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Homotopy Perturbation Method for Fractional Gas Dynamics Equation Using a new local fractional integral Transform.

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Abstract

A user friendly algorithm based on new homotopy perturbation, a new local fractional α - integral transform method is proposed to solve nonlinear fractional gas dynamics equation. Further, the same problem is solved by Adomian decomposition method The results obtained by the two methods are in agreement and hence this technique may be considered an alternative and efficient method for finding approximate solutions of both linear and nonlinear fractional differential equations.This method is a combined form of the new integral transform, homotopy perturbation method, and He's polynomials. The nonlinear terms can be easily handled by the use of He's polynomials. The numerical solutions obtained by the proposed method show that the approach is easy to implement and computationally very attractive.

Keywords: *Homotopy perturbation , local fractional , new integral transform , He's polynomials*

1. Introduction

Fractional calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders. During the last decade, fractional calculus has found applications in numerous seemingly diverse fields of science and engineering. Fractional differential equations are increasingly used to model problems in fluid mechanics, acoustics, biology, electromagnetism, diffusion, signal processing, and many other physical processes .$[1-12]$ There exists a wide class of literature dealing with the problems of approximate solutions to fractional differential equations with various different methodologies, called perturbation methods. The perturbation methods have some limitations; for example, the approximate solution involves series of small parameters which poses difficulty since the majority of nonlinear problems have no small parameters at all. Although appropriate choices of small parameters sometimes lead to ideal solution, in most of the cases unsuitable choices lead to serious effects in the solutions.Therefore, an analytical method is welcome which does not require a small parameter in the equation modeling the phenomenon .Recently, there is a very comprehensive literature review in some new asymptotic methods for the search for the solitary solutions of nonlinear differential equations, nonlinear differential-difference equations, and nonlinear fractional differential equations; $[13]$ The homotopy perturbation method (HPM)was first introduced byHe $[14]$. The HPM was also studied by many authors to handle linear and nonlinear equations arising in various scientific and technological fields . $[15-22]$ The method that we present is a combination of a new local fractional α – integral transform, Homotopy Pertubation Method, and He's polynomials . In this paper, we consider the following nonlinear time fractional gas dynamics equation of the form

$$D_t^{\alpha} U + \frac{1}{2}(U^2)_x - U(1-U) = 0 \quad t > 0 \qquad 0 < \alpha \leq 1$$

with the initial condition

$$U(x,0) = e^{-x}$$

where α is a parameter describing the order of the fractional derivative. The function U(x,t) is the probability density function, t is the time, and x is the spatial coordinate. In the case of $\alpha = 1$ the fractional gas dynamics equation reduces to the classical gas dynamics equation.The gas dynamics equations are based on the physical laws of conservation, namely, the laws of conservation of mass, conservation of momentum, conservation of energy, and so forth .The nonlinear fractional gas dynamics has been studied previously by Das and Kumar .$[23]$ The objective of the present paper is to extend the application of this new method to obtain analytic and approximate solutions to the time-fractional gas dynamics equation.The advantage of the method is its capability of combining two powerful methods for obtaining exact and approximate analytical solutions for nonlinear equations. It provides the solutions in terms of convergent series with easily computable components in a direct way without using linearization, perturbation, or restrictive assumptions. It is worth mentioning that this method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result .

2. . **Mathematical fundamentals** :

2.1 Local Fractional Calculus

Definition 1 : The function f(x) is called local fractional continuous at $x = x_0$ if there is the relation $|f(x) - f(x_0)| < \varepsilon^{\propto}$,$0 < \alpha \leq 1$. with $|x - x_0| < \delta$ for ε > 0 , δ > 0 and ε,δ ϵ R .
It is denoted by $\lim_{x \to x_0} f(x) = f(x_0)$. (1)

Definition 2 : The function f(x) is called local fractional continuous on the interval (a,b) if for ε, δ > 0 and ε,δ ϵ R satisfies the relation $|f(x) - f(x_0)| < \varepsilon^{\propto}$, 0<α≤1.
It is denoted by $f(x) \epsilon C_{\propto}(a,b)$. (2)

Definition 3 : In Fractal space let $f(x) \epsilon C_{\propto}(a,b)$;
Local fractional derivative of f(x) of order α at the point $x = x_0$ is given by

$$D_x^{(\propto)} f(x_0) = \frac{d^{\propto}}{dx^{\propto}} f(x)|_{x=x_0} = f^{(\propto)}(x_0) = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\propto}} ,$$

(3)

Were $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\propto +1)(f(x) - f(x_0))$.
The formulas of local Fractional derivatives of special functions used in the paper are as follows

$$D_x^{(\propto)} a\, g(x) = a\, D_x^{(\propto)} g(x) \quad (4)$$

$$\frac{d^{\propto}}{dx^{\propto}} \left(\frac{x^{n\propto}}{\Gamma(1+n\propto)}\right) = \frac{x^{(n-1)\propto}}{\Gamma(1+(n-1)\propto)} \quad n \epsilon N .$$

Definition 4. The Riemann-Liouville fractional integral operator of order $\alpha > 0$ of a function $f(t) \in C\mu$, and $\mu \geq -1$ is defined as [4]

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau \quad , \quad (\alpha > 0)$$

(5)

$$J^0 f(t) = f(t)$$

For the Riemann-Liouville fractional integral, we have

$$J^{\alpha} t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)} t^{\alpha+\gamma}$$

(6)

Definition 5. The fractional derivative of (t) in the Caputo sense is defined as [25]

$$D_t^{\alpha} f(t) = J^{m-\alpha} D^n f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^m(\tau) d\tau \quad , \quad m-1 < \alpha < m$$

(7)

For the Riemann-Liouville fractional integral and the Caputo fractional derivative, we have the following relation:

$$J_t^{\alpha} D_t^{\alpha} f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0+) + \frac{t^k}{k!}$$

(8)

Definition 6. The α – integral transform of the Caputo fractional derivative is defined as follows

$$K_{\alpha} \quad \{D_t^{\alpha} f(t)\} = \frac{K_{\alpha}\{f(t)\}}{v^{2\alpha n}} - \sum_{k=0}^{m-1} \frac{f^{(k\alpha)}(0+)}{v^{(2(m-k)-1)\alpha}}$$

(9)

2.2 *A new Local Fractional α- integral transform and its inverse formula .*

Definition 7 : Let $\frac{1}{\Gamma(\propto+1)} \int_0^{+\infty} |f(t)|(dt)^{\propto} < K < \infty$.
The α- integral transform f(x) is given by

$$K_{\propto}\{f(t)\} = A_{\propto,f}(v) = \frac{1}{\Gamma(\propto+1)} \frac{1}{v^{\propto}} \int_0^{+\infty} E_{\propto}(-(\frac{t}{v^2})^{\propto}) f(t) (dt)^{\propto}$$

0<α≤1. (10)

were Eα is the Mitag – Lefter function , the integral converges and $v^{\alpha} \epsilon R^{\alpha}$

Definition 8 : The inverse formula of the α- integral transform is given by

$$K_{\propto}^{-1}\{A_{\propto,f}(v)\} = \frac{1}{(2\pi i)^{\propto}} \int_{\beta-i\omega}^{\beta+i\omega} E_{\alpha}((vt)^{\propto}) A_{\alpha}(\frac{1}{\sqrt{v}^{\alpha}}) \frac{(dv)^{\propto}}{\sqrt{v}^{\propto}}$$

, (11)

where $v^{\alpha} = \beta^{\alpha} + i^{\alpha}\omega^{\alpha}$; here i^{α} is fractal imaginary unit of v^{α} dhe $Re(v^{\alpha}) = \beta^{\alpha} > 0$.

2.3 Some Basic properties of the Local Fractional α-integral transform .

Let $K_\alpha\{f(t)\} = A_{\alpha,f}(v)$ and

$K_\alpha\{g(t)\} = A_{\alpha,g}(v)$

the Local Fractional α- integral transform of functions f(t) dhe g(t) , then we have the following formulas

$$K_\alpha\{(af(t) + bg(t)\} = aK_\alpha\{f(t)\} + bK_\alpha\{g(t)\} \quad (12)$$

$$K_\alpha\{f^{(\alpha)}(t)\} = \frac{A_{\alpha,f}(v)}{v^{2\alpha}} - \frac{f(0)}{v^{\alpha}} \quad (13)$$

$$K_\alpha\{f^{(n\alpha)}(t)\} = \frac{A_{\alpha,f}(v)}{v^{2\alpha n}} - \sum_{k=0}^{n-1} \frac{f^{(k\alpha)}(0)}{v^{(2(n-k)-1)\alpha}} \quad (14)$$

$$K_\alpha\{t^{k\alpha}\} = v^{\alpha(2k+1)}\Gamma(\alpha k + 1) \quad (15)$$

3. Homotopy Perturbation α – integral Transform Method

To illustrate the basic idea of this method, we consider a general fractional nonlinear nonhomogeneous partial differential equation with the initial condition of the form

$$D_t^\alpha U(x,t) + RU(x,t) + NU(x,t) = g(x,t) \quad (16)$$

with the initial condition

$$U(x,0) = f(x) \quad (17)$$

where $D_t^\alpha U(x,t)$ is the Caputo fractional derivative of the function $U(x, t)$, R is the linear differential operator, N represents the general nonlinear differential operator, and $g(x, t)$ is the source term.
Applying the α-integral transform (denoted in this paper by K_α) on both sides of the equation , we get

$$K_\alpha\{D_t^\alpha U(x,t)\} + K_\alpha\{RU(x,t)\} + K_\alpha\{NU(x,t)\} = K_\alpha\{g(x,t)\} \quad (18)$$

Using the properties of the transform we have

$$\frac{K_\alpha\{U(x,t)\}}{v^{2\alpha}} - \frac{u(x,0)}{v^{\alpha}} + K_\alpha\{RU(x,t)\} + K_\alpha\{NU(x,t)\} = K_\alpha\{g(x,t)\} \quad (19)$$

Using the initial condition

$$\frac{K_\alpha\{U(x,t)\}}{v^{2\alpha}} - \frac{f(x)}{v^{\alpha}} + K_\alpha\{RU(x,t)\} + K_\alpha\{NU(x,t)\} = K_\alpha\{g(x,t)\} \quad (20)$$

Hence we have

$$K_\alpha\{U(x,t)\} = v^\alpha f(x) - v^{2\alpha} K_\alpha\{RU(x,t)\} - v^{2\alpha}K_\alpha\{NU(x,t)\} + v^{2\alpha}K_\alpha\{g(x,t)\} \quad (21)$$

So that

$$K_\alpha\{U(x,t)\} = v^\alpha f(x) + v^{2\alpha}K_\alpha\{g(x,t)\} - v^{2\alpha} K_\alpha\{RU(x,t) + NU(x,t)\} \quad (22)$$

Operating with the α – integral inverse on both sides of equation we have

$$U(x,t) = G(x,t) - K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\{RU(x,t) + NU(x,t)\}\right\} \quad (23)$$

where

$$G(x,t) = v^\alpha f(x) + v^{2\alpha}K_\alpha\{g(x,t)\} \quad (24)$$

G(x, t) represents the term arising from the source term and the prescribed initial conditions. Now we apply the Homotopy Pertubation method

$$U(x,t) = \sum_{n=0}^{\infty} p^n U_n(x,t) \quad (25)$$

and the nonlinear term can be decomposed as

$$NU(x,t) = \sum_{n=0}^{\infty} p^n H_n(U) \quad (26)$$

for some He's polynomials $H_n(u)$ that are given by

$$H_n(U_0, U_1, \dots, U_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i U_i\right)\right]_{p=0} \quad n = 0,1,2, \dots \tag{27}$$

Substituding the decomposed terms we obtain

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = G(x,t) - p\left(K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\{R\sum_{n=0}^{\infty} p^n U_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(U)\}\right\}\right) \tag{28}$$

which is the coupling of the α – integral transform and the Homotopy Pertubation Method using He's polynomials. Comparing the coefficients of like powers of p, the following approximations are obtained:

$$p^0: \; u_0(x,t) = G(x,t) \tag{29}$$

$$p^1: \; u_1(x,t) = -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\{RU_0(x,t) + H_0(U)\}\right\} \tag{30}$$

$$p^2: \; u_2(x,t) = -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\{RU_1(x,t) + H_1(U)\}\right\} \tag{31}$$

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Proceeding in this same manner, the rest of the components $U_n(x, t)$ can be completely obtained and the series solution is thus entirely determined. Finally, we approximate the analytical solution (x, t) by truncated series

$$U(x,t) = \lim_{N\to\infty} \sum_{n=0}^{N} U_n(x,t). \tag{32}$$

The above series solutions generally converge very rapidly.

4. Solution of the Problem.

Consider the following nonlinear time-fractional gas dynamics equation:

Consider the following nonlinear time-fractional gas dynamics equation:

$$D_t^\alpha U + \frac{1}{2}(U^2)_x - U(1-U) = 0 \quad t > 0 \quad 0 < \alpha \le 1 \tag{33}$$

with the initial condition

$$U(x,0) = e^{-x} \tag{34}$$

Applying the α – integral transform on both sides of , we have

$$\frac{K_\alpha\{U(x,t)\}}{v^{2\alpha}} - \frac{u(x,0)}{v^\alpha} = -K_\alpha\left\{\frac{1}{2}(U^2)_x - U(1-U)\right\} \tag{35}$$

Applying the initial value , we have

$$K_\alpha\{U(x,t)\} = v^\alpha e^{-x} - v^{2\alpha} K_\alpha\left\{\frac{1}{2}(U^2)_x - U(1-U)\right\} \tag{36}$$

Operating with the α – integral inverse on both sides of equation we have

$$U(x,t) = K_\alpha^{-1}\{v^\alpha e^{-x}\} - K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}(U^2)_x - U(1-U)\right\}\right\} \tag{37}$$

Applying the Homotopy Pertubation Method we have

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = e^x - p\left(K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}\sum_{n=0}^{\infty} p^n H_n(U) - \sum_{n=0}^{\infty} p^n U_n(x,t) - \sum_{n=0}^{\infty} p^n H'_n(U)\right\}\right\}\right) \tag{38}$$

where $_n(U)$ and $H'_n(U)$ are He's polynomials that represent the nonlinear terms. So, the He's polynomials are given by

$\sum_{n=0}^{\infty} p^n U_n(x,t) = (U^2)_x$ (39)

The first few components of He's polynomials are given by

$$H_0(U) = (U_0^2)_x$$
$$H_1(U) = (2U_0U_1)_x \quad (40)$$
$$H_2(U) = (U_1^2 + 2U_0U_2)_x$$
.
.
.

For $H_n'(U)$ we find that

$$H_0'(U) = U_0^2$$
$$H_1'(U) = 2U_0U_1 \quad (41)$$
$$H_2'(U) = U_1^2 + 2U_0U_2$$
.
.
.

Comparing the coefficients of like powers of p, we have

$$p^0: \ u_0(x,t) = e^{-x} \quad (42)$$

$$p^1: u_1(x,t) = -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_0(U) - U_0 + H'_0(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_0(U) - e^{-x} + H'_0(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\{-v^{3\alpha} e^{-x}\}$$

$$= \frac{t^k}{\Gamma(1+\alpha)} e^{-x}. \quad (43)$$

$$p^2: u_2(x,t) = -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_1(U) - U_1 + H'_1(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_0(U) - \frac{t^k}{\Gamma(1+\alpha)}e^{-x} + H'_0(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\{-v^{5\alpha} e^{-x}\}$$

$$= \frac{t^{2k}}{\Gamma(1+2\alpha)} e^{-x}. \quad (44)$$

$$p^3: u_2(x,t) = -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_2(U) - U_2 + H'_2(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\left\{v^{2\alpha} K_\alpha\left\{\frac{1}{2}H_0(U) - \frac{t^{2k}}{\Gamma(1+2\alpha)}e^{-x} + H'_0(U)\right\}\right\}$$

$$= -K_\alpha^{-1}\{-v^{7\alpha} e^{-x}\}$$

$$= \frac{t^{3k}}{\Gamma(1+3\alpha)} e^{-x}. \quad (45)$$
.
.
.

Therefore, the series solution is

$$U(x,t) = e^{-x}\left[1 + \frac{t^k}{\Gamma(1+\alpha)} + \frac{t^{2k}}{\Gamma(1+2\alpha)} + \frac{t^{3k}}{\Gamma(1+3\alpha)} + \cdots\right] \quad (46)$$

The approximate solution obtained by the present method is very near to the exact solution. It is to be noted that only the third-order term of this method was used in evaluating the approximate solutions . It is evident that the efficiency of the present method can be dramatically enhanced by computing further terms of (x, t) .

5. Conclusions

In this paper, the homotopy perturbation α-integral transform method is successfully applied for solving nonlinear time fractional gas dynamics equation.Therefore, this method is very powerful and efficient technique for solving different kinds of linear and nonlinear fractional differential equations arising in different fields of science and engineering. Inconclusion, this method is a nice refinement in existing numerical techniques and might find the wide applications.

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