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Some New Families of Face Integer Edge Cordial Graphs

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Abstract

In this paper, face integer edge cordial labeling of duplication of each vertex by an edge in $K_{1,n,n}$, duplication of each edge by a vertex in $K_{1,n,n}$, double triangular snake DT_n, $K_{1,n}\odot P_m$, $(P_n\odot K_1)\odot P_m$ and duplication of each edge by a vertex in $(P_n\odot K_1)$ are presented.

Keywords: *Integer cordial graph, Face integer edge cordial graph, Double triangular snake.*

1. Introduction

By a graph, we mean a simple, finite, planar and undirected unless otherwise specified. A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G, for terms not defined here, we refer to Harary [3]. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [2]. The concept of cordial labeling of a graph was introduced by Cahit [1]. The concept of edge product cordial labeling of graph was introduced by Vaidya et al.[9]. Sedlacek [8] defined a graph to be magic if it had an edge-labeling, with range the real numbers, such that the sum of the labels around any vertex equals some constant, independent of the choice of vertex. In [5], Lih introduced magic labelings of planar graphs where labels extended to faces as well as edges and vertices. In [4], Lawrence et al. introduced the concept of face edge product cordial labeling of graph. In [7], Nicholas et al. introduced the concept of integer cordial labeling of graph. The concept of face integer cordial labeling and face integer edge cordial labeling of graph were introduced by Mohamed Sheriff et al [6].

The present work is focused only on face integer edge cordial labeling of some new families of graphs. The face integer edge cordial labeling of duplication of each vertex by an edge in $K_{1,n,n}$, duplication of each edge by a vertex in $K_{1,n,n}$, double triangular snake DT_n, $K_{1,n}\odot P_m$, $(P_n\odot K_1)\odot P_m$ and duplication of each edge by a vertex in $(P_n\odot K_1)$ are presented. The brief summaries of definition which are necessary for the present investigation are provided below.

Definition: 1.1

A mapping $f : V(G)\rightarrow\{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

Definition: 1.2

A binary vertex labeling f of a graph G is called a cordial labeling if $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$. A graph G is cordial if it admits cordial labeling.

Definition: 1.3

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a product cordial labeling of G if $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and the number of edges respectively labeled with i (i = 0,1). A graph with a product cordial labeling is called a product cordial graph.

Definition: 1.4

For a graph G = (V(G), E(G)), an edge labeling function $f : E(G)\rightarrow\{0,1\}$ induces a vertex labeling function f^* $:V(G)\rightarrow\{0,1\}$ defined as $f^*(v) = \prod f(e_i)$ for $\{e_i \in E(G)/e_i$ is incident to v}. Now denoting the number of vertices of G having label i under f^* as $v_f(i)$ and the number of edges of G having label i under f as $e_f(i)$. Then f is called edge product cordial labeling of graph G if $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. A graph G is called edge product cordial if it admits edge product cordial labeling.

Definition: 1.5

For a planar graph G, the edge labeling function is defined as $g : E(G) \rightarrow \{0,1\}$ and g(e) is called the label of the edge e of G under g, induced vertex labeling function $g^* : V(G) \rightarrow \{0,1\}$ is given as if $e_1, e_2, \ldots, e_m$ are the edges incident to vertex v, then $g^*(v) = g(e_1)g(e_2)\ldots g(e_m)$ and induced face labeling function $g^{**} : F(G) \rightarrow \{0,1\}$ is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f then $g^{**}(f) = g^*(v_1)g^*(v_2)\ldots g^*(v_n)$

$g(e_1)g(e_2)\ldots g(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g^*, $e_g(i)$ is the number of edges of G having label i under g and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 0,1. g is called face edge product cordial labeling of graph G if $|v_g(0)-v_g(1)|\le 1$, $|e_g(0)-e_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$. A graph G is face edge product cordial if it admits face edge product cordial labeling.

Definition: 1.6

Let G be a simple connected graph with p vertices. Let $f:V\to \left[-\frac{p}{2},\ldots,\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor,\ldots,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd be an injective map, which induces an edge labeling f^* such that f(uv) = 1, if $f(u)+f(v) \ge 0$ and f(uv) = 0 otherwise. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. f is said to be integer cordial if $|e_f(0) - e_f(1)| \le 1$. A graph G is called integer cordial if it admits an integer cordial labeling. Here $[-x,\ldots,x] = \{t / t$ is an integer and $|t| \le x\}$ and $[-x,\ldots,x]^* = [-x, \ldots, x] - \{0\}$.

Definition: 1.7

For a planar graph G, the vertex labeling function is defined as $g : V \to \left[-\frac{p}{2},\ldots,\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor,\ldots,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd be an injective map, which induces an edge labeling function $g^* : E(G)\to\{0,1\}$ such that $g^*(uv) = 1$, if $g(u)+g(v) \ge 0$ and $g^*(uv) = 0$ otherwise and face labeling function $g^{**}:F(G)\to\{0,1\}$ such that $g^{**}(f) = 1$, if $g^{**}(f) = g(v_1)+g(v_2)+\ldots+ g(v_n) \ge 0$ and $g^{**}(f) = 0$ otherwise, where $v_1, v_2, \ldots, v_n$ are the vertices of face f. g is called face integer cordial labeling of graph G if $|e_g(0) - e_g(1)| \le 1$ and $|f_g(0)-f_g(1)| \le 1$. $e_g(i)$ is the number of edges of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 0,1. A planar graph G is face integer cordial if it admits face integer cordial labeling.

Definition: 1.8

For a planar graph G, an edge labeling function is defined as $g : E\to\left[-\frac{p}{2},\ldots,\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor,\ldots,\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd be an injective map, which induces vertex labeling function $g^* : V(G) \to \{0,1\}$ such that $g^*(v) = 1$, if $\sum_i g(e_i) \ge 0$ and $g^*(v) = 0$ otherwise, where $e_1, e_2, \ldots, e_n$ are the adjacent edges of the vertex v and face labeling function $g^{**} : F(G) \to \{0,1\}$ such that $g^{**}(f) = 1$, if $g^{**}(f) = g(e_1)+g(e_2)+\ldots+g(e_n) \ge 0$ and $g^{**}(f) = 0$ otherwise, where $e_1, e_2, \ldots, e_n$ are the edges of face f. g is called face integer edge cordial labeling of graph G if $|v_g(0)-v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$. $v_g(i)$ is the number of vertices of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 0,1. A planar graph G is face integer edge cordial if it admits face integer edge cordial labeling.

Definition: 1.9

Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G′ such that $N(v'_k) = \{v_k, v''_k\}$ and $N(v''_k) = \{v_k, v'_k\}$.

Definition: 1.10

Duplication of an edge $e = v_i v_{i+1}$ by a vertex v_k in a graph G produces a new graph G′ such that $N(v_k) = \{v_i, v_{i+1}\}$.

Definition: 1.11

The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

2. Main Results

Theorem 2.1

The duplication of each vertex by an edge in $K_{1,n,n}$ is face integer edge cordial graph, for $n \ge 2$.

Proof.

Let $e_1,e_2,\ldots,e_{2n}$ and $v,v_1,v_2,\ldots,v_{2n}$ be the edges and vertices of $K_{1,n,n}$. Let G be the duplication each edge by a vertex in $K_{1,n,n}$, where $n \ge 2$.

Let $e_1,e_2,\ldots,e_{8n+3}$ and $v,v_1,v_2,\ldots,v_{2n}$, $u,u_1,u_2,\ldots,u_{2n}$, $w,w_1,w_2,\ldots,w_{2n}$ be the edges and vertices of the graph G, where $e_i = vv_i$, $e_{n+i} = u_iv_i$, $e_{2n+i} = w_iv_i$, $e_{3n+i} = u_iw_i$, $e_{4n+i} = v_iv_{n+i}$, $e_{5n+i} = u_{n+i}v_{n+i}$, $e_{6n+i} = w_{n+i}v_{n+i}$ and $e_{7n+i} = u_{n+i}w_{n+i}$, for $1 \le i \le n$, $e_{8n+1} = vu$, $e_{8n+2} = uw$ and $e_{8n+3} = vw$. Let $f,f_1,f_2,\ldots,f_{2n}$ be an interior faces of G, where f = vuwv, $f_i = v_iu_iw_iv_i$ and $f_{n+i} = v_{n+i}u_{n+i}w_{n+i}v_{n+i}$, for $1 \le i \le n$. Then $|V(G)| = 6n+3$, $|E(G)| = 8n+3$ and $|F(G)| = 2n+1$.

Define edge labeling of $g : E(G) \to [-k,\ldots,k]$ as follows.

$g(e_i) = -[i+1]$ for $1 \le i \le n$
$g(e_i) = i - [n+1]$ for $n+1 \le i \le 2n$
$g(e_i) = -i + [n-1]$ for $2n+1 \le i \le 5n$
$g(e_i) = i - [4n-1]$ for $5n+1 \le i \le 8n$
$g(e_{8n+1}) = 1$,
$g(e_{8n+2}) = 0$,
$g(e_{8n+3}) = -1$.

Then induced edge labels are

$g^*(v) = 0$
$g^*(u) = 1$
$g^*(w) = 0$
$g^*(v_i) = g^*(u_i) = g^*(w_i) = 0$ for $1 \le i \le n$
$g^*(v_{n+i}) = g^*(u_{n+i}) = g^*(w_{n+i}) = 1$ for $1 \le i \le n$

Also the induced face labels are

$g^{**}(f_i) = 0$ for $1 \le i \le n$

$g^{**}(f_i) = 1$ for $n+1 \le i \le 2n$

$g^{**}(f) = 1$.

In view of the above defined labeling pattern we have $v_g(0) = v_g(1) + 1 = 3n+2$, and $f_g(1) = f_g(0)+1 = n+1$. Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, G is face integer edge cordial graph.

Example 2.1

The duplication of each vertex by an edge in $K_{1,3,3}$ and its face integer edge cordial labeling is shown in figure 2.1.

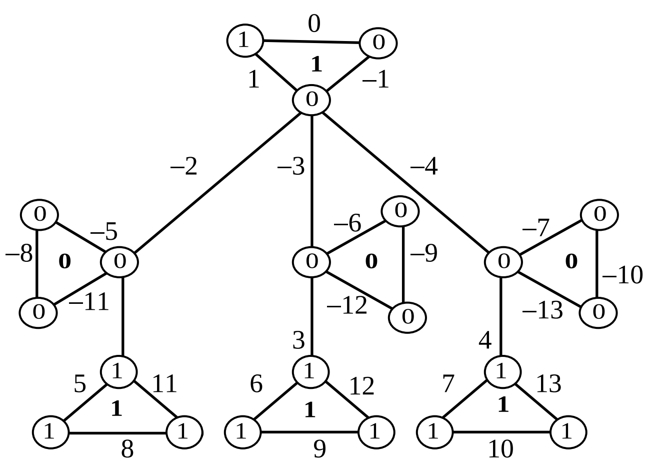

Figure 2.1

Theorem 2.2

The duplication of each edge by a vertex in $K_{1,n,n}$ is face integer edge cordial graph, where $n \ge 2$.

Proof.

Let $e_1,e_2,\ldots,e_{2n}$ and $v,v_1,v_2,\ldots,v_{2n}$ be the edges and vertices of $K_{1,n,n}$. Let G be the duplication each edge by a vertex in $K_{1,n,n}$, where $n \ge 2$.

Let $e_1,e_2,\ldots,e_{6n}$ and $v,v_1,v_2,\ldots,v_{2n},u_1,u_2,\ldots,u_{2n}$ be the edges and vertices of the graph G, where $e_i = vv_i$, $e_{n+i} = vu_i$, $e_{2n+i} = v_iu_i$, $e_{3n+i} = v_iv_{n+i}$, $e_{4n+i} = v_iu_{n+i}$ and $e_{5n+i} = v_{n+i}u_{n+i}$, for $1 \le i \le n$. Let $f_1, f_2, \ldots, f_{2n}$ be an interior faces of G, where $f_i = vv_iu_iv$ and $f_{n+i} = v_iv_{n+i}u_{n+i}v_i$, for $1 \le i \le n$.

Then $|V(G)| = 4n+1$, $|E(G)| = 6n$ and $|F(G)| = 2n$.

Define edge labeling of $g : E(G) \rightarrow [-k,\ldots,k]$ as follows.

$g(e_i) = -i$ for $1 \le i \le 3n$

$g(e_i) = i - [3n]$ for $3n+1 \le i \le 6n$

Then induced vertex labels are

$g^*(v) = 0$

$g^*(u_i) = 0,$ for $1 \le i \le n$

$g^*(v_i) = 0,$ for $1 \le i \le n$

$g^*(u_i) = 1,$ for $n+1 \le i \le 2n$

$g^*(v_i) = 1,$ for $n+1 \le i \le 2n$

Also the induced face labels are

$g^{**}(f_i) = 0$ for $1 \le i \le n$

$g^{**}(f_i) = 1$ for $n \le i \le 2n$

$g^{**}(f_{2n+1}) = 1$.

In view of the above defined labeling pattern we have $v_g(0) = v_g(1) + 1 = 2n+1$, and $f_g(1) = f_g(0) = n$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, G is face integer edge cordial graph.

Example 2.2

The duplication of each edge by a vertex in $K_{1,2,2}$ and its face integer edge cordial labeling is shown in figure 2.2.

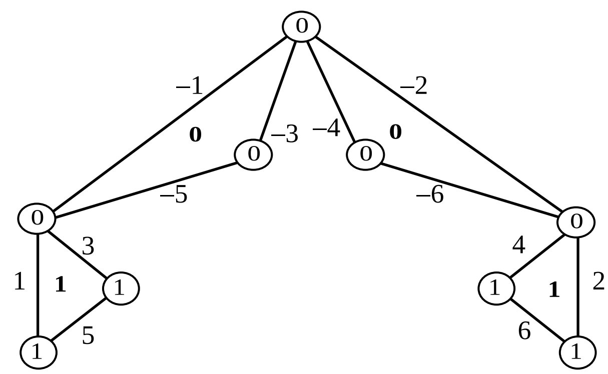

Figure 2.2

Theorem 2.3

Double triangular snake DT_n is a face integer edge cordial graph for $n \ge 3$.

Proof.

Let $e_1,e_2,\ldots,e_{5n-5}$ be edges, $v_1,v_2,\ldots,v_n$, $u_1,u_2,\ldots,u_{n-1}$, $w_1,w_2,\ldots,u_{n-1}$ be vertices and $f_1,f_2,\ldots,f_{2n-2}$ be an interior faces of DT_n, where $e_{2i-1} = v_iu_i$, $e_{2i} = u_iv_{i+1}$, $e_{2n-2+i} = v_iv_{i+1}$, $e_{3n+2i-4} = v_iw_i$, and $e_{3n+2i-3} = w_iv_{i+1}$ for $i = 1,2,\ldots,n-1$, $f_i = v_iu_iv_{i+1}v_i$ for $i = 1,2,\ldots,n-1$ and $f_{i+n-1} = v_iw_iv_{i+1}v_i$ for $i = 1,2,\ldots,n-1$. Let G be the double triangular snake DT_n. Then $|V(G)| = 3n-2$, $|E(G)| = 5n-5$ and $|F(G)| = 2n-2$.

Case (i) : n is odd

Define edge labeling of $g : E(G) \rightarrow [-k,\ldots,k]^*$ as follows.

$g(e_i) = i$ for $1 \le i \le n-1$

$g(e_i) = -i + [n-1]$ for $n \le i \le 2n-2$

$g(e_i) = i - [n-1]$ for $2n-1 \le i \le \frac{5n-5}{2}$

$g(e_i) = -i + \left[\frac{3n-3}{2}\right]$ for $\frac{5n-3}{2} \le i \le 3n-3$

$g(e_i) = i - \left[\frac{3n-3}{2}\right]$ for $3n-2 \le i \le 4n-4$

$g(e_i) = -i + \left[\frac{5n-5}{2}\right]$ for $4n-3 \le i \le 5n-5$

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Then induced edge labels are

$g^*(u_i) = 1$ for $1 \le i \le \frac{n-1}{2}$

$g^*(u_i) = 0$ for $\frac{n+1}{2} \le i \le n-1$

$g^*(v_i) = 1$ for $1 \le i \le \frac{n+1}{2}$

$g^*(v_i) = 0$ for $\frac{n+3}{2} \le i \le n$

$g^*(w_i) = 1$ for $1 \le i \le \frac{n-1}{2}$

$g^*(w_i) = 0$ for $\frac{n+1}{2} \le i \le n-1$

Also the induced face labels are

$g^{**}(f_i) = 1$ for $1 \le i \le \frac{n-1}{2}$

$g^{**}(f_i) = 0$ for $\frac{n+1}{2} \le i \le n-1$

$g^{**}(f_{n-1+i}) = 1$ for $1 \le i \le \frac{n-1}{2}$

$g^{**}(f_{n-1+i}) = 0$ for $\frac{n+1}{2} \le i \le n-1$

In view of the above defined labeling pattern, we have $v_f(0) = v_f(1)+1 = \frac{3n-1}{2}$ and $f_g(0) = f_g(1) = n-1$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus the graph DT_n is face integer edge cordial graph for n is odd.

Case 2: n is even

Define edge labeling of $g : E(G) \to [-k,\ldots,k]$ as follows.

$g(e_i) = i$ for $1 \le i \le n-1$

$g(e_i) = 2$ for $i = n-1$

$g(e_i) = 1$ for $i = n$

$g(e_i) = -i + [n-2]$ for $n+1 \le i \le 2n-2$

$g(e_i) = i - [n-2]$ for $2n-1 \le i \le \frac{5n-6}{2}$

$g(e_i) = 0$ for $i = \frac{5n-4}{2}$

$g(e_i) = -i + \left[\frac{3n-4}{2}\right]$ for $\frac{5n-2}{2} \le i \le 3n-3$

$g(e_i) = i - \left[\frac{3n-4}{2}\right]$ for $3n-2 \le i \le 4n-5$

$g(e_i) = -1$ for $i = 4n-4$

$g(e_i) = -2$ for $i = 4n-3$

$g(e_i) = -i + \left[\frac{5n-4}{2}\right]$ for $4n-2 \le i \le 5n-5$

Then induced vertes labels are

$g^*(u_i) = 1$ for $1 \le i \le \frac{n}{2}$

$g^*(u_i) = 0$ for $\frac{n+2}{2} \le i \le n-1$

$g^*(v_i) = 1$ for $1 \le i \le \frac{n}{2}$

$g^*(v_i) = 0$ for $\frac{n+2}{2} \le i \le n$

$g^*(w_i) = 1$ for $1 \le i \le \frac{n-2}{2}$

$g^*(w_i) = 0$ for $\frac{n}{2} \le i \le n-1$

Also the induced face labels are

$g^{**}(f_i) = 1$ for $1 \le i \le \frac{n}{2}$

$g^{**}(f_i) = 0$ for $\frac{n+2}{2} \le i \le n-1$

$g^{**}(f_{n-1+i}) = 1$ for $1 \le i \le \frac{n-2}{2}$

$g^{**}(f_{n-1+i}) = 0$ for $\frac{n}{2} \le i \le n-1$

In view of the above defined labeling pattern, we have $v_f(0) = v_f(1) = \frac{3n-2}{2}$ and $f_g(0) = f_g(1) = n-1$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Thus the graph DT_n is face integer edge cordial graph for n is even.

Hence the graph DT_n is face integer edge cordial graph for $n \ge 3$.

Example 2.3

The graph DT_4 and its face integer edge cordial labeling is shown in figure 2.3.

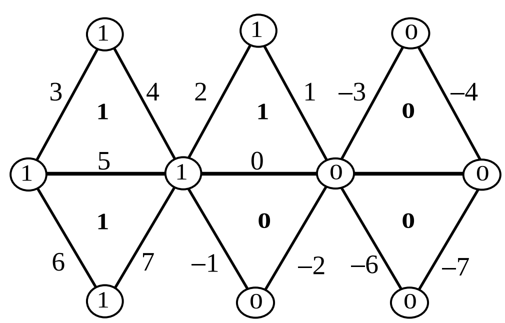

Figure 2.3

Theorem 2.4

$K_{1,n} \odot P_m$ is face integer edge cordial graph except n is even and m is odd.

Proof.

Let $u_1, u_2, \ldots, u_{n+1}$ and $e_1, e_2, \ldots, e_n$ be the vertices and edges of $K_{1,n}$.

Let G be the graph $K_{1,n} \odot P_m$.

The vertex set $V(G) = \{u_i, v_{ij} : 1 \le i \le n+1, 1 \le j \le m\}$, edge set $E(G) = \{e_i, e_{jk} : 1 \le i \le n,\ 1 \le j \le n+1, 1 \le k \le 2m-1\}$ and interior face set $F(G) = \{ f_i : 1 \le i \le (n+1)(m-1)\}$ of G, where $e_i = u_i u_{n+1}$ for $1 \le i \le n$, $e_{jk} = u_j v_{jk}$ for $1 \le j \le n+1$ and $1 \le k \le m$, $e_{j(m+k)} = v_{jk} v_{j(k+1)}$ for $1 \le j \le n+1$ and $1 \le k \le m-1$, $f_i = u_i\, v_{ik}\, v_{i(k+1)}\, u_i$ for $1 \le i \le n+1$ and $1 \le k \le m-1$.

Then $|V(G)| = (n+1)(m+1)$, $|E(G)| = 2(n+1)m-1$ and $|F(G)| = (n+1)(m-1)$.

Define edge labeling of $g : E(G) \to [-k, \ldots, k]$ as follows.

Case 1 : n is odd and m is either odd or even.

$g(e_i) = -i \qquad \text{for } 1 \le i \le \frac{n-1}{2}$

$g(e_i) = 0 \qquad \text{for } i = \frac{n+1}{2}$

$g(e_i) = i - \left[\frac{n+1}{2}\right] \qquad \text{for } \frac{n+3}{2} \le i \le n$

$g(e_{jk}) = -\left[\frac{n-1}{2} + k + (j-1)(2m-1)\right],$

$\text{for } 1 \le j \le \frac{n+1}{2} \text{ and } 1 \le k \le 2m-1$

$g(e_{jk}) = \left[\frac{n-1}{2} + k + \left(j - \left[\frac{n+3}{2}\right]\right)(2m-1)\right],$

$\text{for } \frac{n+3}{2} \le j \le n+1 \text{ and } 1 \le k \le 2m-1$

Then induced vertex labels are

$g^*(u_i) = 0 \qquad \text{for } 1 \le i \le \frac{n+1}{2}$

$g^*(u_i) = 1 \qquad \text{for } \frac{n+3}{2} \le i \le n+1$

$g^*(v_{ij}) = 0, \qquad \text{for } 1 \le i \le \frac{n+1}{2} \text{ and } 1 \le j \le m$

$g^*(v_{ij}) = 1, \qquad \text{for } \frac{n+3}{2} \le i \le n+1 \text{ and } 1 \le j \le m$

Also the induced face labels are

$g^{**}(f_i) = 1 \quad \text{for } 1 \le i \le \left[\frac{n+1}{2}\right](m-1)$

$g^{**}(f_i) = 0 \quad \text{for } \left[\frac{n+3}{2}\right](m-1) \le i \le (n+1)(m-1)$

In view of the above defined labeling pattern, we have $v_f(0) = v_f(1) = \frac{(n+1)(m+1)}{2}$ and $f_g(0) = f_g(1) = \frac{(n+1)(m-1)}{2}$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, $K_{1,n} \odot P_m$ is face integer edge cordial graph for n is odd and m is either odd or even.

Case 2 : n is even

Sub case 2.1 : n is even and m is even.

$g(e_i) = -i \qquad \text{for } 1 \le i \le \frac{n}{2}$

$g(e_i) = i - \left[\frac{n}{2}\right] \qquad \text{for } \frac{n+2}{2} \le i \le n$

$g(e_{1k}) = -[nm+k], \qquad \text{for } 1 \le k \le \frac{m}{2}$

$g(e_{1k}) = nm + \left(k - \frac{m}{2}\right), \qquad \text{for } \frac{m+2}{2} \le k \le m$

$g(e_{1k}) = -\frac{m}{2}[2n+1] - [k-m],$

$\text{for } m+1 \le k \le \frac{3m-2}{2}$

$g(e_{1k}) = 0 \qquad \text{for } k = \frac{3m}{2}$

$g(e_{1k}) = \frac{m}{2}[2n+1] + \left(k - \frac{3m}{2}\right),$

$\text{for } \frac{3m+2}{2} \le k \le 2m-1$

$g(e_{jk}) = -\left[\frac{n}{2} + k + (j-2)(2m-1)\right],$

$\text{for } 2 \le j \le \frac{n+2}{2} \text{ and } 1 \le k \le 2m-1$

$g(e_{jk}) = \left[\frac{n}{2} + k + \left(j - \left[\frac{n+4}{2}\right]\right)(2m-1)\right],$

$\text{for } \frac{n+4}{2} \le j \le n+1 \text{ and } 1 \le k \le 2m-1$

Then induced vertex labels are

$g^*(u_1) = 1$

$g^*(u_i) = 0 \qquad \text{for } 2 \le i \le \frac{n+2}{2}$

$g^*(u_i) = 1 \qquad \text{for } \frac{n+4}{2} \le i \le n+1$

$g^*(v_{1j}) = 0, \qquad \text{for } 1 \le j \le \frac{m}{2}$

$g^*(v_{1j}) = 1, \qquad \text{for } \frac{m}{2} + 1 \le j \le m$

Also the induced face labels are

$g^{**}(f_i) = 0 \quad \text{for } 1 \le i \le \frac{m}{2}$

$g^{**}(f_i) = 1 \quad \text{for } \frac{m+2}{2} \le i \le m-1$

$g^{**}(f_i) = 0 \quad \text{for } m \le i \le \left[\frac{n+2}{2}\right](m-1)$

$g^{**}(f_i) = 1$ for $\left[\frac{n+4}{2}\right](m-1) \le i \le (n+1)(m-1)$

In view of the above defined labeling pattern, we have $v_f(1) = v_f(0)+1 = \frac{(n+1)(m+1)+1}{2}$ and $f_g(0) = f_g(1)+1 = \frac{(n+1)(m-1)+1}{2}$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, $K_{1,n} \odot P_m$ is face integer edge cordial graph for n is even and m is even.

Sub case 2.2(a) : n is even, m is odd and m = 3.

$g(e_i) = -i$ for $1 \le i \le \frac{n}{2}$

$g(e_i) = i - \left[\frac{n}{2}\right]$ for $\frac{n+2}{2} \le i \le n$

$g(e_{11}) = -[nm+2]$,
$g(e_{12}) = 0$,
$g(e_{13}) = nm+1$,
$g(e_{14}) = -[nm+1]$,
$g(e_{15}) = nm+2$,

$g(e_{jk}) = -\left[\frac{n}{2} + k + (j-2)(2m-1)\right]$,

for $2 \le j \le \frac{n+2}{2}$ and $1 \le k \le 2m-1$

$g(e_{jk}) = \left[\frac{n}{2} + k + \left(j - \left[\frac{n+4}{2}\right]\right)(2m-1)\right]$,

for $\frac{n+4}{2} \le j \le n+1$ and $1 \le k \le 2m-1$

Then induced vertex labels are

$g^*(u_1) = 0$

$g^*(u_i) = 0$ for $2 \le i \le \frac{n+2}{2}$

$g^*(u_i) = 1$ for $\frac{n+4}{2} \le i \le n+1$

$g^*(v_{1j}) = 0$, for $1 \le j \le \frac{m-1}{2}$

$g^*(v_{1j}) = 1$, for $\frac{m+1}{2} \le j \le m$

Also the induced face labels are

$g^{**}(f_i) = 0$ for $1 \le i \le \frac{m-1}{2}$

$g^{**}(f_i) = 1$ for $\frac{m+1}{2} \le i \le m-1$

$g^{**}(f_i) = 0$ for $m \le i \le \left[\frac{n+2}{2}\right](m-1)$

$g^{**}(f_i) = 1$ for $\left[\frac{n+4}{2}\right](m-1) \le i \le (n+1)(m-1)$

In view of the above defined labeling pattern, we have $v_f(1) = v_f(0)+1 = \frac{(n+1)(m+1)+1}{2}$ and $f_g(0) = f_g(1)+1 = \frac{(n+1)(m-1)+1}{2}$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, the graph $K_{1,n} \odot P_m$ is face integer edge cordial graph for n is even, m is odd and m = 3.

Sub case 2.2(b) : n is even, m is odd and m > 3.

$g(e_i) = -i$ for $1 \le i \le \frac{n}{2}$

$g(e_i) = i - \left[\frac{n}{2}\right]$ for $\frac{n+2}{2} \le i \le n$

$g(e_{1k}) = -[nm+k]$, for $1 \le k \le \frac{m-1}{2}$

$g(e_{1k}) = 0$, for $k = \frac{m+1}{2}$

$g(e_{1k}) = nm + \left(k - \left[\frac{m+1}{2}\right]\right)$, for $\frac{m+3}{2} \le k \le m$

$g(e_{1k}) = -\frac{m(2n+1)-1}{2} - [k-m]$,

for $m+1 \le k \le \frac{3m-1}{2}$

$g(e_{1k}) = \frac{m}{2}[2n+1] + \left(k - \frac{3m}{2}\right)$,

for $\frac{3m+1}{2} \le k \le 2m-1$

$g(e_{jk}) = -\left[\frac{n}{2} + k + (j-2)(2m-1)\right]$,

for $2 \le j \le \frac{n+2}{2}$ and $1 \le k \le 2m-1$

$g(e_{jk}) = \left[\frac{n}{2} + k + \left(j - \left[\frac{n+4}{2}\right]\right)(2m-1)\right]$,

for $\frac{n+4}{2} \le j \le n+1$ and $1 \le k \le 2m-1$

Then induced vertex labels are

$g^*(u_1) = 1$

$g^*(u_i) = 0$ for $2 \le i \le \frac{n+2}{2}$

$g^*(u_i) = 1$ for $\frac{n+4}{2} \le i \le n+1$

$g^*(v_{1j}) = 0$, for $1 \le j \le \frac{m+1}{2}$

$g^*(v_{1j}) = 1$, for $\frac{m+3}{2} \le j \le m$

Also the induced face labels are

$$g^{**}(f_i) = 0 \quad \text{for } 1 \le i \le \frac{m-1}{2}$$

$$g^{**}(f_i) = 1 \quad \text{for } \frac{m+1}{2} \le i \le m-1$$

$$g^{**}(f_i) = 0 \quad \text{for } m \le i \le \left[\frac{n+2}{2}\right](m-1)$$

$$g^{**}(f_i) = 1 \quad \text{for } \left[\frac{n+4}{2}\right](m-1) \le i \le (n+1)(m-1)$$

In view of the above defined labeling pattern, we have $v_f(1) = v_f(0)+1 = \frac{(n+1)(m+1)+1}{2}$ and $f_g(0) = f_g(1)+1 = \frac{(n+1)(m-1)+1}{2}$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, the graph $K_{1,n} \odot P_m$ is face integer edge cordial graph for n is even, m is odd and m > 3.

Thus, the graph $K_{1,n} \odot P_m$ is face integer edge cordial graph for n is even and m is odd.

Hence, the graph $K_{1,n} \odot P_m$ is face integer edge cordial graph.

Example 2.4

The graph $K_{1,2} \odot P_3$ and its face integer edge cordial labeling is given in figure 2.4.

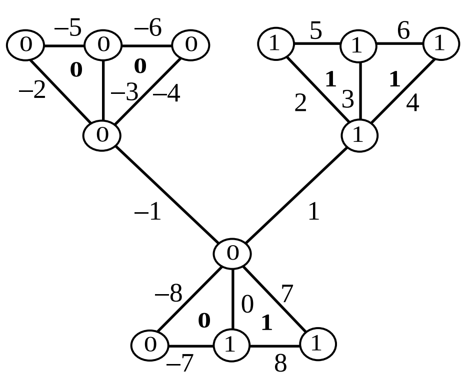

Figure 2.4

Theorem 2.5

$(P_n \odot K_1) \odot P_m$ is face integer edge cordial graph.

Proof.

Let $u_1, u_2, \ldots, u_{2n}$ and $e_1, e_2, \ldots, e_{2n-1}$ be the vertices and edges of $P_n \odot K_1$. Let G be the graph $(P_n \odot K_1) \odot P_m$.

The edge set $E(G) = \{e_i, e_{jk} : 1 \le i \le 2n-1,\ 1 \le j \le 2n$ and $1 \le k \le 2m-1\}$, vertex set $V(G) = \{u_i, v_{ij} : 1 \le i \le 2n,\ 1 \le j \le m\}$, and interior face set $F(G) = \{f_{ik} : 1 \le i \le 2n$ and $1 \le k \le m-1\}$, where $e_i = u_iu_{i+1}$ for $1 \le i \le n-1$, $e_{(n-1)+i} = u_iu_{n+i}$ for $1 \le i \le n$, $e_{jk} = u_jv_{jk}$ for $1 \le j \le 2n$ and $1 \le k \le m$, $e_{j(m+k)} = v_{jk}v_{j(k+1)}$ for $1 \le j \le 2n$ and $1 \le k \le m-1$, $f_{ik} = u_i v_{ik} v_{i(k+1)} u_i$ for $1 \le i \le 2n$ and $1 \le k \le m-1$. Then $|V(G)| = 2n(m+1)$, $|E(G)| = 4nm-1$ and $|F(G)| = 2n(m-1)$.

Define edge labeling of $g : E(G) \to [-k, \ldots, k]$ as follows.

$g(e_i) = -i$ for $1 \le i \le n-1$
$g(e_i) = 0$ for $i = n$
$g(e_i) = i - n$ for $n+1 \le i \le 2n-1$
$g(e_{jk}) = -[(n-1)+k+(j-1)(2m-1)$, for $1 \le j \le n$ and $1 \le k \le 2m-1$
$g(e_{jk}) = [(n-1)+k+(j-n-1)(2m-1)$, for $n+1 \le j \le 2n$ and $1 \le k \le 2m-1$

Then induced vertex labels are

$g^*(u_i) = 0$ for $1 \le i \le n$
$g^*(u_i) = 1$ for $n \le i \le 2n$
$g^*(v_{ij}) = 0$, for $1 \le i \le n$ and $1 \le j \le m$
$g^*(v_{ij}) = 1$, for $n+1 \le i \le 2n$ and $1 \le j \le m$

Also the induced face labels are

$g^{**}(f_{ik}) = 0$, for $1 \le i \le n$ and $1 \le k \le m-1$
$g^{**}(f_{ik}) = 1$, for $n+1 \le i \le 2n$ and $1 \le k \le m-1$

In view of the above defined labeling pattern, we have $v_f(0) = v_f(1) = n(m+1)$ and $f_g(0) = f_g(1) = n(m-1)$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, the graph $(P_n \odot K_1) \odot P_m$ is face integer edge cordial graph.

Example 2.5

The graph $(P_2 \odot K_1) \odot P_3$ and its face integer edge cordial labeling is given in figure 2.5.

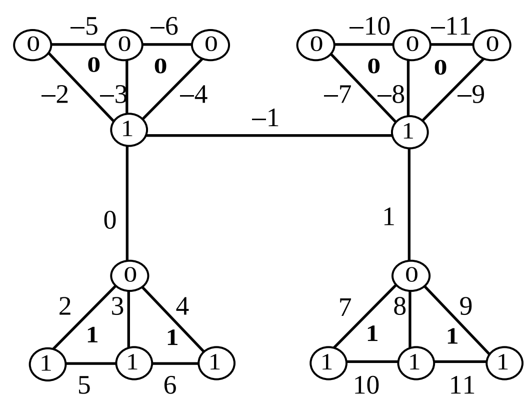

Figure 2.5

Theorem 2.6

The duplication of each edge by a vertex in $(P_n \odot K_1)$ is face integer edge cordial graph, where $n \ge 2$.

Proof.

Let $u_1, u_2, \ldots, u_{2n}$ and $e_1, e_2, \ldots, e_{2n-1}$ be the vertices and edges of $P_n \odot K_1$. Let G be the duplication each edge by a vertex in $(P_n \odot K_1)$, where $n \ge 2$.

The edge set $E(G) = \{e_i : 1 \le i \le 6n-3\}$, vertex set $V(G) = \{u_i, v_j : 1 \le i \le 2n$ and $1 \le j \le 2n-1\}$, and interior face set $F(G) = \{f_i : 1 \le i \le 2n-1\}$, $e_i = u_i u_{i+1}$ for $1 \le i \le n-1$, $e_{n-1+i} = u_i u_{n+i}$ for $1 \le i \le n$, $e_{2n+2i-2} = u_i v_i$ for $1 \le i \le n-1$, $e_{2n+2i-1} = v_i u_{i+1}$ for $1 \le i \le n-1$, $e_{4n+2i-4} = u_i v_{n-1+i}$ for $1 \le i \le n$, $e_{4n+2i-3} = v_{n-1+i} u_{n+i}$ for $1 \le i \le n$, $f_i = u_i u_{i+1} v_i u_i$ for $1 \le i \le n-1$ and $f_{n-1+i} = u_i v_{n-1+i} u_{n+i} u_i$ for $1 \le i \le n$. Then $|V(G)| = 4n-1$, $|E(G)| = 6n-3$ and $|F(G)| = 2n-1$.

Define edge labeling of $g : E(G) \to [-k, \ldots, k]$ as follows.

$g(e_i) = 1+i$ for $1 \le i \le n-1$
$g(e_i) = 0$ for $i = n$
$g(e_i) = -i + n - 1$ for $n+1 \le i \le 2n-1$
$g(e_i) = i - n - 1$ for $2n \le i \le 4n-3$
$g(e_i) = 1$ for $i = 4n-2$
$g(e_i) = -1$ for $i = 4n-1$
$g(e_i) = -i + 3n$ for $4n \le i \le 6n-3$

Then induced vertex labels are

$g^*(v) = 0$
$g^*(u_i) = 0$, for $1 \le i \le n$
$g^*(v_i) = 0$, for $1 \le i \le n$
$g^*(u_i) = 1$, for $n+1 \le i \le 2n$
$g^*(v_i) = 1$, for $n+1 \le i \le 2n$

Also the induced face labels are

$g^{**}(f_i) = 0$ for $1 \le i \le n$
$g^{**}(f_i) = 1$ for $n \le i \le 2n$
$g^{**}(f_{2n+1}) = 1$.

In view of the above defined labeling pattern we have $v_g(0) = v_g(1) + 1 = 2n+1$, and $f_g(1) = f_g(0) = n$.

Then $|v_g(0) - v_g(1)| \le 1$ and $|f_g(0) - f_g(1)| \le 1$.

Therefore, G is face integer edge cordial graph.

Example 2.6

The duplication of each edge by a vertex in $(P_2 \odot K_1)$ and its face integer edge cordial labeling is shown in figure 2.6.

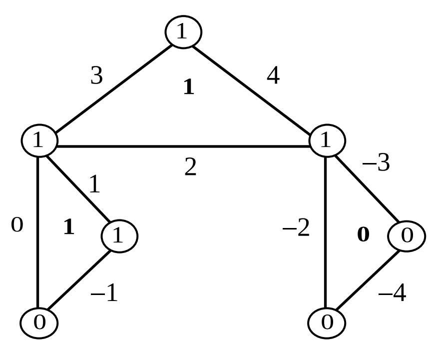

Figure 2.6

4. Conclusions

In this paper, we present the face integer edge cordial labeling of duplication of each vertex by an edge in $K_{1,n,n}$, duplication of each edge by a vertex in $K_{1,n,n}$, double triangular snake DT_n, $K_{1,n} \odot P_m$, $(P_n \odot K_1) \odot P_m$ and duplication of each edge by a vertex in $(P_n \odot K_1)$.

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