \sin y$ during derivation of the theorem $z_1 z_2 = r_1 r_2[\cos(x+y) + i \sin(x+y)]$. when calculating angle between vectors the formula $\vec{a}.\vec{b}=|\vec{a}||\vec{b}| \cos x$ is used. In describing the motion of a projectile, formulas like $y = x \tan \theta - \frac{gx^2}{2v^2 cos^2 \theta}$ and $R = \frac{v^2 \sin 2\theta}{g}$ are used as the equation of the trajectory and horizontal range respectively. Students with misunderstandings in trigonometry will perform poor in such concepts. Since it has been noted that failure in trigonometry also affect negatively the performance of students in other topics, there is need to carry out the research specifically on the teaching of trigonometry aiming at finding if divergent or convergent based instructional strategies will improve the level of achievement of students. Thus, finding the best way of teaching trigonometry does not only improve students' performance in trigonometry but also in other topics which require trig concepts as pre-requisites and this will consequently improve the pass rate of 'A' level maths students.

Trigonometry is not only a problem at Pamushana but is also a national problem as evident from the 2004 advanced level maths examinations reports below:

Question 4 (Paper 1)

(i) Sketch and label, on the same diagram, the graphs of $y = 2sinx$ and $y = cos2x$, for the interval $0 \leq x \leq \pi$ [4]

(ii) Hence state the number of solutions of the equation $2 \, sin \, x = cos2x$ in the interval $0 \leq x \leq \pi$ [1]

Comments

(i) Although there were some excellent solutions to the question, candidates generally showed lack of ability in sketching trigonometric graphs. Many automatically sketched graphs in the range 0 to 2π instead of 0 to π, though this lost time rather than marks. Many others preferred to draw accurate graphs, and again this was penalised only in time. Many weaker candidates failed completely to recognise the difference between the sketches of $y = 2sinx$ and $y = sin2x$ and similarly between $y = 2cosx \; and \; y = cos2x$.

(ii) The majority of candidates failed to realise the link between this part of the question and the sketches drawn in part (i). Many ignored the word 'hence' and attempted to solve the equations by various methods. The majority of such candidates gave the solutions to the equation rather than the number of solutions, thereby gaining no credit. The failure to recognise that the number of solutions was the same as the number of intersections of the two graphs was surprising.

From the above it is clear that pupils nationally are taught through rote learning and drilling because how can you draw graphs in a range not required by the examiner? Also, the teaching methods which are being used do not foster originality and flexibility since students fail to read the number of times in which the graphs intercept. Hence the need to carry out this project

Question 6 (Paper 1**)**

The function $f: x = 5 \, sin^2 \, x + 3cos^2 \, x$ is defined for the domain $0 \leq x \leq \pi$.

(i) Express $f(x)$ in the form $a + b\,sin^2\,x,$ stating the values of a and b [2]

(ii) Hence find the values of x for which $f(x) = 7sinx.$ [3]

(iii) State the range of $f.$ [2]

Comments

(i) This was well answered with most candidates realising the need to replace cos^2 x by $1 - sin^2$x.

(ii) Virtually all candidates realised the need to use part (i) to obtain a quadratic equation for $sinx$. There were however errors in factorising the quadratic and the more serious error of solving $7sinx - 2sin^2x = 3$ as $sinx = 3\ or\ 7 - 2sinx = 3$ was often seen from weaker candidates Obtaining answers to $sin^{-1}(\frac{1}{2})$ in radians presented difficulty with a significant number of attempts being left in degrees.

(iii) This proved to be the most difficult question on the paper with only a handful of candidates realising that the minimum value of f occurred when sin^2x = 0 and the maximum occurred when sin^2x = 1.

From the above, it is evident that pupils who are poor in solving quadratic equations also face challenges in finding solutions of trig equation. Teachers should ensure that pupils understand all pre-requisite concepts before they teach new concepts. The idea of finding minimum value of a function was another challenge. So, how best such a concept can be taught is one of the major aims of doing this project.

It is evident that trigonometry is one of the difficult topics in the syllabus, and is also linked to trigonometry like, the iterative methods of finding roots of equations, differentiation and integrations to mention but a few. Thus, there is need to find the best instructional strategies to handle concepts in this topic. Are they divergent or convergent?

1.1.2 Instructional Strategies Used To Teach Trigonometry at Pamushana

The syllabus is silent about instructional strategies to be used by teachers. However, from the responses given by students, indicates that teachers should use computer simulation methods especially on graphs so that they can have an audio visual aspect of transformations required from original graphs to the transformed one. This is a clear indicator that teachers are using the traditional teaching styles of resorting to the chalkboard and talk. There is evidence that teachers have not yet embraced technology in their instructional methods and techniques.

From the teachers’ response, most of their lessons are conducted using the lecture method and discovery method. This is also supported by students who indicated that teachers should attend all lessons and not appoint some students to do presentations for this topic especially when introducing a new concept. Students also indicate the need for teachers to use guided discovery methods in which they lead them in finding solutions to questions and show them how to drive the trigonometric identities in the formula booklet. However there is conflict of interest in this issue as teachers know that question do not test how to derive identities but how to apply identities. So, teachers wonder why we should teach derivation of such identities. Some teachers indicate the need to finish their syllabus in time to give ample time for revision as one of the reasons why they do not bother teaching derivation of identities. However, teachers should give vast amounts of work which requires application of identities and show pupils how to use the formulas.

Teachers lack pre-requisite concept of trigonometry and they do not marry the concepts in trig to related topics. This was highlighted by some students who argued that when teachers deliver lessons, they must give a close reference of trigonometric concepts to related topic like functions and should give relevant examples and show how trigonometry is applied to solve everyday life problems. This would enable students to appreciate the topic. As it stands, the instructional methods used do not lead to appreciation of the topic as it is being taught divorced from the real world.

From the students' response, teachers are not using interactive methods such as group work and pair work. They argued that teachers should put students in groups. Students said this topic should be given more time and teachers must be patient with student as it involves a lot of memorisation. Students also argued that teachers should give them more homework, and they should make syllabuses available.

The analysis of teachers' questionnaire seems to indicate that teachers are not making reference to the syllabus especially on assessment objectives, specification grid and syllabus aims. It can be noted that since teachers are not showing how identities are derived, they are not fulfilling assessment objective (c) of evaluating mathematical models, including an appreciation of the assumptions made, and interpret, justify and present the results from a mathematical analysis in a form relevant to the original problem.

With regards to the specification grid, teachers are not using Bloom's taxonomy when preparing test items. It is surprising to see that most of the midyear exams have trigonometric concepts which only require knowledge, comprehension, understanding and application. Trigonometric concepts which require higher order objectives such as analysis, synthesis and evaluation, thus depriving students practise such skills. Consequences are normally seen when pupils write public exams which are normally standardised. Teachers at Pamushana in the Mathematics department have considered the following as measures in trying to eliminate the problem

1.1.3 Invention to Challenges in Trigonometry by Teachers at Pamushana

Teachers in the department held workshops led by the National Association of Secondary Heads (NASH) on subject panels at Better Schools Programmes in Zimbabwe (BSPZ) Bikita. On the last held workshop in June 2014, the following was the agenda:

1) Strategies put in place by teachers to make pupils pass exams.
2) Grey areas noted by markers when marking various papers and way forward on those areas.
3) Hints or exam techniques.
4) Study techniques.
5) Motivation of students.
6) Exam preparation.
7) Mid-year exam item writers.

Mr Matenga (Pamushana High) and Mr Masasire (Silvera High) core presented the workshop and they highlighted the following:

Syllabus coverage-Teachers were urged to complete the syllabus as early as first term. This will ensure that ample time is given to the revision process. However, teachers urged that with the calibre of students they recruit it's impossible (same of them have Cs at 'O' level as indicated by student questionnaire).

Identify problem topics and cover all curriculum objectives for each topic- This is ensured by consulting different textbooks and considering a variety of past exam materials.

Content sequencing-They highlighted that order of topics in the syllabus is not the correct order. For example, graphs and coordinate geometry should be taught after functions or differentiation should be taught after trigonometry.

Work presentation-Teachers should encourage students to show their working neatly and logically and to show all working. For instance, when solving trig equations, if students get $\sin x = 3$, they should comment that the solution is invalid and should be discarded. Failure to mention this leads to loss of method marks.

Degree of accuracy-Teachers should encourage pupils to always refer to the instructions with regards the degree of accuracy on the cover page if the question does not specify the degree of accuracy. Students normally underestimate solution. In instances when they are required to leave answers in exact form they don't realise that the answer should be in surd form, in terms of π .

Attempting all sections in the exam-They urged students to attempt all questions in an exam. The presenters gave an example of pupils doing paper 4. They cited that they don't attempt questions in the mechanics section. Same students even don't attempt trig questions in paper 1.

Team teaching-Teachers were encouraged to have exchange programmes where-by they share topics to teach depending on aptitude and competence. This however normally fails since there is competition in schools as to who produces best results.

Marking-Teachers were urged to give immediate feedback to students. During marking, teachers should award method marks and not just place a mark at the end of each solution. The idea of quick feedback failed because of large numbers of pupils in maths class (60 pupils in the current upper sixth class at Pamushana. With such a big class teachers don't show methods marks and just place the marks at the end of each solution. Teachers were discouraged from over relying on past exam papers when giving students tests. Presenters gave the 2012 November paper 1, which was biased towards concepts on differentiation and integration. Teachers were urged that when preparing a test, they should make sure every topic is set and identify the weight given to each topic so as to avoid skipping other topics

Midyear exam harmonisation-Item writers (teachers) are selected who set different papers of the syllabus. A harmonised exam and harmonised marking scheme will be administered to all Bikita secondary schools. Teachers had agreed to mark the papers collectively at a specific centre of choice. During marking teachers would not mark answer scripts of pupils from their respective current stations. This was a measure to avoid bias towards certain students whose performance might be pre determined by the teacher. Mr Matenga indicated that since he became a

ZIMSEC marker, students who came first in his local test are not always the highest at ZIMSEC. However this harmonisation of marking schemes did not succeed since the NASH failed to raise enough resources (money) for this to happen and some of the impacts of the interventions are highlighted below.

1.1.4 Outcome of the Interventions to Student Achievement in Trigonometry at Pamushana High

Many of the intervention measures done by teachers at Pamushana High improved the performance of students to a lesser extent. This is because the interventions did not address the methodology that is the instructional strategies used to teach trigonometry at Pamushana high. From the failures of the invention measures it can be commented that the achievement of students in trigonometry is largely affected by the teaching methods in current use.

Although the current upper-six students managed to complete the whole syllabus in February 2015, the fact that they were doing paper four, seemed to demand so much of their concentration to such an extent that they have forgotten some of the content done in paper one during 2014. Content in trigonometry is no exception since some of the students cannot recall the general solutions for equations in sine, cosine or tangent. The idea of completing the syllabus early helps if during the course of doing the optional paper (paper 4 in the case of Pamushana) teachers constantly give exams which test concepts in paper 1. Since at Pamushana teachers together with the students forget about paper 1 and focus on paper 4 for a long time, students' memories tend to decrease with time, thus the idea of completing the syllabus early fails.

Although teachers identified differential equation, integration, iterative method and trigonometry as problem topics, little was done on the best methods of teaching such concepts and consequently the idea did not improve the performance of students to named areas. There is need to look into the lesson delivery methods used by teachers and have an overall assessment of the teaching methods considering alternative methods like the divergent or convergent based instructional strategies.

The idea of proper content sequencing was done, as the teachers sat down and drafted a new school based syllabus. This helped to a larger extent as it reduces confusion to students. It is often seen that teachers realise that a certain topic was supposed to have taught first as they will be checking on the assumed knowledge at the introductory stage of the lesson. Comments from teachers such as "ouch, we should have done that concept first before doing this concept", are often herd when the sequence of topics is not proper. This obviously confuses students and reduces their zeal to learn the new concept. Proper content sequencing also saves time for teachers during lesson preparation.

Since students were encouraged to present their work properly and follow instruction pertaining to the degree of accuracy especially on angles, their performance improved. However, students can only present the content they have, so without changing teaching method the depth of content mastered by student will remain shallow and thus some students leave questions un-attempted or partially attempted.

The idea of team teaching was done and teachers in the department had to share different topics in the syllabus, however it was done in paper 4 where one teacher would teach discrete random variables and the other teaches continuous distributions while the other teaches the normal distribution, linear combinations of random variables and sample and another teacher takes the mechanics section. The same idea needs to be done in the pure maths section especially in the revision session. The other factor which reduces the effectiveness of team teaching is that there is tendency to complete between teachers as to who produces better results and with the idea of incentives which are now performance based thus many teachers normally don't prefer to share the money thus teachers will tent to minimise the team teaching as they know that at the end of the day the reward is given to the one with the current stream.

Harmonisation of midyear exams also helped in improving the performance of students. Since the question papers were revised, they were standard papers which satisfy blooms taxonomy and they prepared the student for the final public exams in a commendable way. However, due to lack of funds affecting NASH, the harmonisation of the marking sessions did not happen. Thus little effort was made in the identification of common errors made by students in the entire district. As answer script were marked by teachers with the stream, bias was inevitable. All intervention made by teachers at Pamushana lacked capacity to guarantee improvement of the teaching strategies and thus they did improve the achievement of students to a lesser extent. The research considers Kolb's experimental learning theory to be the most suitable when trying to carry out effective lessons in the teaching of trigonometry

This project adopts most of the steps proposed by Kolb, in his experimental learning theorem, especially during the period in which the two experimental groups are being taught trigonometric concepts one with divergent based instructional strategies and the other one with convergent based instructional strategies.

1.1.5 Theoretical Framework

According to Illeris and Knud (2004), a learning theory is a conceptual framework describing how information is absorbed, processed, and retained during learning. Cognitive, emotional, and environmental influences, as well as prior experience, all play a part in how understanding, or a world view, is acquired or changed, and knowledge and skills retained. This project follows steps cited by Kolb in his experimental learning theorem.

1.1.5.1 Kolb's Experiential Learning Theory

First Kolb showed that learning styles could be seen on a continuum running from:

1. Concrete experience: being involved in a new experience
2. Reflective observation: watching others or developing observations about own experience
3. Abstract conceptualization: creating theories to explain observations
4. Active experimentation: using theories to solve problems and make decisions

Although Kolb thought of these learning styles as a continuum that one moves through over time, usually people come to prefer, and rely on, one style above the others. It is these main styles that instructors need to be aware of when creating instructional materials.

1.1.5.2 Types of Learners

Concrete Experience (CE):

It is a receptive, experience based approach to learning that relies for a large part on judgements based on feelings. CE individuals tend to be empathetic and people oriented. They are not primarily interested in theory; instead they like to treat each case as unique and learn best from specific examples. In their learning they are more oriented towards peers than to authority and they learn best from discussion and feedback with fellow CE learners. Labs, field work, videos and observations are some of the instructional strategies suitable for them.

Reflective Observation (RO):

It is a tentative, impartial and reflective approach to learning. They rely on careful observation of others and/or like to develop observations about their own experience. They like lecture format learning so they can be impartial objective observers. Self-reflection exercises, journals, brainstorming are some of the instructional strategies suitable for them.

Abstract Conceptualisation (AC):

It is an analytical, conceptual approach to learning: logical thinking, rational evaluation.
These learners are oriented to things rather than to people. They learn best from authority-directed learning situations that emphasize theory. They don’t benefit from unstructured discovery type learning approaches. Lectures and papers are some of the instructional strategies suitable for them.

Active Experimentation (AE):

AE is an active, doing approach to learning that relies heavily on experimentation. These learners learn best when they can engage in projects, homework, small group discussion. They don’t like lectures, and tend to be extroverts. Simulations, case studies and homework are some of the instructional strategies suitable for them

1.1.5.3 Kolb's Learning Styles

1) Accommodator Learning Style

Accommodator’s dominant learning abilities are Concrete Experience (CE) and Active Experimentation (AE). This person’s greatest strength lies in doing things and involving oneself in the experience. This person can be more of a risk-taker and tends to adapt well in specific circumstances. This person tends to solve problems in an intuitive trial and error manner, relying often on other people’s information rather than their own analytic ability. It is suited for action-oriented jobs (business, marketing, sales). These learners are good with complexity and are able to see relationships among aspects of a system. A variety of methods are suitable for this learning style, particularly anything that encourages independent discovery allowing the learner to be an active participant in the learning process instructors working with this type of student might expect devil's advocate type questions, such as "What if?" and "Why not?"

2) Assimilator

Assimilator’s dominant learning abilities are Abstract Conceptualization (AC) and Reflective Observation (RO). They are motivated to answer the question, "What is there to know?" They are good at creating theoretical models, less interested in people and more concerned with abstract concepts. This learning style is more characteristic of basic sciences and mathematics. They like accurate, organized delivery of information and they tend to respect the knowledge of the expert. They aren't that comfortable randomly exploring a system and they like to get the 'right' answer to the problem. Instructional methods that suit Assimilators include: lecture method, followed by a demonstration exploration of a subject in a lab, following a prepared tutorial (which they will probably stick to quite closely) and for which answers should be provided These learners are perhaps less 'instructor intensive' than those of other learning styles. They will carefully follow prepared exercises, provided a resource person is clearly available and able to answer questions.

3) Converger

Converger’s dominant learning abilities are Abstract Conceptualization (AC) and Active Experimentation (AE). They are motivated to discover the relevancy or "how" of a situation and their greatest strength lies in the practical application of ideas. Application and usefulness of information is increased by understanding detailed information about the system's operation. They are relatively unemotional and prefer to deal with things rather than people. They like to specialize in the physical sciences and this learning style is characteristic of many engineers. Instructional methods that suit Convergers include interactive, hands-on, not passive, instruction (labs, field work) computer-assisted instruction problem sets or workbooks for students to explore.

4) Diverger

Diverger’s dominant learning abilities are Concrete Experience (CE) and Reflective Observation (RO). Their greatest strength lies in imaginative ability. This person is very good at viewing concrete situations from many perspectives. They prefer to have information presented to them in a detailed, systematic, reasoned manner. Flexibility and the ability to think on your feet are assets when working with Divergers. Counsellors, managers are typical professions they are well suited to. Instructional methods that suit Divergers include lecture method, hands-on exploration and brainstorming.

This research is going to use Kohlberg’s instructional model as a template. It is going to assume that learning experiences of a child should be a continuum which involves each of the four learning styles namely, concrete experience, reflective observation, abstract conceptualization and active experimentation. The model is going to help in the sense that weather the researcher uses divergent based instructional strategies or convergent based instructional strategies they should cater for the above named learning styles. Since the research is of the opinion that every child should follow the continuum, analysis between the divergent or convergent instructional strategies will be done to establish which one between them allows for the smooth transition of learning experiences of learners in the continuum.

1.2 Statement of the Problem

In Zimbabwe, trigonometry is a topic which is encompassed in Zimbabwe School Examinations Council (ZIMSEC) advanced level mathematics syllabus (9164) for the period 2013 – 2017. It is one of the areas where pupils face problems. The performance of candidates at Pamushana and nationally in questions which require knowledge and skills of trigonometry or in other questions which need integration of trigonometric concepts is consequently a problem. Unfortunately, concepts in trigonometry are pre-requisite to many topics in the named syllabus and these topics include graphs and coordinate geometry in two dimension, functions, differentiation, integration and numerical methods. From the researcher's experience as a teacher and the November 2004 and 2005 reports from the Zimbabwe Schools Examinations Council (ZIMSEC) on Mathematics examinations, it was realised that at least two-thirds of candidates at A level displayed inability to handle problems in trigonometry and questions which require trigonometric concepts and other related concepts. Out of many causes of student failure, the main reason might be the teaching strategies used to teach trigonometry. Therefore, if that is the case, it might be the convergent- based instructional approach or divergent-based instructional approach causing such problems and thus, the researcher is thrilled in finding the best way to effectively and efficiently reach the mind of learners' during delivery of trigonometric lessons hence, the need for this research.

1.3 Research Questions

Does divergent teaching method affect learning outcomes differently from convergent teaching method on students' achievement in trigonometry?

1.3.1 Sub Problems

i) What is the correlation in students' performance in trigonometric concept gained either by convergent or divergent based instructional strategies?
ii) Which one between divergent or convergent- based instructional strategies produces better students' performance in trigonometry at advanced level?
iii) What are the students' and teachers' views in the acquisition and imparting of concepts in trigonometry?

1.3.2 Hypothesis

A divergent teaching method is more effective than a convergent teaching method in learning trig concepts.

1.4 Assumptions of the Study

The high failure rate of pupils in Trigonometry is largely caused by instructional strategies used to develop such concepts. The main problem students meet is to apply trigonometric skills and to manipulate the knowledge gained in trigonometry to solve problems that are, ideally, unfamiliar. This shows that there are some grey areas in the teaching and learning of trigonometry. Secondly students had covered all concepts in trigonometry and topics which require trig concepts as pre-requisite knowledge

1.5 Significance of the Study

The evidence and other information gathered in this research could be important for the Ministry of Education. Curriculum officers and teachers could use the findings of this research as a basis for policies and practices and pedagogical techniques that would address the difficulties faced by students in solving questions which require concepts on trigonometry. The findings would encourage educational leaders to strongly consider setting a curriculum framework that addresses effective teaching of Mathematics at Advanced level.

The research findings will be benefit at workshops led by the NASH on subject panel at BSPZ Bikita. Presenters' of such workshops will have literature to refer to when doing their presentation. The information presented will help Bikita district to raise its pass rate.

1.6 Delimitations of the Study

The research was carried out at one school only (Pamushana High School) in Bikita district (Zimbabwe). The participants of the research were the Upper sixth Maths students at the mentioned school.

1.7 Research Ethics

During the research process, the researcher was obliged to protect the autonomy, safety, privacy, and welfare of human research subjects. Autonomy was ensured when the participants were given the right to determine what activities they wanted or did not participate in. Implicitly, full autonomy was evidenced when individuals were made to understand what they were being asked to do, and make a reasoned judgment about the effect participation had on them, and made a choice to participate free from coercive influence. The investigator provided a potential research participant with full disclosure about the nature of the study, the risks, benefits and alternatives, and an extended opportunity to ask questions before deciding whether or not to participate so as to protect autonomy of participants. This process is called the "informed consent process".

The investigator maximise benefits for the individual participant and/or society, while minimising risk of harm to the individual. Maximizing potential benefits was predicated on sound experimental design, thus by employing Kohlberg' experimental learning theorem and ensure that the learning session of students would lead then migrate through the four learning continuum, this lead to effective and efficient transfer of trig concepts to students and hence improved their achievements.

The researcher ensures justice by equitable selecting of participants, by avoiding participant populations that may be unfairly coerced into participating. The maths students and maths teachers which were involved in this study were having immediate benefit from the study.

2. Research Methodology

The study's ultimate goal is to seek for the most effective and efficient instructional approach (divergent or convergent based) which leads to the improvement of students' achievements in trigonometry at advanced level.

A Quasi- experimental methodological framework used in this study, its merits and its demerits are highlighted below. Mixed method paradigms (quantitative and qualitative approaches) have been used in the analysis, discussion and interpretation of the results. The following is the description of data gathering methods, which are, pre- test (T_1), the divergent and convergent instructional methods (M_1 and M_2respectively) to be used as treatment for pupils and the post-tests (T_2) and questionnaires for both students and teachers. Procedures for selection of participants, the sample size and procedure for data collection were also considered.

2.1 Quasi-Experimental Design

The Quasi-experimental design is a type of evaluation which aims to determine whether a program or intervention has the intended effect on a study's participants. It involves selecting groups, upon which a variable is tested, without any random pre-selection processes. For example, to perform an educational experiment; a class might be arbitrarily divided by alphabetical selection or by seating arrangement. The division is often convenient and, especially in an educational situation, causes as little disruption as possible. After this selection, the experiment proceeds in a very similar way to any other experiment, with a variable (Instructional method) being compared between different groups, or over a period of time.

Its merits especially for this project is that, it was difficult to for pre-selection and randomization of groups and that it can be very useful in generating outcomes for general trends on how performance of students is affected upon the use of either divergent -based instructional strategies or convergent based instructional strategies

Quasi-experimental design is often integrated with individual case studies; the figures and results generated often reinforce the findings in a case-study, since this project was a case of Pamushana High, the design was considered necessary. It allows some sort of statistical analysis to take place and hence the researcher uses the correlation, ANOVA and the t-Test.

In addition, without extensive pre-screening and randomization needing to be undertaken, the quasi-experimental design adopted helps to reduce the time and resources needed for experimentation.

However one of the draw backs of this design particularly for this project is the fact that one group of children may have been slightly more intelligent or motivated. Without some form of pre-testing, it is hard to judge the influence of such factors. In this project the named problem was minimised by having a stratified sampling in which girls were considered alone and boys were also considered alone during sample selection, thus the gender balance can reduce the problem.

In this case a pre-test and post-test are used for investigating the effect on achievements of students taught using convergent and students taught using divergent teaching methods on learning trigonometry at advanced level. There are two experimental groups and one non treatment control group. This can be schematically shown as it follows where G_1 to G_3 represent groups 1 to 3. T_1 Represents the pre-test and T_2 represents the post-test and M_1 and M_2 stands for the treatment, namely, the divergent based instructional strategies and convergent based instructional strategies respectively.

$$G_1 \rightarrow T_1 M_1 T_2$$
$$G_2 \rightarrow T_1 M_2 T_2$$
$$G_3 \rightarrow T_1 \text{ - } T_2.$$

2.2 Research Paradigm

The methodological framework for this study adopted the mixed method approach for the data collection. This approach integrates aspects which are found in both the quantitative and qualitative paradigm. This mixed approach is most suitable for this project as it enables triangulation of the inferences made from pre-test, post test and the responses given by both students and teachers in questionnaire with regards to their views in the acquisition and imparting of knowledge in trigonometry.

The use of quantitative research statistically measures performance of participants. Walliman (2001) stated that the quantitative research approaches provide objective and unbiased results. Creswell (2003) mentioned that quantitative research is directly from the sample to provide a basis for making inference about the larger population. Walliman (2001) said, key features of many quantitative studies are the use of instruments such as tests to collect data.

For triangulation of data, the qualitative paradigm was also used. According to Creswell(1994) say, a qualitative study is defined as an inquiry process of understanding a social or human problem, based on building a complex, holistic picture, formed with words, reporting detailed views of informants and conducted in a natural setting. The qualitative approach is an approach of data collection which deals with feelings and non-quantifiable elements by VanderStroep et al, (2010) the qualitative method is concerned with process rather than simply on products or outcomes that is, concerned with how understandings are formed, how meanings are negotiated and how roles are developed.

In this research, questionnaires were used for that purpose especially in section, with open ended questions which requires students to point out what they can do in order to overcame challenges they face they face in trigonometry and where teachers are required to give any recommendation, comments or suggestion on teaching and learning of concepts in trigonometry. This gives an in-depth exploration of questions for better understanding of underlying behaviours of teachers and students during imparting and acquisition of trig concepts.

2.3 Population

The target population, as defined by Chimedza (2003), is the collection of all units of population from which a sample is to be collected. The population of this research are the ‘A’ level maths students, all the five maths teachers who are currently in the department and the administrators of Pamushana High in Bikita district. The school has 60 upper sixth and 55 lower sixth maths students. In most cases, trigonometry is dealt with in lower sixth as students will be doing paper 1 topics and in upper sixth form, students will be doing concepts which require trig as a pre requisite. It is for this reason that upper sixth students were used in the study.

2.4 Sampling Techniques

A sample is a subset of the population selected to represent the whole population. Two sampling methods were considered in this research, the stratified and the systematic random sampling.

Stratified random sampling procedure was used to make sure both female and male pupils are included. Crawshaw and Chambers (2002) said stratified sampling is used when the population is split into distinguishable layers or strata that are quite different from other, which are upper sixth girls and upper sixth boys. Advantages of stratified random sampling are, to increase a sample's statistical efficiency, to provide adequate data for analyzing the various subgroups and useful when the researcher wants to study subgroups. In the current upper sixth form, there are 22 girls and 38 boys a total of 60 students. Since there are 3 groups to be considered for the research, it implies that the sample should comprise of 15 girls and 15 boys.

The three pairs of 5 girls and three pairs of 5 boys were selected separately using the systematic random sampling. The k^{th} term will differ from each one group to another and it also differs within different gender (boys or girls) in the same group. The following is how the systematic random sampling was done to place participants in each group.

Group 1(Treatment Group)

In G_1 for girls the researcher considers all multiples of 4(that is girls number 4; 8; 12; 16 and20) and for boys all multiples of 7 were considered (that is boy number 7; 14; 21; 28and 35)

Group 2(Treatment Group)

In $G_2$for girls the researcher considers from the remaining girls all multiples of 3(that is 3; 6; 9; 15 and 21) and for boys the researcher considers all multiples of 6(6; 12; 18; 24 and30).

Group 3 (Non Treatment Control Group)

In G_3 for girls, the researcher considers from the remaining girls, all even numbers (that is 2; 10; 14; 18 and 22) and for boys the researcher considers from the remaining boys all multiples of 5(5; 10; 15; 20 and 25)

The systematic random sampling was most suitable since each participant was selected according to a predetermined sequence which originated by chance and hence it supports the research design (quasi-experimental) which also involves selecting groups, upon which a variable is tested, without any random pre-selection processes. Each group is made up of 5 girls and 5 boys, which means the sample consists of 30 upper sixth maths students.

Chimedza (2003) stated that the recommended minimum size of the sample of the target population is 10%. However, it is not the size of the sample that matters most but its representatives. Therefore, 26.1%, was a fair for the study.

2.5 Research Instruments

Research instruments is a phrase which encompasses all the different tools which can be used for data gathering and collection in a study and it can also include mathematical packages used for data analysis. In this study, the research instruments used are tests and questionnaires (for students and for teachers).

2.5.1 Test

Pupils are to be given one the pre- test and three post-test. The average of the post-tests was considered for all statistical inferences to be made in this research. The concept to be tested comes from trigonometry for the advanced level syllabus. The two tests assessed pupils' ability to solve trig equations, apply reference angle to non acute angle, prove trig identities, graph trig functions and their transformations, apply trig to model real life scenarios, find the shaded area in terms of sine and cosine and finding the maximum and minimum values of a trig function. Chakanyuka (2000) said a test is a standardised situation that provides an individual with a score. The purpose of a test is to determine how much a pupil has achieved. The tests were administered by teachers in the Mathematics department in interval 9 mouths. Experienced markers and item writers set the tests and were verified by teachers who are ZIMSEC examiners. Pupils will write tests which assesses pupils' performance in trigonometry.

2.5.2 Questionnaire

Chimedza (2003) defined a questionnaire as a document containing a lot of pertinent questions for statistical enquiry. There were two different questionnaires, one for the pupils and one for the teachers. These were intended to get both teachers' and pupils' perceptions towards the acquisition and imparting of concepts in trigonometry. There is an open–ended question at the end of each questionnaire to give opportunity to discuss their experience regarding their knowhow in trigonometry

2.5.2.3 Pilot Testing

According to Crewell (2003), a pilot study is a small study carried out prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate. This mini-research is intended to identify deficiencies of the measuring instruments or the procedure to be followed in the actual. Fifteen students and three teachers at Pamushana high were involved in piloting of the questionnaires. Necessary corrections for example wording of certain items were made due to feedback from the pilot study. This exercise was very helpful as it makes researcher aware of any possible unforeseen problems that may emerge during the main investigation.

2.6 Data Collection Procedure

For the control group (G_3) there were only a pre-test at the beginning and a post-test after nine months and there wasn't any treatment for them. For the convergent group (G_2) after a pre-test at the beginning, then there were 14 sessions of teaching/learning trig concepts. With reference to Kolb's experimental learning theory, the convergent group was classified as having either abstract conceptualisation ability or active experimentation ability, thus the interactive, hands on, not passive instructions and computer- assisted instructions were used with the help of workbooks (trig formula sheets) for students to explore. Grouping will be employed during transformation of trig functions in which transformation with similar effects are put together. The students were encouraged to memorize skills associated with particular trig concepts and the researchers tried to review the skills every session and ask students about what they had learnt so far.

The filtering technique will be applied when solution of trig equations are filtered so as to get those which are in the required interval. Prioritisation will be mastered when students are taught how to select the best identity to reduce a certain trig expression. Then after 14 sessions they received post-tests.

The divergent group (G_1) also received a pre-test, and then they started learning trig concepts for 14 sessions. With reference to Kolb's learning theory, the divergent group consist of students with learning abilities classified as either the concrete experience or the reflective observation. The researcher used the lecture methods, hands on exploration, self discovery and brainstorming as proposed by Kolb. According to Lipoff (2013) divergent learning occurs through play. This makes students think deeply and try to be creative; therefore, the researchers also divided the group into two sub-groups and tried to make a competition after teaching the points. For each question the students suggested several solutions and tried to solve the problem through collaboration by saying their opinion. During the sessions, brainstorming is used when proving identities; scamper is applied when finding missing sides through mnemonic devices like CHASHOTAO. Visual connections are used during transformation of graphs; forced connections are used when solving trig identities; excursion is applied when finding shaded areas in terms ofsin x or $\cos x$; idea box are used during proving identities

Marking points of the solution were discussed one by one and at last having a complete set of different methods to be used to solve the same problem. Matching the parts of different methods would be done as a means of reinforcing concepts and attracting students. Each session was accompanied with new points of solving novel problems and new games designed to catch the interest of students. Then students received the post-test after 14 sessions.

2.7 Data Representation and Analysis

Convergent and divergent teaching methods are the independent variables and pre-test and post-test are the dependent variables in this study. The one way ANOVA statistical procedure and t-Test was used to analyse the effects of the named independent variables to the achievement of students in trig concepts. If the F-statistic is less than the F-critical or the T-calculated is less than the T-critical we do not reject the Null hypothesis but if the vice-verse is true we do not accept the Null hypothesis at 5% level of significant. The hypothesis was carried at 5% significant level because it's a reasonable very low level which if used and is true to the sample, the results can be safely inferred to the population since their probability error is marginally very low(0.05).

2.7.1 t-Test

The t-Test is going to be used to test the hypothesis on the effectiveness of the convergent or divergent teaching methods. The scores of those students in the pre-test will be matched to scores in the post-test. Thus the t-Test to be carried out falls under the category for the dependent samples (matched pairs), in which the investigator is concerned with the difference in a pair of related observations that is scores in the pre-test and in the post-test. In this case, the t-Test is based on the difference of each student's scores (post-test scores minus pre-test score) denoted byd_i. The mean difference ($\overline{d}$) is calculated as follows

$$(\overline{d}) = \frac{1}{n}\sum_1^n d_i.$$

Were n is the total number of students in each group. The hypothesis will beH_0:$\mu_d = 0$ $versus$ $H_1 : \mu_d > 0$ at 5% $significant\ level$ Where μ_d= 0 implies there is no difference between the mean difference ($\overline{d}$) between scores in the post–test and scores in the pre-test and $\mu_d > 0$ means the mean difference is greater than zero (implying the teaching method was effective in improving the achievements of student). The test statistics to be used are the T-critical (T_{crit}) and T-calculated (T_{cal}).

$$T_{cal} = (\overline{d} - \mu_d)/(\frac{S_d}{\sqrt{n}})$$

Were s_d is the standard deviation of the mean difference d_i

$$T_{crit} = t_\alpha(n-1)$$

Were α is the level of significant and $n-1$ is the degree of freedom

If $T_{cal} > T_{crit}$ we do not the null hypothesis and conclude that the teaching method has a positive effect on students' achievements. If $T_{cal} < T_{crit}$ we do not reject the null hypothesis and conclude that the teaching method did not significantly improve the performance of student in trigonometry.

2.7.2 One Way Analysis of Variance (ANOVA)

The ANOVA is going to be used since; there are two independent random samples (students in the convergent group and students in the divergent group) of sizes n_1 and n_2 with mean μ_1 and μ_2. Then if $\mu_1 = \mu_2$, the random variable (test-scores)

$$F_{statistic} = \frac{MSTR}{MSE}$$

Has the F distribution with $df = (k-1, N-k)$ where N denotes the total number of students in all the groups,k is the number of groups, df is the degree of freedom, $k-1$ is the degree of freedom associated with the different treatments, $N-k$ is the degree of freedom associated with errors within samples, $MSTR$ is the mean square or variance estimate explained by the different treatments and MSE is the square or variance estimate that is unexplained (due to chance)

Table 2.1 is a summary of how the F-statistics is calculated

Table 2.1: One Way ANOVA Table

Source of variance	**Degree of freedom ($d.f$)**	**Sum of squares($S.S$)**	**Min sum of square($M.S$)**	**F-statistic**
Treatment	$k-1$	$SSTR$	$MSTR = \frac{SSTR}{k-1}$	$F_{cal} = \frac{MSTR}{MSE}$
Error	$N-k$	SSE	$MSE = \frac{SSE}{N-k}$	
Total	$N-1$	SST		

The critical value is F-critical denoted by F_{crit}

$$F_{crit} = F_{\alpha}(K-1\,;\,N-k)$$

The hypothesis will be: $H_0 \colon \mu_2 = \mu_1$ $versus$ $H_1 : \mu_2 > \mu_1 at\ 5\%\ significant\ level.$ Were μ_2 denotes the mean for convergent group and μ_1 is the mean for the convergent group.

If $F_{statistic} > F_{crit}$ we do not accept the Null hypothesis and conclude that the divergent teaching method was more effective than convergent teaching methods and if $F_{statistic} < F_{crit}$ we do not reject the Null hypothesis and conclude that there is no significant difference between the levels of achievement of students in the divergent group compared to that one of students in the convergent group.

2.7.3 Spearman's Rank Method

The correlation coefficient between the performances of students who have received convergent based instructional strategies and those who have received divergent- based instructional strategies in the leaning of trig concepts will be calculated using the Spearman rank method. A scatter diagram for the scores in the pre-test and post-test of students in all groups will be drawn in order to have a visual image of how the scores are spread.

The Spearman's Rank Order coefficient is used to investigate nature of correlation existing between students' achievement in trig concepts taught using convergent or divergent teaching methods. The corresponding values that were compared were ranked according to the relative position in the group. The difference between the ranks were calculated and then squared. According to this method the correlation coefficient indicated by Γ or the Greek letter ρ(rho) and was calculated using the formula:

$$\Gamma = 1 - \frac{6\sum d^2}{n(n^2-1)}.$$

Were d= difference between ranks and n= number of pairs of values compared. The correlation coefficient must be a value between -1 and 1 inclusive. If Γ = + 1, this indicates that a positive and strong correlation between the sets of data compared If $\Gamma = -1$,this also indicates a strong correlation between the compared sets of data through with a negative slope If $\Gamma = 0$, there is no linear relationship between the values

Advantages of Spearman Rank Correlation coefficient: Spearman correlation is less sensitive than the Pearson correlation to strong outliers that are in the tails of both samples.

3. Discussion of Results

In this study data from the pre-test scores and post test scores of students, responses from student and teachers' questionnaire are to be presented, described, analyzed and give some statistical inferences.

For all statistical tests and inferences done in this project, the average mark for the post-test for each participant was used against his/her pre-test score. An overview of the mean score of students in the control group and the two experimental groups (convergent and divergent) and dispersion of these scores, descriptive indicators (mean, standard deviation, minimum and maximum of scores) are presented below.

Table 3.1: Description of test Scores of Three Groups of Convergent, Divergent and Control (Pre-Test and Post-Test)

Group		Pre-Test	Post-Test
Control	**Number**	10	10
	Minimum	11	19
	Maximum	49	50
	Mean	31.1	37.8
	Median	31	41.5
	standard deviation	10.13739	9.953224
Convergent	**Number**	10	10
	Minimum	30	34
	Maximum	48	62
	Mean	39.5	47
	Median	41.5	48
	Standard deviation	5.681354	7.874008
Divergent	**Number**	10	10
	Minimum	27	43

	Maximum	58	74
	Mean	42.4	58.6
	Median	43	59
	Standard deviation	9.901739	11.09755
Total	**Number**	30	30
	Minimum	11	19
	Maximum	58	74
	Mean	37.6666667	47.8
	Median	38	48
	Standard deviation	9.80265	12.77228

As is evident from table 3.1 the minimum and maximum scores of the control group in the pre-test are respectively 11 and 49 and in the post-test are 19 and 50. The mean scores of the pre-test for the control group are 31.1 with a SD of 10.13739 and the mean score of the post-test is 41.5 with a SD of 9.953224. The minimum score is the smallest score obtained by students in the control group, while the maximum score is the highest score obtained by students in the control group either in the pre-test or post-test. The mean score for the control group is the average of the score which was computed by summing the scores and dividing by the number of students set for the test (10) either in the pre-test or post-tests. The median is the middle test score obtained when either the pre-test scores or the post-test scores where arranged in order of size. The standard deviation is a measure of dispersion (a measures of the extent to which the individual test score items vary) which is given by the square root of the variance.

The maximum and minimum scores of the convergent teaching method are 48 and 30 in the pre-test and 62 and 34 in the post-test, respectively. The mean score of students taught by the convergent teaching method is 39.5 with a SD of 5.681354 in the pre-test and a mean score of 47 with a SD of 7.874008 in the post-test. The maximum and minimum scores of the divergent teaching method are 58 and 27 in the pre-test and 74 and 43 in the post-test, respectively. The mean score of students taught by the divergent teaching method is 42.4 with a SD of 9.901739 in the pre-test and a mean score of 58.6 with a SD of 11.09755 in the post-test.

3.1 Inferential Statistics on Tests

In statistics, the term inferences refer to the conclusions which are drawn directly from the gathered data and information. In this research inferences done include hypothesis testing, determining the correlation coefficient between the performance of students in the three groups and analysis of both students and teachers' responses from their questionnaires.

Hypothesis 1: The Convergent teaching methods have a positive effect on learning of trig concepts.

Table 3.1 is made up of the pre-test and post-test scores for all the students in the divergent group. These scores are the one used for carrying out the hypothesis testing.

Table 3.2: Scores Obtained By Student in Group 3 of Convergent (Pre-Test and Post Test)

Pre-test scores	30	32	42	42	44	42	40	41	34	48
Post-test scores	34	39	47	49	53	51	45	49	41	62

$H_0:\mu_d = 0 \; versus \; H_1 : \mu_d > 0 \; at \; 5\% \; significant \; level$ Where $\mu_d = 0$ implies there is no difference between the mean difference ($\overline{d}$) of scores in the post–test and scores in the pre-test and $\mu_d > 0$ implies that the mean for the post-test is greater than that one of the pre-test.

$$T_{cal} = (\overline{d} - \mu_d)/(\frac{S_d}{\sqrt{n}})$$

$$T_{cal} = (7.5 - 0)/\frac{2.83823}{\sqrt{10}}$$

$$= 8.35629$$

$$T_{crit} = t_\alpha(n-1)$$

$$T_{crit} = t_{0.05}(9)$$

=1.89.

Therefore, since 8.35629 falls in the rejection region, we do not accept H_0 and conclude that at 5% level, there is significant evidence to show that the convergent teaching method has a positive effect on learning of trig concepts.

Hypothesis 2*:* The divergent teaching methods have a positive effect on learning of trig concepts

Table 3.2 is made up of the pre-test and post-test scores for all the students in the convergent group. These scores are the one used for carrying out the hypothesis testing.

Table 3.3: Scores Obtained By Student in Group 1 of Divergent (Pre-Test and Post Test)

Pre-test scores	35	40	49	27	36	32	49	46	52	58
Post-test scores	48	55	68	43	50	48	66	63	71	74

$H_0{:}\mu_d = 0\ versus\ H_1 : \mu_d > 0\ at\ 5\%\ significant\ level$ Where $\mu_d = 0$ implies there is no difference between the mean difference ($\overline{d}$) of scores in the post–test and scores in the pre-test and $\mu_d > 0$ implies that the mean for the post-test is greater than that one of the pre-test.

$$T_{cal} = (\overline{d} - \mu_d)/(\frac{S_d}{\sqrt{n}})$$

$$T_{cal} = (16.2 - 0)/\frac{1.93218}{\sqrt{10}}$$

$$= 26.5135$$

$$T_{crit} = t_\alpha(n-1)$$
$$T_{crit} = t_{0.05}(9)$$

=1.89.

Therefore since 26.5135 falls in the rejection region, we do not accept H_0 and conclude that at 5% level, there is significant evidence to show that the divergent teaching method has a positive effect on learning of trig concepts.

From hypothesis 1 and 2 it can be noted that both convergent and divergent teaching methods had a positive effect in the achievements of students. These results are also supported by Tomar and Sharma (2005) who argue that both a convergent approach and a divergent approach are needed. Tomar and Sharma (2005) advocated for a teaching method which strikes a balance between the divergent and convergent teaching methods. This method is called guided-divergent, meaning that it is more structured and less flexible than divergent teaching. However it is less narrow and limiting than convergent teaching.

Hypothesis 3: Divergent teaching method is more effective than a convergent teaching method in learning trig concepts.

H_0:$\mu_2 = \mu_1$ $versus$ $H_1 : \mu_2 > \mu_1 at$ 5% $significant\ level$. Where μ_2 denotes the mean for convergent group and μ_1 is the mean for the convergent group

Table 3.4 is made up of the post-test scores of the students in the convergent group and post-test score of students in the convergent group. These scores are the one used for carrying out the hypothesis testing.

Table 3.4: Scores of Students in the Two Groups of Convergent and Divergent In the Post-Test.

	Convergent	**Divergent**
Post-test scores	34	48
	39	55
	47	68
	49	43
	53	50
	51	48
	45	66
	49	63
	41	71
	62	74
$\boldsymbol{n_i}$	10	10
$\boldsymbol{T_i}$	470	586

Table 3.4 shows n_i which denotes the number of students in each group and T_i is the sum of scores in each group.
The ANOVA table 3.5 was constructed from scores in table 3.4

Table 3.5: Analysis of Variance (ANOVA) table

Source of variance	**Degree of freedom ($\boldsymbol{d.f}$)**	**Sum of squares($\boldsymbol{S.S}$)**	**Min sum of square($\boldsymbol{M.S}$)**	**F-statistic**
Treatment	1	672.4	672.4	7.2674

Error	18	1666.4	92.5778	
Total	19	2339.2		

$$F_{crit} = F_{\alpha}(K-1\,;\ N-k)$$
$$F_{crit} = F_{0.05}(1\,;\ 18)$$
$$= 4.41$$

Since 7.2674 falls in the rejection region, we do not accept H_0 and conclude that at 5% level, there is significant evidence to show that the divergent teaching method was more effect on learning of trig concepts than convergent teaching methods.

From hypothesis 3, it can be deduced that in trigonometry, the divergent method was more effective than the convergent method. Although Guilford and Jackson (1961) found both convergent and divergent students to be equal in scholastic performance, in trigonometry the divergent students have better performance. This can be explained by looking at the nature of question or the type of question which are frequently examined in trigonometry; it is more often that examiners prefer questions which require divergent thinking. Again hypothesis 3 contradicts with the third conclusion made by Jo Ann (2003) who studied the relation between creativity and convergent and divergent methods of teaching spelling. He found that there is no significant difference in achievement between children taught by a convergent method and children taught by a divergent method. This could have been true in spelling because, spelling need a bit of rote learning in which students can memorise things but in trigonometry concepts are learnt to be understood not memorised.

The scatter diagram was drawn to establish if there is a correlation between the teaching method and the level of achievement of students in trigonometric-concepts

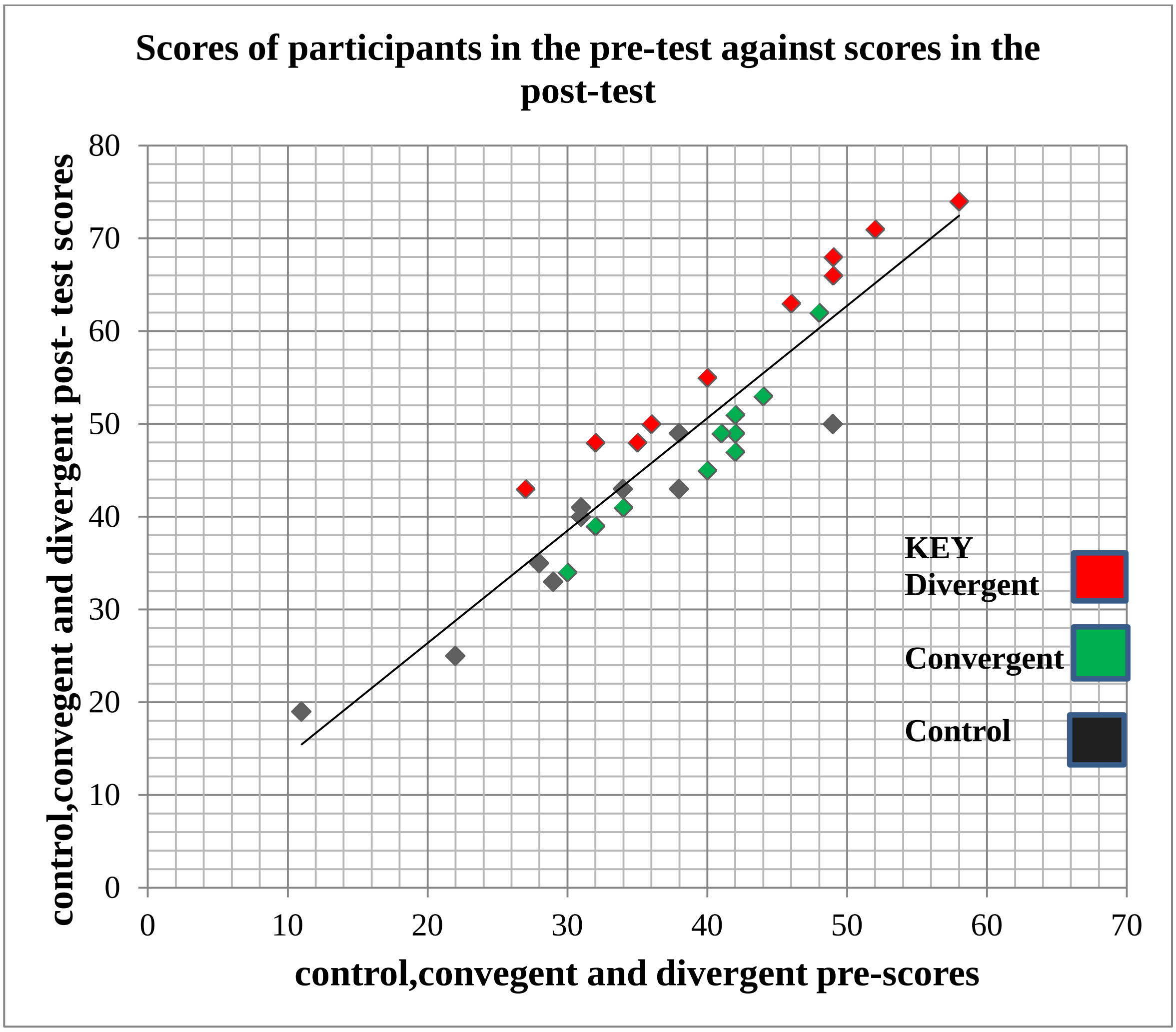

Fig 3.1 Relationship between scores of control, convergent and divergent groups (pre- test and post-test)

The scatter diagram represents a relationship between the participants' performance in the pre-test and post-test given after a teaching method was applied to them for seven months. This is a diagram that reflects the pupils' performance during the data collection period.

In fig 3.1, the red plot on the top extreme right corner, indicated that a student in the divergent group scores 68 in the pre-test against the mark of 74 the same student scores in the post-test. It is evident from fig 3.1 that the scatters for students in the divergent group are on above the line of best-fit, while those of students in the convergent group are below. This is an indicator that students in the divergent group performed better. This is supported by the results of hypothesis 3 which states that the divergent teaching method is more effective than the convergent teaching methods.

The scattered nature of the points shows that there is a very strong positive correlation between the level of achievement (post-test scores) and the teaching method used to deliver the trig concepts

A very strong positive correlation may be explained as follows. Firstly the convergent group shows a relatively high improvement of scores in the post-test compared to those they got in the pre-test. On the same note, participants' in the divergent group also showed a great improvement of scores in the post-tests as compared to those they got in the pre-test. For the control group(with no treatment), their scores in their post-tests were more or less similar to those in the pre-test, which may support the idea that practice makes perfect and the notion given by students from responses of their questioner that the topic needs time for them to collaborate and assimilate the concepts. Spearman's Rank correlation was calculated as follow.

$$\Gamma = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(10)}{30(30^2-1)}$$

$$= 0.9977753059$$

This shows a very strong positive correlation between the achievement of pupils and the teaching (convergent and divergent) method used to teach them. Nadezhda (2010) and Abedi and Lord (2001) found similar results where there is some strong positive correlation between the performance in worded problems solved by either convergent approach or divergent approach at college students. In addition, Koe (1967) studied the effects of convergent and divergent teaching methods on students' achievements on two mathematical problem-solving tasks. The aim was to investigate the interaction between the convergent and divergent teaching methods and the thinking style (either convergent or divergent) of the learner. It was noted that in this study, divergent thinkers scored higher than convergent thinkers, which contradicts with the findings from Koe(1967) study in which the vice- versa was noted. This maybe because the nature of solutions to worded problems needs more of convergent thinking, while nature of solutions in trigonometry needs more of divergent thinking. Interesting to note is the fact that Koe also did find a strong positive correlation between the achievements of students which are either convergent or divergent thinkers which agrees with findings in this study.

3.2 Students' Questionnaire Analysis

Analysis of the students' questionnaire was done considering the personal and general information first, and then section B, lastly the open-ended question..

3.3 Section B of the students' questionnaire

The statements in this section were designed to investigate students' feelings towards acquisition of trigonometric concepts

Fig 3.2 shows the distribution of participants responses to statements addressing theme one, that is, pupils feelings towards trigonometry.

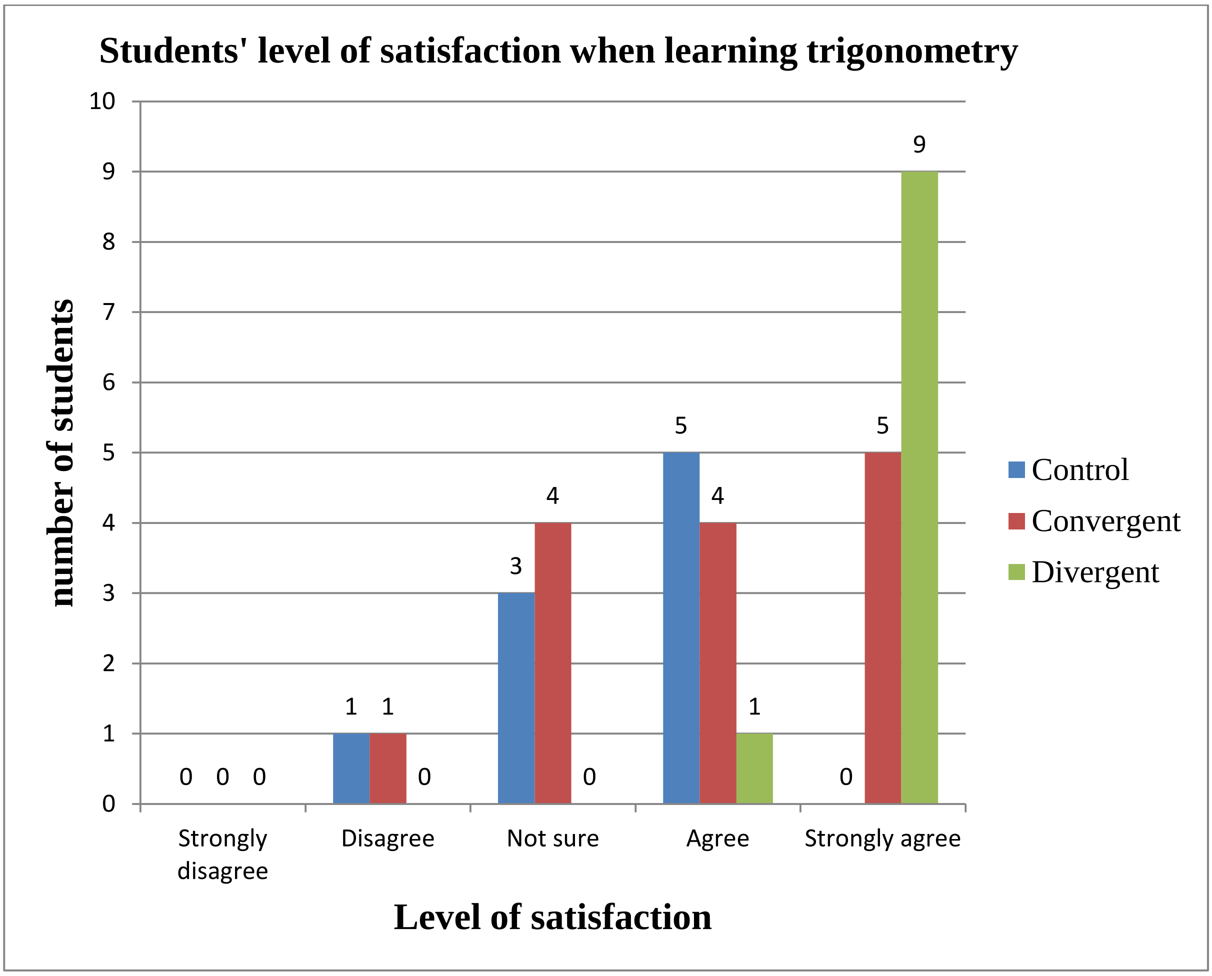

Fig 3.2 Number of student in each group and their level of enjoyment when learning trigonometry

It can be deduced from fig 4.2 that in either group no student strongly dislike trigonometry. In the convergent group about half of the students were not sure if they enjoyed learning trigonometry or not. Interesting to note is the fact that almost everyone in the divergent group strongly liked trigonometry. It can be inferred that the teaching method somehow affected the attitude of students towards trigonometry.

3.4 Pupils' responses to trigonometric-concepts which are difficult.

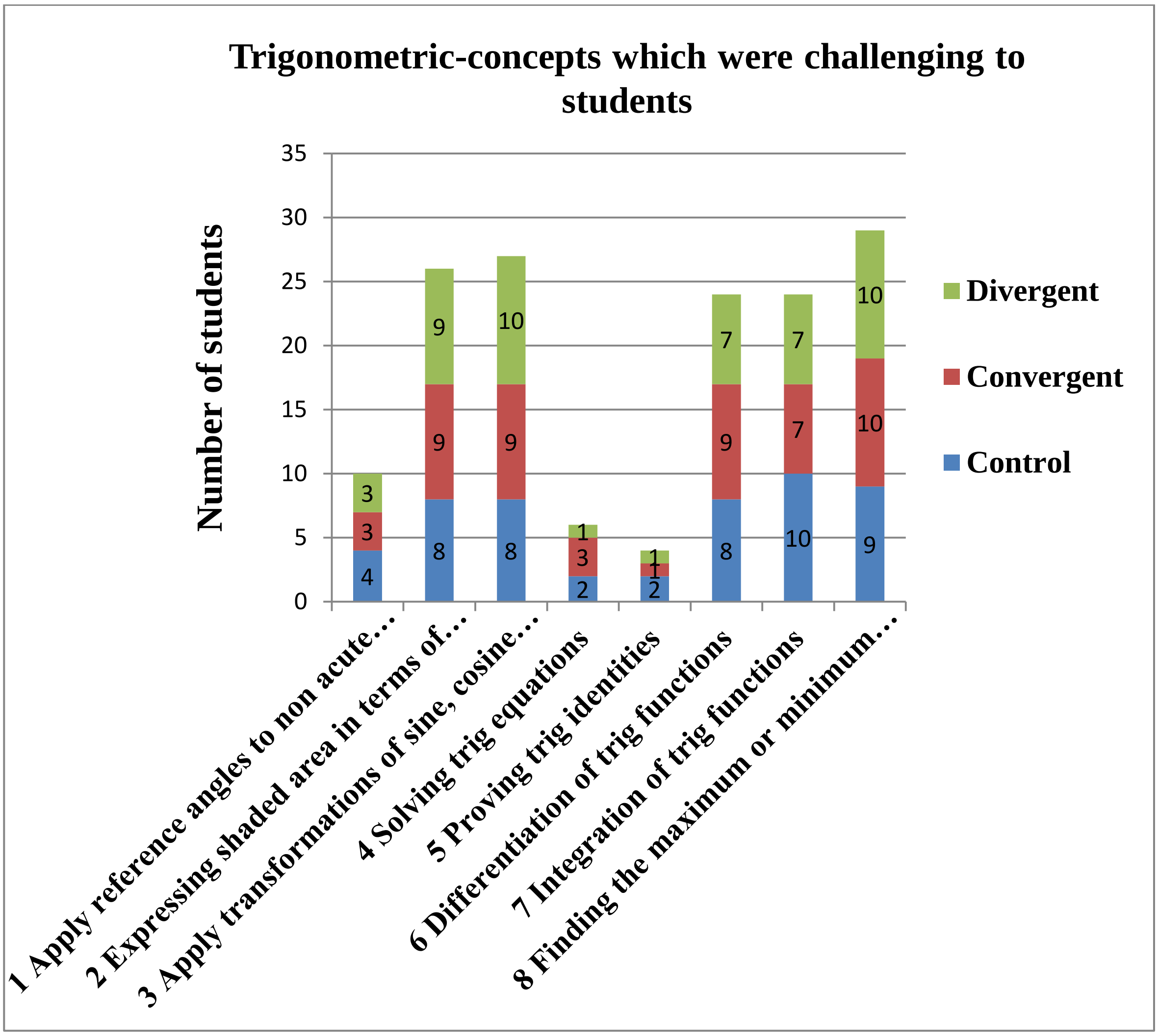

Fig3.3: Students from each group and their indication on trigonometric-concept which they faced difficulties.

From fig 3.3, concept 8; 3; 2; 7 and 6 were the most difficult in that order. More should be done to reduce the difficultness of finding the maximum and minimum values of trig functions, applying transformations of trig-functions, expressing shaded area in terms of sine or cosine functions, integration and differentiation of trig-functions.it can also be noted that for the mentioned topics more or less equal numbers from each group were recorded. More of computer-aided instructions must be used in such concepts as indicated by students' response in the instructional strategies they prefer the concepts must be taught using.

Table 3.6: Number of students preferring a certain teaching strategy to be applied against a given trigonometric-concept

		Instructional strategy					
		Computer-aided instructions	**Lecture method**	**Group work**	**Self discovery**	**brainstorming**	**Guided discovery**
Concepts	**1**	0	20	19	11	14	28
	2	7	15	15	17	16	28
	3	28	7	18	16	5	27
	4	0	20	18	10	21	19
	5	0	14	13	15	23	19
	6	1	24	8	12	9	28
	7	0	25	7	12	9	28
	8	28	8	11	9	16	27

Key

15

Means 15 participants' indicating that trigonometric-identities (concept 5) should be taught using self-discovery instructional strategy

7

Means 7 participants' preferring lecture method to be used when teaching the concept of applying transformations of trigonometric-functions (concept 3)

It is important to highlight that as inferences are being made from table 3.6, a small number of students indicating that the instructional method to be used on a certain concept, does not necessarily mean the concept is unpopular with students but it implies that the instructional strategy is less suitable and less applicable to that concept. For example zero students indicated that computer-aided method is used in teaching integration simply implies that the method is unsuitable for integration of trigonometric-functions.

However from table 3.6 by considering the three instructional methods with highest frequencies, it can be noted that, the best methods for teaching the concepts indicated by students as most difficult are:

For teaching how to find the maximum and minimum values of tri-function, use the computer-aided instructions (28 students), guided discovery (27 students) and brainstorming (16 students). Teaching transformations of trigonometric-functions must be done using computer-aided instruction (28), guided discovery (27) and group work (18), while expressing the shaded area in terms of sine or cosine functions should be done using guided discovery (28), self-discovery (17) and brainstorming (16), to analyze just a few concepts and their most suitable instructional strategies. It can be inferred that the students have indicated that those most difficult concepts must be taught using methods which falls under the divergent –based instructional strategies, this support the research findings that convergent-based teaching methods must be fused with divergent once, if the complete cognitive development which reach up to the standard of examiners is to be realized.

Finally, it can be noted that the use of technology in trigonometry supplies not only the time but also the context for the design and analysis of algorithms (Hirsch and Schoen 1990). The opportunities are rich, running from simple calculator-based iterations of functions, such as the cosine and the analysis of fixed points, through graphics programs, to computer methods for calculating nth roots of complex numbers.

3.5 Section B of teachers' questionnaire

Every teacher in the department likes student who uses the convergent thinking approach while most teachers also dislike students who uses a divergent thinking approach, this complies with findings by Strot (1985) ,who analyzed the attractiveness of students with convergent and divergent learning styles to teachers with convergent and divergent learning styles. He mentioned that Convergers have been described as favoured over divergers by teachers.

3.6 Teachers' responses to the open ended question

The last item on the questionnaire are two open-ended questions. One which requires teachers to choose a teaching method and highlight how it can be utilized to teach certain tri-concepts and the other one which gives them the opportunity to add any recommendations, comments or suggestion on teaching and learning of concepts in trigonometry.

Most teachers show familiarity with teaching methods like the guided discovery, lecture method brain storming and group work. A few have mentioned the use of computer-aided instructional methods. However teachers have recommended students to engage in a lot of practice and attempt as many questions as they can. Some teachers pointed out the fact that during lessons in trigonometry, teachers should use methods which are more pupils centred that teacher centred.

4. Conclusion

The study's ultimate goal was to seek for the most effective instructional approach (divergent or convergent based) which leads to the efficient improvement of math students in trigonometry at advanced level.

It was observed from the study, that the divergent-based instructional strategy was more effective and efficient in the improvement of students' achievements in trigonometry. However, the convergent-based instructional strategy cannot be completely dismissed since it can help students understand a particular topic and the basic information. The convergent based instructional strategy alone does not ensure retention, there is need to fuse it with divergent instructional strategies. Students in the divergent group liked and enjoyed solving trigonometric-questions more than their fellow students in the convergent and control group. Generally a lot of students favoured the idea of using computer-aided instructions during lessons. Across the three groups, transformation of trigonometric-functions and finding the maximum and minimum values of trigonometric-function were a great challenge. Almost all participants indicated the need for the use of real-life (modelling) example when teaching trigonometry.

Teachers were also indicating a bias toward students who are convergent thinkers than divergent thinkers', maybe it is because divergent thinkers are inquisitive. Some teachers do not like being asked questions but rather do the asking (Tabula-Russo). It can also be deduced that teachers are using more of convergent-based instructional approaches than divergent once. Teachers are also reluctant in using modern technology. Maybe they have not been trained enough in the use of technology.

5.5 Recommendations

In conjunction to the findings and conclusions of this study, the following recommendations were made:

- Convergent approaches should be used to introduce the foundational knowledge needed to solve problems.
- Teachers and learners of mathematics at advanced level should embrace the divergent approach toward the imparting and acquisition of knowledge especially with the current situation with item writers of public exams who seem to favour challenging and tricky question which can only be addressed with little if no difficulties to someone who is a divergent thinker.
- Computer-aided learning is especially appropriate for helping students learn complex concepts assuming students already posses the relevant foundational knowledge.
- The instructions necessary for the students to complete a divergent activity must be spelled out as much as possible without providing the right answer. This is called a guide-divergent approach by Tomar and Sharma (2005).
- Although in most cases when solving a math problem only one answer is considered correct, the divergent part of mathematics is that in most cases there are more than two ways to reach to that particular solution.
- Teachers are encouraged to use technology, because technology in trigonometry supplies not only the time but also the context for the design and analysis of algorithms (Hirsch and Schoen 1990). The opportunities are rich, running from simple calculator-based iterations of functions, such as the cosine and the analysis of fixed points, through graphics programs, to computer methods for calculating nth roots of complex numbers.

References

[1] Akbari Chermahini, S. (2011). *Creative Mood Swings: Divergent and Convergent Thinking Affect Mood in Opposite Ways.* Psychological research: 76(5) 634-640 proficiency. Tehran: University of Social Welfare and Rehabilitation Sciences.

[2] Brown, B and Ryoo, K (2008) "*Teaching Science as a Language: A "Content-First" Approach to Science Teaching*" *Journal of Research in Science Teaching* **45** (5): 529–53

[3] Chakanyuka, S (2000). *Measurement and evaluation Research*. Zimbabwe Open University, Harare.
Chimedza, C. (2003). *Research method:* Zimbabwe Open University. Harare.

[4] Colburn,A (2000).*An Inquiry Primer. Science scope* Retrieved January 31, 2010 from: http://www3 .nsta.org/maininews/pdf/ss0003 42

[5] Conference Board of the Mathematical Sciences (CBMS): *The Mathematical Sciences Curriculum K-12: What Is Still Fundamental and What Is Not.* Report to NSB Commission on Precollege Education in Mathematics, Science, and Technology. Washington, D.C.: CBMS, 1983.

[6] Creswell, J.W. (2003).*Research design qualitative quantitative and mixed methods Approaches*. Thousand Oaks: CA Sage Publications.

[7] Eckert, Paul, George K, Cameron N, and Charles V, E. *Graphing Calculators in the Secondary Mathematics Curriculum.* Lansing, Mich.: Michigan Council of Teachers of Mathematics, 1989.

First Author
Misheck Tsododo is a holder of an MScED in Mathematics and BSc (Hons) in Mathematics and Computer Science. He is currently lecturing at Mutare Teachers College. His research interest is in time series analysis and forecasting (modelling) and Mathematics Education. He has published and co-authored one research paper and presented three papers at conferences.

Second Author
Cosmas Mutoto is a holder of BScED (Hons) in Mathematics and Diploma in Education. He is currently teaching at Pamushana High School (Zimbabwe).